Phase noise and its time domain counterpart time–jitter are major concerns in oscillators. In biological and neural systems, phase noise plays a fundamental role explaining synchronization/desynchronization processes which, in turn, influence neural information processing. In electronic systems, phase noise is responsible for errors in bit transmission rate, modulation and demodulation of data. Therefore characterizing phase noise in oscillators is of paramount importance.

The physics of oscillators subject to white Gaussian noise is rather well understood. However, real world random fluctuations are characterized by finite, non null time-correlation, where the power spectral density is concentrated at low frequency. A more realistic description of such a perturbation is represented by an exponentially correlated process, known as colored noise.

In this contribution we present a novel approach for phase noise analysis in nonlinear oscillators subject to colored noise. The model we study can be decomposed into two parts: the oscillator dynamics and the stochastic process modeling noise. We consider oscillators of generic order \( N \), subject to colored noise that can be either modulated (multiplicative) or unmodulated (additive). We do not impose any restrictions on the noise intensity. The colored noise is modeled as an Ornstein-Uhlenbeck process (OUP). OUP is the solution of a linear stochastic differential equation (SDE) with an un-modulated (additive) white Gaussian noise. OUP has exponentially decaying expectation value and correlation, and it is characterized by a Lorentz (Cauchy) distribution. The only assumption we use is that noise correlation time is small, although not vanishing small (in the vanishing small limit, OUP reduces to white noise).

Using a method proposed in [1], [2] the system with colored noise is first transformed into an equivalent system subject to white Gaussian noise. That is, the \( N + 1 \) dimensional SDEs describing oscillator dynamics and OUP are reduced to an \( N \) dimensional system of SDEs describing the evolution of an equivalent nonlinear oscillator subject to white Gaussian noise. The solution of the reduced oscillator model converges weakly to the solution of full system, in the sense that all of the statistical properties of the exact (strong) solution are retained by the reduced system. In most practical applications this information is quite adequate. The advantage in considering the reduced system is twofold. First, not only numerical simulations are greatly simplified, because it is not necessary to solve the SDE for the OUP, but also expected quantities and other useful information can be found by direct computation on the reduced system applying stochastic calculus. Second, the transformation resolves the Itô-Stratonovich dilemma [3]: even if the original system (oscillator + noise modelling) has un-modulated (additive) noise, any order reduction and/or nonlinear coordinate transformation introduces noise modulation, making the noise source multiplicative [4], [5]. As a consequence, the resulting SDE yields different results depending on whether it is interpreted as a Stratonovich or an Itô SDE.

Using Floquet theory, the reduced system with white noise is then transformed into a phase-amplitude model [6]. The phase variable describes a random walk process along a direction tangent to the limit cycle of the unperturbed oscillator, while the amplitude describes motion transversal to the cycle. Using projection operator method the phase-amplitude model is then reduced to phase oscillator model, analogous to the famous Kuramoto model [7].

The main findings that we provide are the following: 1) the transformation into an equivalent system with white Gaussian noise highlights the effect of finite noise correlation time. 2) We derive an accurate phase reduced model. We show that other model previously proposed in literature can be obtained from the new one introducing increasing degrees of approximation.

REFERENCES