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### SCHEDULE AT A GLANCE

**TUESDAY, JUNE 18th**
- 08:00-08:30: Registration (Front Desk)
- 08:30-09:00: Open Ceremony (Auditorium)
- 09:00-09:45: Plenary 1 (Auditorium)
- 09:45-10:45: Session 7A: Devices 3 (Auditorium)
- 09:45-10:45: Session 7B: Fundamental Theory 1 (Room 1)
- 10:15-11:25: Session 3A: Devices and Materials 1 (Auditorium)
- 11:15-12:15: Session 3B: Fundamental Theory 2 (Room 1)
- 12:30-14:00: Launch Break
- 14:00-15:30: Session 5A: Devices 2 (Auditorium)
- 15:00-17:30: Session 5B: Biological Systems 1 (Room 1)
- 18:30-20:30: Welcome Reception: Salle des Chevaliers, Chateau de Neuchatel

**WEDNESDAY, JUNE 19th**
- 08:00-09:00: Plenary 2 (Auditorium)
- 09:00-09:45: Session 6A: Devices 1 (Auditorium)
- 09:00-09:45: Session 6B: Fundamental Theory 3 (Auditorium)
- 09:00-09:45: Session 7A: Devices 3 (Auditorium)
- 09:00-09:45: Session 7B: Fundamental Theory 1 (Room 1)
- 10:15-11:25: Session 18A: Devices and Circuits 1 (Auditorium)
- 11:15-12:15: Session 9A: Materials 2 (Auditorium)
- 11:15-12:15: Session 9B: Circuits 1 (Room 1)
- 14:00-15:30: Session 12A: Circuits and DSP 1 (Auditorium)
- 16:00-17:30: Session 12B: Biological Systems 2 (Room 1)
- 18:30-19:15: Cultural Visit: Musée d’art et d’histoire

**THURSDAY, JUNE 20th**
- 08:00-09:00: Plenary 3 (Auditorium)
- 09:00-09:45: Session 14A: Devices and Materials 3 (Auditorium)
- 09:00-09:45: Session 14B: Fundamental Theory 5 (Room 1)
- 09:45-10:45: Session 16A: Devices and Noise Measurements 1 (Auditorium)
- 09:45-10:45: Session 16B: Devices and Materials 2 (Room 1)
- 10:45-11:15: Coffee break/Poster Session A
- 10:45-11:15: Coffee break/Poster Session C
- 11:05-12:35: Session 3A: Devices and Materials 1 (Auditorium)
- 11:15-12:45: Session 9A: Materials 2 (Auditorium)
- 11:15-12:45: Session 9B: Circuits 1 (Room 1)
- 14:00-15:30: Session 4A: Fundamental Theory 3 (Auditorium)
- 14:00-15:30: Session 4B: Fundamental Theory and Materials 1 (Room 1)
- 16:00-17:40: Session 12A: Circuits and DSP 1 (Auditorium)
- 16:00-17:40: Session 12B: Biological Systems 2 (Room 1)
- 18:30-19:00: Excursion: Visit to the International Watch Museum in La Chaux-de-Fonds

**FRIDAY, JUNE 21st**
- 08:00-09:00: Plenary 4 (Auditorium)
- 09:00-09:45: Session 18A: Devices and Circuits 1 (Auditorium)
- 09:45-10:45: Session 18B: Devices and Optoelectronics 1 (Room 1)
- 10:45-11:15: Coffee break/Registration
- 10:45-11:15: Coffee break/Poster Session C
- 11:05-12:35: Session 3A: Devices and Materials 1 (Auditorium)
- 11:15-12:45: Session 9A: Materials 2 (Auditorium)
- 11:15-12:45: Session 9B: Circuits 1 (Room 1)
- 12:05-12:30: Closing Ceremony
- 12:30-14:00: Lunch Break
Introduction

The International Conference on Noise and Fluctuations (ICNF) is a biennial event that brings together researchers interested in theoretical and experimental aspects of fluctuations across a wide spectrum of scientific and technological fields. Since the development of the theory of Brownian motion, the science of fluctuation has been one of the most important parts of physics. The investigation of noise and fluctuation is indispensable for the understanding of the physical processes in various microscopic and macroscopic systems. This conference will be a great opportunity for noise researchers operating in very different areas of scientific and technological endeavor to come together and create the basis for continued and renewed cross-fertilization and collaboration. The International Conference on Fluctuation Phenomena started in 1968 and moved all over the world. For the first time the conference is taking place in Switzerland: the 25th edition (ICNF 2019) will be held in Neuchâtel (Switzerland), from June 18 to June 21, 2019.

About Neuchâtel

Neuchâtel is located on the northwestern shore of the Lake of Neuchâtel, in West Switzerland. On the side of the Jura Mountains, the environment is characterized by remote, windswept settlements and deep, rugged valleys. Neuchâtel is also in the heartland of the celebrated Swiss watchmaking industry.

EPFL

The Ecole Polytechnique Fédérale de Lausanne (EPFL) was founded in 1853 and became a national school in 1969. It is considered to be among the world’s most prestigious universities of technology.

EPFL Neuchâtel campus

At the heart of the Microcity pole of innovation, the Canton of Neuchâtel is hosting an important part of EPFL’s Microengineering Institute (IMT). This institute’s research activities cover topics such as health, microsystems, photovoltaic and watchmaking. The installation of EPFL Neuchâtel campus, in 2013, is part of the EPFL’s strategy to bring research activities closer to industrial interests at all stage of the innovation process. They benefit from the proximity of other research institutions and high-tech manufacturing societies active in micro- and nano-technologies and advanced manufacturing.
Chairs & Committees

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Biological Systems: Sandro Carrara
Circuits and Systems: Alessandro Pezzotta
Electron Devices: Gjøs Bosman
Finance: Michael Small
Instrumentation: Massimo Macucci

Imagers: Assim Boukhayma
Materials: Arindam Ghosh
Photonic Devices: Shiva Kumar
Reliability: Gilson Wirth
Stochastic Processes: Ping Ao
Theory: Luca Varani

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Prof. SHOJI KAWAHITO
Imaging Devices Laboratory, Research Institute of Electronics, Shizuoka University.

Ultra-Low-Noise Design of CMOS Image Sensors toward Photoelectron-Counting-Based Wide Dynamic Range Imaging

Tuesday, June 18th
09:00-09:45
Auditorium

Shoji Kawahito (F’09) received the Ph.D. degree from Tohoku University, Sendai, Japan, in 1988. He joined Tohoku University as a Research Associate in 1988. From 1989 to 1999, he was with the Toyohashi University of Technology, Toyohashi, Japan. From 1996 to 1997, he was a Visiting Professor with ETH Zurich, Zürich, Switzerland. Since 1999, he has been a Professor with the Research Institute of Electronics, Shizuoka University, Shizuoka, Japan. Since 2006, he has been the Chief Technology Officer of Brookman Technology Inc., Shizuoka, a university spin-off company for CMOS imager developments. He has authored over 300 papers in peer-reviewed journals and international conference proceedings. His current research interests include CMOS imaging devices, sensor interface circuits, and mixed analog/digital circuits designs.

Dr. Kawahito is a fellow of the Institute of Image Information and Television Engineers and a member of the Institute of Electronics, Information and Communication Engineers (IEICE) and the International Society for Optics and Photonics. He has received plenty of awards, including the Outstanding Paper Award at the IEEE International Symposium on Multiple-Valued Logic in 1987, the Special Feature Award in LSI Design Contest at the Asia and South Pacific Design Automation Conference in 1998, the Beatrice Winner Award for Editorial Excellence at the IEEE International Solid-State Circuits Conference (ISSCC) in 2005, the IEICE Electronics Society Award in 2010, the Takayanagi Memorial Award in 2010, and the Walter Kosonocky Award in 2013. He served as a Technical Program Committee Member of ISSCC from 2009 to 2012, and the Program Committee Chair of the 2011 International Image Sensor Workshop. He was the Chair of the SSCS Japan Chapter from 2013 to 2014.
Alper Demir received the BS degree from Bilkent University in Turkey and the MS and PhD degrees from the University of California at Berkeley in the USA. Dr. Demir previously spent time at Motorola, Cadence Design Systems, Bell Laboratories Research, MIT, and UC Berkeley. He has been with Koç University in Istanbul as a faculty member since 2002. His work on noise won several best paper awards: 2002 Best of ICCAD Award, 2003 and 2014 IEEE/ACM William J. McCalla ICCAD Best Paper Awards, and the 2004 IEEE Circuits & Systems Society Guillemin-Cauer Award. He was named an IEEE Fellow in 2012 for his contributions to stochastic modelling and analysis of phase noise.
Prof. ENRICO RUBIOLA  
*Université de Franche Comté & Department of Time and Frequency of the CNRS FEMTO-ST Institute, Besançon, France*

**The Origin and the Measurement of Phase Noise in Oscillators**

*Thursday, June 20th  
09:00-09:45  
Auditorium*

Enrico Rubiola is full professor at the Université de Franche Comté and deputy director of the Department of Time and Frequency of the CNRS FEMTO-ST Institute, Besançon, France. Since 2012, he is the PI of the Oscillator IMP project, a platform for the measurement of short-term frequency stability and spectral purity. Formerly, he was a full professor at the Université Henri Poincaré, Nancy, France, a guest scientist at the NASA JPL, a guest professor at the Università di Parma, Italy, and an assistant professor at the Politecnico di Torino, Italy. 

He graduated in electronic engineering at the Politecnico di Torino in 1983, received a Ph.D. in Metrology from the Italian Minister of University and Research, Roma (1989), and a Sc.D. degree from the Université de Franche Comté in 1999.

Enrico’s interests are high-spectral purity oscillators from low RF to optics, general time and frequency metrology, phase noise, amplitude noise, noise in digital systems, frequency synthesis, spectral analysis, wavelet (Allan-like) variances, microwave photonics, precision electronics form dc to microwaves, and precision instrumentation. He has developed innovative instruments for AM/PM noise measurement with ultimate sensitivity, and a variety of dedicated signal-processing methods. He introduced the "Leeson effect" and "I\(I\) and \(\Lambda\) frequency counters," and the full theory underneath, and he received the IEEE W. G. Cady Award (2018) for his contributions. Enrico founded the European Frequency and Time Seminar (http://efts.eu) in 2013, and he has been chairing it since. A wealth of articles, reports, conference presentations, and lectures for PhD students and young scientists are available on the Enrico’s home page http://rubiola.org.
Prof. ALEXANDER BALANDIN
University of California, Riverside

Low-Frequency Noise in Low-Dimensional van der Waals Materials

Friday, June 21st
09:00-09:45

Auditorium

Alexander A. Balandin received his BS and MS degrees Summa Cum Laude in Applied Mathematics and Applied Physics from the Moscow Institute of Physics and Technology (MIPT), Russia. He received his second MS degree and PhD degree in Electrical Engineering from the University of Notre Dame, USA. From 1997 till 1999, he worked as a Research Engineer at the Department of Electrical Engineering, UCLA. In 1999 he joined UCR, where he is presently Distinguished Professor of Electrical and Computer Engineering, University of California Presidential Chair Professor of Materials Science, Director of the Phonon Optimized Engineered Materials (POEM) Center, Director of UCR Nanofabrication Facility, and Associate Director of DOE EFRC Spins and Heat in Nanoscale Electronic Systems (SHINES) Center. Professor Balandin is a Founding Chair of UCR campus-wide Materials Science and Engineering (MS&E) Program. His research expertise covers a wide range of nanotechnology, materials science, electronics, low-frequency noise, phononics and spintronics fields. He is recognized as a pioneer of the graphene thermal research field. His current research interests include charge-density-wave effects in two-dimensional materials and their device applications, low-frequency current fluctuations and electronic noise in materials and devices, Brillouin – Mandelstam and Raman spectroscopy of advanced materials, practical applications of graphene in thermal management, electronics and energy conversion. Professor Balandin is a recipient of The MRS Medal from the Materials Research Society, and Pioneer of Nanotechnology Award from IEEE Society for his graphene and nanotechnology research. He is an elected Fellow of eight professional societies: MRS, APS, IEEE, OSA, SPIE, IOP, IOM3 and AAAS. He is among the Clarivate Analytics and Thomson Reuters Highly Cited Researchers (Physics). He serves as Deputy Editor-in-Chief of the Applied Physics Letters. For more information, visit his group web-site: http://balandingroup.ucr.edu/
Conference Technical Program

Day 1

Session 1: Plenary 1

Ultra-Low-Noise Design of CMOS Image Sensors toward Photoelectron-Counting-Based Wide Dynamic Range Imaging
Shoji Kawahito

Session 2A: Materials 1

Detection of Shot Noise in Solar Cells and LEDs Using Cross-Correlation Current Noise Spectroscopy
Kevin Davenport, Andrey Rogachev and Mark Hayward

Low frequency noise deviation from Schottky theory in p-n junctions
Jean-Guy Tartarin, Jacques Graffeuil and Laurent Escotte

Session 2B: Fundamental Theory 1

Low-Frequency Noise of Magnons
Sergey Rumyantsev, Michael Balinskiy, Fariborz Kargar, Alexander Khitun and Alexander Balandin

Fluctuation-Dissipation-Dispersion Relation for Slow Processes and Quality Factor for Oscillation Systems
Viacheslav Belyi

Session 3A: Devices and Materials 1

Charge carrier dynamics of strongly-correlated electrons in low-dimensional molecular metals studied by fluctuation spectroscopy
Jens Müller

Lorentzian noise approach for 1D transport studies
M. Petrychuk, I Zadorozhnyi, Yu Kutovyi, S Karg, H Riel and Svetlana Vitusevich

Prehistory probability distribution of ion transition through graphene nanopore
Carlo Guardiani, William Gibby, Miroslav Barabash, Dmitry Luchinsky, Igor Khovanov and Peter McClintock

Session 3B: Fundamental Theory 2

Resistivity Characteristics and Noise Spectroscopy of Composites with Carbon Fiber Felts
Marina Tretjak, Sandra Pralgauškaite, Jonas Matukas, Ieva Kranauskaitė, Jan Macutkevič, Jūras Banys, Vanessa Fierro, Alain Celzard and Blagoy Karakashov

Session 4A: Fundamental Theory 3

Design and realization of ultra-low noise cryoHEMTs for cryogenic readout electronics
Yong Jin, Quan Dong, Laurent Couraud, Antonella Cavanna, Ulf Gennser, Christian Ulysse and Edmond Cambril

Shot noise and squeezing in the conduction channel of a Field Effect Transistor at ultra-low temperature
Anthoni Manseau, Édouard Pinsolle, Christian Lupien and Bertrand Reulet

Discussion of the flicker noise origin at very low temperature and polarisation operation
Dimitri Boudier, Bogdan Cretu, Eddy Simoen, Anabela Veloso and Cor Claeyss

Generation-recombination noise of magnetic monopoles in spin ice
Alexey Klyuev, Mikhail Ryzhkin and Arkady Yakimov
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Sumit Kumar, Sudha Chauhan and Rajib Jha

Cosmic Nonstationarity of the Coherent Gravodynamic Quantum 1/f Effect
Peter H Handel and Erika Splett

Temporal stability measurements of a cooled infrared type II superlattice (T2SL) focal plane array detector
Vignesh Arounassalame, Jean Ngheim, Maxence Guenin, Eric Costard, Philippe Christol and Isabelle Ribet-Mohamed

Characterization methods of low frequency RTS noise in HgCdTe infrared detectors
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Session 12A: Circuits and DSP 1

Pulse Detection with a Multi-state System
Roy Howard

Noise and Linearity of High-Speed SiGe HBT Cells in CE and CB Configuration
Paulius Sakalas, Anindya Mukherjee and Michael Schroter

Implementing software defined noise generators
Robert Mingesz and Dénès Faragó

Radon Transform and Dynamic Stochastic Resonance based Technique for Line Detection from Noisy Images
Rajib Kumar Jha and Badal Soni

Suprathreshold Stochastic Resonance for Gamma Noise with Watermarking Application
Sumit Kumar, Nancy Chauhan and Rajib Jha

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Effect of Clustering on the Fluctuation in Binding Activity of Sugar Chains to Influenza Viruses
Toshio Kawahara

Statistical theory of mixed-valence selectivity in biological ion channels
William Gibby, Miraslau Barabash, Carlo Guardiani, Dmitry Luchinsky and Peter McClintock

Interplay between channel and shot noise in subthreshold voltage fluctuations of neural membranes
Beatriz G. Vasallo, Javier Mateos and Tomás González

1/f DNA Hydrogen-Bond Energy Noise
Nazarii Boichuk, Yuriii Kutovyi, Nicolas Lobato-Daizier, Anthony Genot, Teruo Fujii, Andreas Offenhäusser, Svetlana Vitusevich and Nicolas Clément

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Day 3

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The Origin and the Measurement of Phase Noise in Oscillators
Enrico Rubiola

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Bulk induced 1/f noise in topological insulators
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Trapping Investigation of the GaN HEMT Devices Using the Low Frequency Noise Characterization
Mohamed Bouslama, Jean-Christophe Nallatamby and Michel Prigent

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Takayuki Kobayashi

Thomson Scattering in Inhomogeneous Plasmas: The Role of the Fluctuation-Dissipation Theorem
Viacheslav Belyi

Quantum 1/f Noise —a Decoherence Phenomenon
Peter H Handel
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Low frequency noise of GaSb layers on GaAs substrate
Lukasz Ciura, Andrzej Kolek, Krzysztof Czuba, Iwona Sankowska and Agata Jasik

Using the velocity auto-correlation function to characterize functional “noise” in bio-molecules
Marlon Sidore, Yoann Meriguet, Anastasiiia Kudashova, Mathias Lechelon, Matteo Gori, Marco Pettini, Jeremie Torres and Luca Varani

Energy and Area Aware Digital Fingerprint Generator Using Intrinsic Randomness
Jawar Singh, Sandeep Kumar Pandey, Rajib Kumar Jha and Pramod K Tiwari

Effects of mechanical stress on electrical parameters and noise of supercapacitors
Arkadiusz Szewczyk, Łukasz Lentka and Janusz Smulko

Session 16A: Devices and Noise Measurements 1

Noise in electrical double-layer capacitors (EDLCs)
Janusz Smulko, Arkadiusz Szewczyk and Łukasz Lentka

Ultra-low noise, single JFET voltage pre-amplifier for low frequency noise measurements
Graziella Scandurra, Carmine Ciofi and Gino Giusi

Low frequency noise characterization and modeling of SiGe HBT featuring LASER annealing in a 55-nm CMOS node
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Shoji Kawahito

Research Institute of Electronics, Shizuoka University, Hamamatsu, 432-8011, Japan

E-mail: kawahito@idl.rie.shizuoka.ac.jp

Abstract

An ultimate goal of imaging devices is to realize wide dynamic range imaging without the influence of noise other than photon shot noise and other problems like motion artifact. The use of very high photoelectron conversion gain is a typical technique for reducing the influence of readout noise. However, the high conversion gain limits the full-well capacity of the pixel, leading to the reduction of dynamic range. This contradiction can be solved by reducing the readout noise to be 0.1e-, using photoelectron-counting-type analog-to-digital converters and integrating the digitized signal in a digital photoelectron counter per pixel. This talk presents the recent progress of ultra-low-noise CMOS image sensors using high conversion gain pixel and multiple-sampling-based low-noise ADCs and future prospects toward photoelectron-counting-based wide dynamic range CMOS image sensors by attaining the readout noise of 0.1e-.
Detection of Shot Noise in Solar Cells and LEDs Using Cross-Correlation Current Noise Spectroscopy

Kevin Davenport  
Department of Physics and Astronomy  
University of Utah Salt Lake City, UT, USA  
kevin.davenport@utah.edu

Mark Hayward  
Department of Physics and Astronomy  
University of Utah Salt Lake City, UT, USA

Andrey Rogachev  
Department of Physics and Astronomy  
University of Utah Salt Lake City, UT, USA

Abstract—Using cross-correlation techniques, we are able to extend the bandwidth of typical noise measurements and go beyond the standard 1/f characterization of heterostructured silicon and perovskite solar cells, as well as organic light-emitting diodes. With this method, we are able to resolve the frequency-independent contribution to the noise spectra. In all studied systems, we find this term to be roughly proportional to current and, therefore, attributed to shot noise. The extracted Fano factor, \( F \), from the equation \( S = 2eIF \), was found to be in the range 0.2-1.

We argue that in the solar cells, this shot noise originates from discreteness of electron-hole pair generation by photons and from discreteness of hopping sites along a percolative transport path. In organic LEDs, the shot noise is likely due to bottleneck effects related to bulk percolative transport.

Keywords—noise, cross-correlation, solar cell, organic LED, shot noise, Fano factor

I. INTRODUCTION

Noise spectroscopy has proven an effective tool over the last several decades for studying transport dynamics and defect states [1,2] in both organic and inorganic materials and devices [3,4]. However, much of this literature focuses on low-frequency flicker noise below 10kHz [5]. The primary limitation is twofold: (i) the background noise of the front amplifiers in single-channel spectrum analyzers set the noise floor, limiting viewable signal, and (ii) in devices with planar geometry, their capacitance leads to a high-frequency roll-off in voltage noise measurements, which is often indistinguishable from 1/f noise.

We have designed and constructed a two-channel spectrum analyzer based on the current cross-correlation technique [6,7]. This allows us to resolve details in noise spectra well below the noise floor. The transimpedance stage has been built in close proximity (>1cm) to the device under test, greatly reducing parasitic input capacitance, thus increasing bandwidth. In an improvement over our apparatus used earlier in noise [8,9] and impedance [10,11] studies of organic LEDs, the first amplification stage has been built inside a liquid nitrogen flow cryostat with optical access. This allows noise spectra to be collected under different light intensities, device currents, and temperatures.

II. EXPERIMENTAL

Here we report the study of three different types of devices: a series of high-efficiency silicon solar cells, the structure of which is shown in Fig. 3; methylammonium lead triiodide perovskite solar cells, shown in Fig. 5; and a series of super yellow PPV co-polymer organic light emitting diodes, shown in Fig. 7. All devices were studied under vacuum, using current noise measurements as detailed in [7] under zero bias conditions. Illumination was provided by a high-power, broad spectrum light-emitting diode powered using batteries. For the solar cells, a yellow LED (\( \lambda = 585-595\text{nm} \)) was used. For the super yellow samples, a blue LED (\( \lambda = 450-465\text{nm} \)) was used to excite above the bandgap. The output of the transimpedance stage was first sent through a custom-built two-stage, low-noise amplifier and then into a National Instruments 6366 USB DAQ card. The cross-correlation was performed in a computer with a purpose-built LabVIEW procedure.

III. RESULTS AND DISCUSSION

A. Detection of Shot Noise

We found that in the three studied series of devices, all contributions to noise spectra can be accounted for by some (or all) of the terms in the following equation:

\[
S = S_1 + \frac{S_2}{f^n} + S_3 \times \text{Re} \left[ \frac{1}{1 + (i\omega \tau)^c} \right] + S_4 f^c. \tag{1}
\]

Here, the first term represents a frequency-independent noise contribution made up of shot noise, thermal noise, and residual background noise. The second term represents the contribution of 1/f-like flicker noise. The third term is a generic generation-recombination noise which allows for a distribution of recombination times, \( \tau \); note that if \( b = 0 \), this term takes on the familiar Lorentzian form for a single recombination time. Finally, the fourth term represents an upturn due to sample capacitance which cannot be removed by the cross-correlation method. Ideally, \( c = 2 \) [7], though we often find a value that is between 1.5 and 2 due to noise gain peaking [14].

An important technical achievement of our work is the ability to resolve shot noise (\( S_1 \) term) in planar devices with large capacitances. The total shot noise in these devices can be thought of as generated by a series of noise sources, each self-shorted by its resistance, \( R_s \) (Fig. 1) [12]. These resistances model the energy barriers encountered by charge carriers at the interfaces between materials, or between hopping sites along a percolative transport path.
Fig. 1. Shot noise sources envisioned as a collection resistive Kirchhoff loops, each corresponding to an internal energy barrier. These could be seen as tunneling barriers, such as at the interfaces between materials, or as percolation hopping sites.

From Kirchhoff’s law, the total noise current is given as

$$I_T = \sum_{n} i_n R_n,$$  

where $R_n$ is the total resistance of a given device. As a first approximation, the current noise sources are uncorrelated and each of them generates full scale shot noise power with the Schottky value $S_n = 2eI$. The total shot noise seen by the electrodes then is

$$S_T = \left( I_T^2 \right) = \sum_{n} \left( i_n^2 \right) \left( R_n R_T \right)^2 = 2eI \sum_{n} \left( R_n R_T \right)^2 = 2eIF,$$  

where the Fano factor, $F$, characterizes the reduction of the noise. Thus, the addition of sources to the network actually suppresses the overall shot noise signal. For example, for $N$ identical noise sources in series, $F = 1/N$.

B. Shot noise in silicon solar cells

Fig. 2a below shows the current noise spectra of a high-efficiency silicon solar cell. At the bottom of the Fig. 3 the structure of the device is shown. The bulk of this device is a 145 µm-thick layer of n-type crystalline silicon (Cz-Si) grown via the Czochralski process. Very narrow layers of p-doped amorphous Si and n-doped nanocrystalline Si (energy gap 1.6 eV) allow efficient light propagation into the Cz-Si where the generation of electron-hole pairs takes place. These layers have a high concentration of defects; to separate them from the Cz-Si, two passivating layers of hydrogenated intrinsic amorphous Si (i-α-Si:H) have been deposited on both sides. The p-n junction of the device extends over three layers: the n-Cz-Si, i-α-Si:H, and p-α-Si:H. Finally, a thin layer of ITO and gold contacts have been deposited on either side. The total area of the device is 1 cm². In our experiment, the sample was illuminated from the side of the p-type material, however illumination from the either side yields similar efficiency.

The noise spectra shown in Fig. 1 were taken in the short circuit configuration; the legend indicates light-induced DC current in the device. The solid lines show Eq. 1 fit to the data.

The mid-frequency feature, seen from approximately 1 kHz to 100 kHz, is the result of the combination of the frequency-independent and generation-recombination noise contributions; this feature is completely hidden in the single channel data. Similar to [8], all of the terms in Eq. 1 are necessary to acquire a good fit to the data.

Fig. 2. A.) Cross-correlated current noise spectral density curves for an amorphous silicon photovoltaic cell at 0V bias under different illuminations at 300K. Black curves represent a fit to Eq. 1. The red dashed line represents the residual capacitive upturn. B.) The frequency-independent term extracted from the fit, showing a Fano Factor of $F = 0.44$.

Fig. 2b shows the magnitude of the frequency-independent term plotted as a function of photocurrent, the linear dependence of which indicates that this term is due to shot noise. From the fit, shown in black, we find a Fano factor of $F = 0.44$.

Fig. 3. Equivalent circuit and device structure (not to scale) of the silicon solar cells studied. The main generators of shot noise are likely the highly-resistive intrinsic amorphous silicon layers surrounding the bulk, one of which is additionally modified by the presence of a p-n junction.
Based on the model presented in section A, we can identify three main sources of shot noise (Fig. 3). Out of these, the likely dominant sources are the thin, intrinsic silicon layers which serve to passivate surface defect states on the bulk Cz-Si layer; these intrinsic layers are much more resistive than the doped layers, 10^4 S/cm compared to 10^1 S/cm. If the two layers were identical, a Fano factor of $F = 0.5$ would be expected. Deviation from this value can occur for two reasons. First, one of the intrinsic layers lies inside the device p-n junction, modifying the layer’s net resistance and thus contributing asymmetrically to the Fano factor. Second, the internal resistance of the bulk layer increases the value of the total device resistance, $R_v$, which pushes the Fano factor below 0.5.

C. Shot noise in perovskite solar cells

Fig. 4a shows current noise spectra, along with their fits to Eq. 1, for a perovskite solar cell. The structure of the device is shown in Fig. 5. A TiO$_2$ matrix was deposited on top of an FTO-templated glass substrate, upon which approximately 100 μm of the organic-inorganic hybrid perovskite methylammonium lead triiodide was added via spin coating. The hole transport material spiro-OMeTAD was then deposited, also using spin coating, followed by the thermal evaporation deposition of the gold contact. Each glass substrate contained four separate devices, each ~25mm$^2$.

Comparing these data to those in Fig. 2a, we see the absence of the generation-recombination term seen in the silicon samples.

The magnitude of the $S_1$ term extracted from the fits here is roughly the same order of magnitude ($\sim 10^{-22}$) as that seen in the silicon samples, however the extracted Fano Factor is $F = 0.67$. The equivalent circuit for the perovskites (Fig. 5) is simpler than that of silicon due to a less complex internal structure.

It has been observed that performance degradation can occur in these devices due to the modification of the spiro-OMeTAD transport layer [13], either through direct modification such as photo-oxidization, or by diffusion of Au from the anode contact. Therefore, it seems likely that this layer is responsible for the generation of shot noise in these devices. The fact that $F > 0.5$ supports this claim as a greater number of barriers would drop this below 0.5, as seen in the silicon devices.

D. Shot noise in super yellow PPV co-polymer LEDs

Fig. 6a shows current noise spectra collected from a super yellow LED operated as a photovoltaic cell. The structure of the device is indicated in Fig. 7. Approximately 100 nm of PEDOT:PSS was spin coated onto an ITO-patterned glass substrate, followed by 50 nm of super yellow PPV co-polymer. An anode of calcium and aluminum was then deposited via thermal evaporation. The device active area is ~1mm$^2$.

The properties which make for a good LED tend to be at odds with the desired properties of a good photovoltaic. LEDs are designed to promote exciton formation and recombination, whereas the desirable trait in a photovoltaic is rapid and efficient exciton dissociation. This is reflected in the data by the fact that the observed short circuit current seen in the super yellow devices was many orders of magnitude lower than either of the solar cell samples, even at much higher illumination intensities.

The small currents led to a lower signal-to-noise ratio, making curve fitting more difficult. The dashed lines in Fig. 6a represent a fit to the mid-frequency plateau region which corresponds to the upper bound of the frequency-independent term. The qualitative shape of the curves suggests that the generation-recombination term is not present, similar to the perovskites.
Fig. 6. Cross-correlated current noise spectral density curves for a super yellow PPV co-polymer LED, operated as a photovoltaic, at 0V bias under different illuminations at 300K. The dashed lines represent a fit to the noise plateau corresponding to the frequency-independent noise term, $S_i$, plotted as a function of photocurrent, showing a Fano Factor of $F = 1$.

From these data, the Fano factor is found to be close to unity, as shown in Fig. 6b. We note that the magnitude of $S_i$ in these devices is three orders of magnitude smaller ($\sim 10^{-25}$) than in the previous samples.

Fig. 7. Device structure and equivalent percolation network of the super yellow organic light emitting diodes. Shot noise generation is likely dominated by cross-correlating technique on two channels. The transport in polymeric organic devices occur via both intra-chain and inter-chain electron hopping along least resistive paths in a percolative network. If the barriers for all jumps are the same, the expected Fano factor is $F = 1/N$, where $N$ is the number of hops the carriers take between electrodes [12].

Estimating the distance of a single hop to be $\sim 1$nm, and using a thickness of $\sim 50$nm, we would then expect $F \approx 0.02$, much lower than the observed value near unity. We can explain this by assuming that the percolation path is dominated by a single hop, shown in Fig. 7 as the resistor in bold, whose “bottle-neck” resistance dominates the entire percolative chain.

IV. CONCLUSION

In summary, the current cross-correlating technique allowed us to resolve mid-frequency noise contributions, namely shot noise and generation-recombination noise. Importantly, we see that the analysis of shot noise and the corresponding Fano factor can yield information about interfacial dynamics and charge transport. In particular, the shot noise associated with percolation in organic materials, such as the super yellow PPV co-polymer, hint at using this method to directly probe mesoscopic transport.

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VI. REFERENCES

Low Frequency Noise Deviation from Schottky theory in p-n junctions

J. Graffeuil
LAAS-CNRS and Universite de Toulouse, Universite Paul Sabatier
Toulouse, France
graffeui@laas.fr

L. Escotte
LAAS-CNRS and Universite de Toulouse, Universite Paul Sabatier
Toulouse, France
escotte@laas.fr

J.G. Tartarin
LAAS-CNRS and Universite de Toulouse, Universite Paul Sabatier
Toulouse, France
tartarin@laas.fr

Abstract—Theories on linear white noise sources such as thermal noise or shot noise are well established and massively used for low noise device modeling and circuit design. However, it has been experienced that diffusion noise in a large variety of pn diodes (transistors) can deviate from the expected value given by the Schottky theorem or by the Van der Ziel representation commonly used. In this work, more than ten types of pn junctions have been investigated, all featuring an increase of the diffusion noise floor in the low frequency band when operated under low d.c current conditions. These specific conditions certainly explain why such phenomenon has not been reported earlier; however, this noise degradation becomes a problem as many systems make use of pn diodes for low signal photodetection (PPD or CCD), operating at very low (dark) current. For the first time, we report current spectral densities deviations from the Schottky theorem at low frequency; a focus on the experimental workbench is given to remove any doubt regarding the opportunity to analyze data under concern. Then, low frequency noise spectra are presented for various diodes and pn junctions, and a model is proposed. Impedance spectroscopy is also used to support this study.

Keywords—Low frequency noise, diffusion noise, noise frequency dispersion.

I. INTRODUCTION

Low frequency noise of junctions is still of great interest as it is massively used for the noise modelling of solid state devices such as diodes and transistors. The well-established theory formulated by W. Schottky one hundred years ago (and first demonstrated in ideal vacuum tubes [1]) still applies and lays down the basic principles of electronic sensitivity for receivers, detectors or any electronic system facing the case of minimum detectable signal. From this theory, a d.c current \(I\) flowing through a barrier of potential will generate a current noise spectral density as

\[
\gamma = \frac{S_n}{2q(I+2I_s)}
\]

where \(S_n\) is the noise spectral density, \(I_s\) the saturation current of the DUT, \(q\) the electron charge and \(\gamma\) the Fano factor. According to the Fano factor, the noise spectral density can vary by up to 2.5 orders of magnitude, depending on the overall current, on the frequency and on the temperature. In this paper, a new low frequency spectral signature is presented, never previously reported to the authors’ knowledge. Still using the normalized \(\gamma\) expression of the noise floor (but not referred to as Fano factor), this factor deviates from unity beyond a given frequency, and stabilizes at a value ranging from one to more than six as depicted in figure 1 (and more than ten in specific conditions). This noise degradation phenomenon has only been observed on silicon p-n diodes at low frequency and for very low biasing conditions. Various emitter-base or collector-base junctions have been investigated in silicon transistors such 2N2222, 2N2905, BC557 for the most commonly used devices. In section 2, the Low Frequency Noise (LFN) experimental workbench is presented, and measurements are performed with various pre-amplification schemes in order to validate reported data. Given the very low noise spectral densities involved, particular attention is paid to the instrumentation used for the characterization of the LFN, and to the deembedding of the noise from various contributions (amplifiers termination versus frequency, noise contribution and correlation of amplifier noise sources) to the total noise. This warrants a rigorous characterization of the Devices Under Test (DUT) noise. Calibration steps and measurements are presented in this section. The third section is dedicated to the modeling and analysis of the observed phenomenon versus d.c current. Diode impedance spectroscopy is used in order to substantiate LFN data from a newly proposed model.

Fig. 1: Evidence of the diffusion noise floor increase at low frequencies (deviation of the normalized representation from unity), for three various silicon technologies. Nota: emitter-base diodes (collector open) are investigated

II. LOW FREQUENCY DEVIATION FROM THE EXPECTED DIFFUSION NOISE FLOOR.

A. Experimental setup

Various experimental workbenches for LFN measurements have been developed over the past five decades [3][4]; the experimental workbench allows LFN measurement of a 2-port DUT current (or voltage) noise sources from 1 Hz to 1 MHz. For the study case under concern, only diodes are measured; the current spectral density \(S\) of the DUT is obtained from a transimpedance low noise preamplifier (model 5182 from EG&G Instruments) or also a voltage low noise preamplifiers (model 5184 from Perkin Elmer Instruments or model SR560 from Stanford Research Systems). According to the selected preamplifier, different appropriate noise deembedding techniques are used to get the...
S, current spectral density of the DUT from the overall noise measure $S_{\text{meas}}$ as depicted in Figure 2. As long as the DUT is a diode operating at very low current, hence featuring a high dynamic resistance, using a current-voltage preamplifier is more convenient as it presents the lowest impedances to the DUT and therefore sinks most of the DUT noise current. The schematic representation is depicted in Figure 2. On the left part of the figure, d.c biasing battery powered circuit makes use of a by-pass capacitor featuring elevated $C_d$ to consider this circuit as a short circuit over the whole frequency bandwidth of measurement (from 10 Hz to 1 MHz). The DUT (diode) is represented by a parallel association of $R_d$ with $C_d$. The DUT’s package capacitive coupling and the coaxial cable capacitance are accounted for with $C_d$ connected at the output of the DUT. Then the measurement apparatus makes use of a current-voltage preamplifier connected to a Fast Fourier Transform signal analyzer. LFN sources or the preamplifier are represented at the input terminal of the preamplifier ($S_{\text{in}}$ and $S_{\text{in}}$ respectively for the voltage and current noise sources). The parallel association of $R_{\text{in}}$ and $C_{\text{in}}$ accounts for the frequency variation of the input impedance of the preamplifier ($S_{\text{in}}$ and $S_{\text{in}}$). It is easy to express $S_{\text{in}}$ as in equation 2.

$$S_{\text{in}}(f) = S_{\text{in}}(f) \left( \frac{S_{\text{in}}(f)}{S_{\text{in}}(f)} \right)^{-2} - S_{\text{in}}(f) - 2\text{Im}(S_{\text{in}}(f)) - S_{\text{in}}(f) \right)$$

Then it is easy to express $S_i$ as in equation 2.

$$S_i(f) = S_{\text{in}}(f) \left( \frac{S_{\text{in}}(f)}{S_{\text{in}}(f)} \right)^{-2} - S_{\text{in}}(f) - 2\text{Im}(S_{\text{in}}(f)) - S_{\text{in}}(f) \right)$$

2- The low d.c biasing conditions at which the phenomenon appears implies low $S_i$ noise current levels. Therefore high sensitivity and high gain are needed for the voltage-current preamplifier, at the cost of a reduced bandwidth. Low amplifier noise, that warrants accuracy, and wide frequency bandwidth cannot be achieved simultaneously (as a consequence of the well-known properties of any current amplifier), and tradeoff with various experimental conditions are developed and compared. Moreover, the selection of a high gain (and a high sensitivity that corresponds to a low noise current $S_{\text{in}}$) is associated with an increase in the input impedance of the preamplifier EG&G 5182. This means that a lower fraction of the diode noise current is collected by the amplifier, but fortunately $R_e$ also increases with $f$ decreasing that makes the measurement still possible. It must be noticed that, even if the amplifier gain selection for the better accuracy over a given bandwidth at a given bias is always a challenge, it can nevertheless be handled (see the recovery of spectra obtained for different frequencies and gains in Figure 1).

Figure 3 compares measurements performed with different preamplifier configurations (current amplifier, voltage amplifier and two current amplifiers for cross-correlation measurements) at a not too low current (in order to warrant voltage amplifier measurements) on the same device. Although the DUT noise deembedding from the measurements is very different for each technique, the final results are similar and demonstrate the same noise enhancement. Voltage amplifiers are not suitable for those measurements as the DUT impedance must remain much lower than that of the amplifier, and this is only achieved at elevated d.c current where the investigated phenomenon is no more present. Correlation technique has also been used with no noticeable improvement.

3- The impedances used in equation 1 and 2 must be accurately characterized versus frequency for each gain selection of the preamplifier ($R_{\text{in}}$ and $C_{\text{in}}$) or d.c biasing of the DUT ($R_{\text{in}}$ and $C_{\text{in}}$) obtained from impedance spectroscopy, see section III) in order to warrant a correct noise deembedding. Furthermore, noise of parallel R-C cells using discrete elements is measured as a test; resistance and capacitor values are chosen in the range of the measured diode $R_{\text{in}}-C_{\text{in}}$ values. As expected, measured $\gamma$ normalized values of the R-C cell are nearly equal to unity over all the measured bandwidth, as depicted in Figure 3 for a given R-C configuration. This last calibration test gives consistency to the measurements.
B. LFN deviation from diffusion noise floor

More than ten silicon devices have been investigated, with a special focus on many 2N2222, 2N2905 and BC557 devices. Figure 4 plots the evolution of the $\gamma$ values versus frequency, and for many d.c currents. Whatever the considered DUT, the same trend is observed: the corner frequency shifts towards higher frequencies when increasing d.c current. The plateau magnitude also increases with $I$, but then the frequency band limitation does not allow to develop any conclusions about the probable decrease of this plateau beyond a second frequency corner.

Similar trends on the variation of $\gamma$ versus frequency have been observed for different technologies. Deviation from the unity expected value can be large (up to more than 10). As a result of this noise increase, significant errors can appear on CCD detector equipment’s if this phenomenon is not properly accounted for. In the next section, a model is developed that correctly describes LFN measurements. Junction impedance spectroscopy is performed in order to substantiate this model.

III. BEHAVIOUR VERSUS D.C CURRENT

From the LFN spectra measurements, a model is proposed that fits all the measured noise spectra of the devices under test, as illustrated by the dotted line representation from Figure 4 a) b) c). This model is depicted in Figure 5: serial $R_{\text{add}}$-$C_{\text{add}}$ elements are added to the conventional diode equivalent network in order to match both the LFN and impedance spectroscopy measurements. Moreover, the variation of these additional electrical elements with $I$ shows similar features for the three devices under test as shown in Figure 6. An inversely proportional relation of $R_{\text{add}}$ with $I$ for all the tested devices suggests that $R_{\text{add}}$ is closely related to the material, as for the main diode resistance $R_d (=nU_T/I)$ in figure 5 (crosses in Figure 6).

The capacitances share the same behavior with the biasing d.c current in Figure 6 (in spite of a slightly higher value for the 2N2222 diode): once again, it must be mentioned that the LFN measurements have been performed over different frequency bands, still featuring the same increase from the conventional noise floor (diffusion noise).

Moreover, it must be noticed that $R_{\text{add}}$ produces thermal noise in the proposed model. Thus $R_{\text{add}}$-$C_{\text{add}}$ cannot be interpreted as a time constant or as a trap. Therefore, the spectral current noise $S_{I}(f)$ can be modelled by Equation 3:

$$S_{I}(f) = S_{I_d}(f) + \frac{(e_{\text{add}})^2 S_{e_{\text{add}}}}{(1+(R_{\text{add}}C_{\text{add}})^2) f^2}$$  \hspace{1cm} \text{Eq. 3}$$

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Figure 7 compares $R_{add}$ and $C_{add}$ versus $I$, either obtained from noise measurements or from impedance spectroscopy measurements, for the emitter-base diode of the 2N2222 transistor. Good agreement is found between the two techniques. It validates the proposed model of Figure 5 that can therefore be used to describe accurately the diffusion noise enhancement in silicon diodes, even if no data allows us to conclude about the higher frequency behavior (as a decrease of $\gamma$ is expected!).

IV. CONCLUSIONS

In this paper, a deviation from the conventional diffusion noise theory has been evidenced below 1 MHz in various silicon junction devices operating below 1 µA. As the noise level under consideration is very low, its measurement needs special considerations that have been evoked in this paper. An electrical and noise model is proposed for this noise floor degradation. The model is well substantiated since elements provided by the diode impedance spectroscopy closely fit those obtained from noise data. This newly observed excess white noise can be of great impact on the noise floor of photodetectors considering that the dark current in silicon devices range between 0.001 and 100 nA, i.e. within the biasing range under concern in this study. However, some issues still need to be solved; if the excess diffusion noise grows up from a specific frequency, it should be expected to return to the conventional level above a second specific frequency. Despite sustained efforts, no experimental result has been obtained on this last point so far.

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Low-Frequency Noise of Magnons

Sergey Rumyantsev  
Nano-Device Laboratory (NDL),  
Department of Electrical and  
Computer Engineering,  
University of California  
Riverside, California, USA  
Center for Terahertz Research and Applications (CENTERA),  
Institute of High Pressure Physics, PAS  
Warsaw, Poland  
roumis4@gmail.com

Michael Balinskiy  
Nano-Device Laboratory (NDL)  
and Phonon Optimized  
Engineered Materials (POEM)  
Center, Department of Electrical and  
Computer Engineering,  
University of California  
Riverside, California, USA  
m-balinskiy@ece.ucr.edu

Fariborz Kargar  
Nano-Device Laboratory (NDL)  
and Phonon Optimized  
Engineered Materials (POEM)  
Center, Department of Electrical and  
Computer Engineering,  
University of California  
Riverside, California, USA  
Fkargar001@ucr.edu

Alexander Khitun  
Nano-Device Laboratory (NDL)  
and Phonon Optimized  
Engineered Materials (POEM)  
Center, Department of Electrical and  
Computer Engineering,  
University of California  
Riverside, California, USA  
akhitun@ece.ucr.edu

Abstract—The low-frequency noise of magnetization waves, i.e., magnons, was measured in the yttrium iron garnet (YIG) waveguides. This type of noise, which originates from the fluctuations of the physical properties of the YIG crystals, has to be taken into account in the design of YIG-based RF generators and magnonic devices for data processing, sensing and imaging applications. It was found that the amplitude noise level of magnons depends strongly on the power level, increasing sharply at the onset of nonlinear dissipation. The noise spectra of both the amplitude and phase noise have the Lorentzian shape with the characteristic frequencies below 100 Hz.

Keywords — magnon, magnetization wave, phase noise, amplitude noise, random telegraph signal noise, RTS

I. INTRODUCTION

The majority of devices for information processing and sensing applications are based on the charge transfer in different media, e.g., semiconductors, metals, or vacuum. Recently, a completely different approach — termed magnonics — received significant attention. It is based on manipulation of the spin currents carried by the magnetization waves — magnons — in electrical insulators [1-7]. Spin currents in insulators avoid Ohmic losses and, therefore, Joule heating. A number of new devices based on magnon propagation have already been proposed and demonstrated for data processing, sensing and imaging applications [8-10]. The operation frequency of magnonic devices ranges from the low GHz to THz frequencies. The key material for these devices is yttrium iron garnet (YIG). Despite the strong interest to magnonic devices, their low-frequency noise characteristics remained largely unexplored [11].

YIG has also been used for filters and resonators, operating at frequencies up to 26 GHz [12-15]. Noise properties of RF generators based on YIG spheres and delay lines were studied in details [16-20]. The noise of a generator depends on many factors, and the generator scheme may include several noise sources. The YIG crystal is only one of them. However, to the best of our knowledge, the noise properties YIG material have not been rigorously addressed yet. We have recently reported on the low-frequency amplitude noise in YIG waveguides [11]. The low-frequency noise is a ubiquitous phenomenon, present in all kinds of electronic materials and devices [21-33]. The low-frequency noise in magnonic devices is also an important metric, which deserves much more attention. In this paper, we report the results of the measurements of the low-frequency noise of magnons in a YIG waveguide, focusing on the phase noise.

II. EXPERIMENTAL DETAILS

The YIG-film of 9.6 μm thickness and 1.5 mm × 13.5 mm in dimensions was grown on the gadolinium gallium garnet (GGG, Gd3Ga5O12) substrate by the liquid phase epitaxy. The Ti/Au antenna for spin wave (SW) excitation and detection were fabricated on the surface of YIG-film waveguide (see inset in Figure 1). The devices were placed in a magnetic field
created by the permanent neodymium magnet. Depending on the orientation of the magnetic field, the spin waveguide structure supports either the magneto-static surface spin waves (MSSWs) or backward volume magneto-static spin waves (BVMSWs). The MSSW can propagate either on the top surface of the YIG waveguide (surface waves) or at the interface between the YIG waveguide and GGG substrate (interface waves). The strength of the magnetic field corresponded to the ferromagnetic resonance (FMR) frequency of about 5 GHz. Antenna 1 or 3 were used to excite spin waves and antenna 2 was used as a receiver (see Figure 1).

In order to confirm the generation and propagation of magnon current through the electrically insulating waveguide we measured the S-parameters of the waveguide as a function of frequency and magnetic field. The measured S – parameters were compared with known dispersion laws for BVMSW and MSSW. Good agreement with the theory confirmed the type of propagating magnons, and allowed for tuning the \( f_p \) space parameters for the magnon noise studies. These data also confirmed that the signal is not a result of direct electromagnetic coupling between antennas.

Propagating in the waveguide, magnon current acquires variations in the amplitude and phase due to the fluctuations of the physical properties of the YIG thin film. In order to measure these fluctuations, the commercial Schottky diode (33330B Keysight Technologies Inc.) was connected to the antenna 2 (see Figure 1). The DC detected signal from the diode was amplified by the low noise amplifier and analyzed by the FFT spectrum analyzer.

### III. Results and Discussions

Fourier transform of the signal from the diode yields the spectrum of the amplitude fluctuations. The amplifier noise of the magnons propagating along the interface between YIG waveguide and GGG substrate was the highest, and the lowest was the noise of the volume magnons. The high noise level of the interface magnons was explained by the YIG/GGG interface roughness, resulting in stronger fluctuations of the material parameters that govern magnon current propagation. The dependence of noise on power for interface and surfaces magnons reveals one or more maxima. The positions of the noise peaks correspond to the change of the slope of \( S_{21} \) dependence on the power.

The spectra of the amplitude fluctuations had the shape of the Lorentzian, \( S_0=1/(1+x^2/\omega^2) \) with the characteristic corner frequency \( f_c<100-1000 \text{Hz} \). In the time domain, the magnon noise revealed itself as a random telegraph signal (RTS) noise. Very small changes in the input power of \( ~0.1 \text{dB} \) led to the significant changes in the RTS noise and its spectrum. RTS noise is well known in electronic devices and charge density materials and devices [34-38]. Observation of RTS noise in the large magnon waveguides can be explained by the individual discrete macro events which contribute to both the noise and magnon dissipation processes [11].

Fluctuations of the speed of the magnon wave lead to the fluctuations of the phase of the wave. In order to measure the phase noise, the delay line itself with the three antennas, as shown in Figure 1, was used as a phase detector. For this purpose, the output of the external generator was split and fed to antennas 1 and 3, correspondingly (see Figure 1). The line for the antenna 3 included an attenuator and a phase shifter. Using the attenuator, the power on the output antenna 2 was adjusted to be equal when powered separately in each of two configurations. The resulting power on antenna 3 was equal to \(-2 \text{dBm}\). These two signals are merged at the plane, which corresponds to the antenna 3 (see Figure 1). Due to the interference, when both antennas 1 and 3 are powered, the power on the output is a function of the phase difference of these two signals. This phase difference is defined by the distance between antennas 2 and 3 and by the external phase shifter. Fluctuations of the magnon wave speed contribute to the phase difference, and are converted to the voltage fluctuations detected at the antenna 2. The symbols in Figure 2 show the DC voltage on the detector, which is proportional to the output power at the antenna 2 as a function of the phase shift adjusted by the external phase shifter. The solid line is a fit with \( A\cos^2(\Psi/2) \) function (\( A \) is a fitting parameter). The derivative, \( R \), of this function is a conversion coefficient, which determines the spectral noise density of the voltage fluctuations: \( S_v=S_f x R^2 \) (\( S_f \) is the spectral noise density of the phase fluctuations).

![Fig. 1. Schematic view of the YIG waveguide with three antennae.](image)

![Fig. 2. DC voltage on the detector as a function of the phase shift adjusted by the phase shifter. The symbols show experimental data while the solid line is a fit with \( A\cos^2(\Psi/2) \) function.](image)
Figure 3 shows the spectral noise density $S_v$ at $f=10$ Hz as a function of the phase difference. The blue symbols and line show the measured spectral noise density at $f=10$ Hz; the red symbols and dashed line show the level of the background noise. As one can see, the noise is minimal at the phase differences $\Psi=0$ and $\Psi=\pi$. The noise at its maximum, i.e., at $\Psi=\pi/2$ is more that an order of magnitude higher. The conversion coefficient, $R$, has its maximum at this phase. Therefore, we can conclude that this amplitude noise is predominately due to the conversion of the phase noise.

Figure 4 shows the calculated phase noise $S_{\Phi} = S_v/R^2$. As seen, within the measurement accuracy, the phase noise is independent on $\Psi$, as expected.

Although RTS noise was not found in the phase fluctuations, the noise spectra of the phase noise within the frequency range $10$ Hz < $f$ < $1$ kHz also had the form of the Lorentzian with the characteristic frequency within $10$ Hz – $100$ Hz (see Figure 5). With the excitation power of $\sim 2$ dBm on one of the antennas, the phase noise at the frequency of $10$ Hz was measured to be around $-68$ dB/Hz.

We attributed the measured phase noise to the magnetization wave phase velocity fluctuations. The velocity fluctuations can be estimated as $S_v/V^2 = S_v/V_{\text{int}}^2$, where $V_{\text{int}}$ is the phase difference gained by the spin wave between the antennas, which can be measured by the vector network analyzer. $V_{\text{int}}$ was measured by method of substitution used “unwrapped phase” format of VNA. Total phase margin at operation frequency was first measured with our device, $\Psi_{\text{DUT}}$, and then with electrically short SMA connector which substituted our device, $\Psi_{\text{cont}}$. Interested value $\Psi_{\text{int}}$ was calculated as difference $\Psi_{\text{DUT}} - \Psi_{\text{cont}}$. We found $\Psi_{\text{int}}=93$ rad, which yields $S_v/V^2 \approx 2 \times 10^{-11}$ Hz$^{-1}$ at the frequency of the analysis $f=10$ Hz. This value of the velocity fluctuations can be used to estimate the phase fluctuations in the waveguides of an arbitrary length.

**IV. CONCLUSIONS**

In conclusion, the low-frequency noise of magnons, propagating as magneto-static surface spin waves, was measured in YIG waveguides at the frequencies $f<1$ kHz. The noise spectra had the Lorentzian shape with the characteristic frequencies below 100 Hz. At these frequencies, the noise of magnons sets the limit for data processing, sensing and imaging applications. It can also contribute to the phase noise of RF devices based on YIG crystals.
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Fluctuation - Dissipation - Dispersion Relation for Slow Processes and Quality Factor for Oscillation Systems

Viacheslav V. Belyi
Theoretical Department
IZMIRAN, Russian Academy of Sciences
Trotsk, Moscow, Russia
email address: slava.belyi(at)gmail.com

Abstract—We have generalized the Fluctuation-Dissipation Theorem to the systems with slowly varying parameters. The important conclusion of this analysis is to reveal that the spectral function of the fluctuations is determined not only by dissipation but also by the derivatives of the dispersion. The non-Joule dispersion contribution is characterized by a new non-local effect originating from an additional phase shift between the force and the response of the system. That phase shift results from the parametric control to the system. Finally, an electrical oscillation circuit is considered as a concrete example. In that system, it is shown that the dispersive contributions strongly affect the Q factor.

Index Terms—non-equilibrium fluctuations, FDT, Q-factor

I. INTRODUCTION

Any oscillating system is characterized by two main parameters: the proper frequency and the quality factor. The latter is inversely proportional to the width of the spectral line of the parameter fluctuations. In thermodynamic equilibrium, fluctuations are determined by the system temperature Θ and the dissipation. The first fluctuation-dissipation relation between the diffusion coefficient and the dissipative friction coefficient was derived independently by Einstein and Smoluchowski in their theory of Brownian motion [1], [2]. Later, this relation was established by Nyquist [3] for electric circuits and was experimentally confirmed by Johnson [4]. The Nyquist-Johnson relation was extended by Callen and Welton [5] to a general class of dissipative thermodynamic equilibrium systems (see also [6]). In the classical case the spectral function of the fluctuations has the form:

\[ \langle x^2 \rangle_\omega = \frac{2\Theta \omega}{\Im \alpha(\omega)} \]  

where \( \alpha(\omega) \) is the response function, and \( \Theta \) is the temperature in energy units. The linear response theory and the fluctuation-dissipation theorem for arbitrary dynamic systems was developed by Kubo [7], Mori [8] and Zwanzig [9]. In the Kubo method the response of the density matrix to the external field is calculated, whereas the Mori-Zwanzig technique introduces a projection operator to the space of variables that describe macroscopic states of the system. Generally, the system parameters may depend on both time and space. Inhomogeneities in space and time on scales greater than the fluctuation scales will certainly also contribute to fluctuations. Recently, in the context of plasma physics, and using the Langevin approach and the time-space multiscale technique, it has been shown that the amplitude and the width of the spectral lines of the electrostatic field fluctuations and the electron form factor are determined not only by the imaginary part of the dielectric susceptibility but also by the derivatives of its real part [10]. As a result of the inhomogeneity, these properties become asymmetric with respect to the inversion of the sign of the frequency. In the kinetic regime, the form factor is more sensitive to space gradients than the spectral function of the electrostatic field fluctuations. This asymmetry of lines can be used as a diagnostic tool to measure local gradients in the plasma.

In this communication we generalize the fluctuation-dissipation theorem for slowly varying processes. Using the momentum method and the time multiscale technique, a generalized Callen-Welton formula is derived. The width and the amplitude of the spectral lines of the fluctuations are determined not only by the dissipation but also by the derivatives of the dispersion. These two effects have a comparable influence for systems with a high quality factor. The non-Joule dispersion contribution is characterized by a new non-local effect originating from an additional phase shift between the force and the response of the system. This phase shift results from the parametric control to the system. As an application we consider a LC-circuit. It is shown that the spectral function of the current depends not only on the real part of the impedance (dissipation) but also on the derivatives of its imaginary part (dispersion). It is also shown that at finite time intervals one can increase drastically the quality factor by the simultaneous increasing the inductance and decreasing the capacity.

II. RESULTS

Let us consider an arbitrary system whose evolution is described by the following equation:
\[ \left( \frac{\partial}{\partial t} + L(t) \right) G(t, t') = 0, \quad t > t', \quad (2) \]

where \( L(t) \) is generally a non self-conjugate, linear operator in the Hilbert space. This operator varies slowly in time. The term “slowly” means that the control parameter undergoes only a small change during the period of the system motion. \( G(t, t') \) may be the Heisenberg operator. Then \( L(t) \cdot G(t, t') \) will be the commutator with the Hamiltonian. In other cases \( G(t, t') \) could be a density matrix, and \( L(t) \) would appear as the Liouville operator. Finally, for \( G(t, t') \) we can take the two-time correlator \( G(t, t') = \langle \delta f_{nm}(t) \delta f_{n'm'}(t') \rangle \) of the deviation from the referent state \( f_{nm}(t) \) of the density matrix in the energy representation \( \delta f_{nm}(t) \) [11], [12]. In such a case \( L(t) \) takes into account the self-consistent field and collisions. The time dependence in \( L(t) \) manifests itself in the referent state and in the terms containing the external force. The slow scale is much larger than the characteristic fluctuation time.

We can therefore introduce a small parameter \( \mu \), which allows us to describe fluctuations on the basis of a multiple time scale analysis. Obviously, fluctuations vary on both “fast” and “slow” time scales. The solution of the linear equation (2) can be expressed through the Green’s function or the propagator \( U(t, t') \) of Eq. (2) as:

\[ G(t, t') = U(t, t') \cdot G(0), \quad (3) \]

where in the case of the kinetic fluctuations, the one-time moment \( G(0) \) is given by

\[ G(0) = \langle \delta f_{nm}(t') \delta f_{n'm'}(t') \rangle = \delta_{nn'} \delta_{mm'} \frac{f_n(\mu t') + f_m(\mu t')}{2}. \quad (4) \]

If the operator \( L \) does not depend on time, the dependence on time of the Green’s function appears only through the interval \( t - t' \). However, when we consider an operator \( L(\mu t) \) slowly varying in time, and when we take non-local effects into account, the time dependence of \( U(t, t') \) is more subtle [13], [14].

\[ U(t, t') = U(t - t', \mu t'). \quad (5) \]

Here we want to stress that the non-local effects appear due to the slow time dependences \( \mu t' \). At first order, the expansion of Eq. (5) with respect to \( \mu t' \) leads to

\[ U(t, t') = (1 - \mu t' \frac{\partial}{\partial \mu t'}) U(\tau, \mu t); \quad \tau = t - t'. \quad (6) \]

Let us introduce the resolvent operator \( \mathbf{R}(z) \) which can be defined formally as the Laplace transform of the propagator \( U(\tau) \):

\[ \mathbf{R}(z) = \int_0^\infty U(\tau) \exp(iz\tau) d\tau; \quad z = \omega + i0 \quad (7) \]

The Laplace transform of Eqs. (3,6) gives

\[ G^+(z) = (1 + i \frac{\partial^2}{\partial \omega^2}) \mathbf{R}(z) \cdot G(0). \quad (8) \]

For sake of convenience we omit \( \mu \) from that equation and throughout this communication, keeping in mind that the time derivatives are taken with respect to the slowly varying variables. Thus in first approximation the expression for the spectral function of the fluctuations is

\[ G(\omega) = 2 \Re (1 + i \frac{\partial^2}{\partial \omega^2}) \mathbf{R}(z) \cdot G(0). \quad (9) \]

The spectral density of the fluctuations of the internal parameters of the system in local equilibrium can be defined as usual [12], [15].

\[ \langle \delta A \delta B \rangle_\omega = A \cdot G(\omega) \cdot B \]

\[ = h [\Im \alpha_{AB}(\omega) + \frac{\partial^2}{\partial \omega^2} \Re \alpha_{AB}(\omega)] \coth(h\omega/2\Theta). \quad (10) \]

where

\[ \alpha_{AB}(\omega) = i\hbar \sum \tilde{R}_{nmnm}(z) A_n B_m (f_m - f_n) \quad (11) \]

is the response function for diagonal resolvent [12]. In the classical limit \( \hbar \to 0 \) the generalized Callen-Welton formula (10) takes the form

\[ \langle \delta A \delta B \rangle_\omega = [\Im \alpha_{AB}(\omega) + \frac{\partial^2}{\partial \omega^2} \Re \alpha_{AB}(\omega)] \frac{2\Theta}{\omega}. \quad (12) \]

In deriving Eqs. (10,12) we assumed the system to be in a local equilibrium state, so that the characteristic time for parameters variation exceeds the relaxation time of the distribution function. When expanding the Green’s function in Eq. (6) in terms of the small parameter \( \mu \), there appears an additional term at first order. It is important to note that the imaginary part of the response function is now replaced by the real part. If the quality factor of the system is of the order of \( 1 \) (it can be a broad-band system or a process near the zero frequency), the real and imaginary parts of the response function are of the same order and the correction is negligibly small. But in the case of systems with a high quality factor, for which the real part of the response function is greater than the imaginary part, the second small parameter appears to be inversely proportional to the quality factor. An example of such system with a high quality factor could be plasma fluctuations near the Langmuir frequency when the quality factor is inversely proportional to the small plasma parameter [10]. When this small parameter is comparable with \( \mu \), the second term in Eqs. (10,12) may have an effect comparable to the first term. This will be shown in the next example. At the second order in the expansion in \( \mu \), the corrections appear only in the imaginary part of the response function, and they can reasonably be neglected. It is therefore sufficient to retain the first order corrections to solve the problem. The same derivatives of the dispersion, as in Eqs. (10,12), appear in the geometrical optics approximation [16] and play an important role in defining the adiabatic invariant in a dispersive medium [17].
As an example we consider the electrical oscillation circuit which can be used to model many oscillation processes in nature. We assume that all the circuit elements (resistance \( R \), inductance \( L \), and capacity \( C \)) have the same temperature \( \Theta \), which can change adiabatically. Therefore the system parameters \( R, L, C \) will vary slowly in time. Moreover the change of these parameters may also be mechanical, due to the action of external forces, by \"hand\". It is this case that we will consider when evaluating the quality factor of LC-circuit.

The thermal motion of the charged particles in the circuit give rise to thermal oscillations which can be considered to be equivalent to Brownian motion. The corresponding Langevin equation is

\[
\frac{dq}{dt} = J; \quad L(\mu t) \frac{dJ}{dt} + R(\mu t) J + \frac{q}{C(\mu t)} = E, \tag{13}
\]

where \( q \) is the electric charge, \( J \) is the current, and \( E \) is the Langevin source. It can be treated as the e.m.f. equivalent of the two-time correlator of the electric current \( G_j(t,t') \) as

\[
G_j(t,t') = U(t,t')G_j(0), \quad \tag{14}
\]

where \( U(t,t') \) is the propagator of the set of equations (13), and where the initial condition \( G_j(0) \) for the local equilibrium state is

\[
G_j(0) = \frac{(L/C)^{1/2}}{2\pi\Theta} \int J^2 \exp \left( -\frac{LJ^2 + q^2/J}{2\Theta} \right) dq dJ = \frac{\Theta(\mu t')}{L(\mu t' )}. \tag{15}
\]

Applying the procedure above, we obtain the following expression for the spectral function of the current in the circuit.

\[
(J^2)_{\omega} = 2Re(1 + \frac{d^2}{d\omega d\omega}) R(z) \frac{\Theta}{L} = \frac{2[Re Z(\omega) + \frac{\Theta}{\pi\Theta} \frac{d}{d\omega} \text{Im} Z(\omega)] \Theta}{\text{Im}^2 Z(\omega) + [Re Z(\omega) + \frac{\Theta}{\pi\Theta} \frac{d}{d\omega} \text{Im} Z(\omega)]^2}, \tag{16}
\]

where \( Z(\omega) = R - i(L\omega - 1/C\omega) \) is the complex impedance.

In deriving Eq. (16) we assumed that the time variations of the parameters in the resonant take place at scales much greater than the oscillation period, and the local equilibrium initial state (15) is achieved when \( R \) is greater than \( \frac{dL}{dt} \). The second restriction can be relaxed by introducing the nonequilibrium initial correlator of the current \( G_j^{eq}(0) \). In this case the Eq. (16) takes the form

\[
(J^2)_{\omega} = \frac{2[Re Z(\omega) + \frac{\Theta}{\pi\Theta} \frac{d}{d\omega} \text{Im} Z(\omega)] \Theta}{\text{Im}^2 Z(\omega) + [Re Z(\omega) + \frac{\Theta}{\pi\Theta} \frac{d}{d\omega} \text{Im} Z(\omega)]^2}. \tag{17}
\]

Using the Langevin equations (13) the expressions for the spectral function of the current takes the form

\[
(J^2)_{\omega} = \frac{(E^2)_{\omega}}{\text{Im}^2 Z(\omega) + [Re Z(\omega) + \frac{\Theta}{\pi\Theta} \frac{d}{d\omega} \text{Im} Z(\omega)]^2}. \tag{18}
\]

The comparison of Eqs. (16) and (18) gives for the spectral density of the e.m.f.

\[
(E^2)_{\omega} = 2[Re Z(\omega) + \frac{\Theta}{\pi\Theta} \frac{d}{d\omega} \text{Im} Z(\omega)] \Theta = 2[R - \frac{dL}{dt} + \frac{1}{\omega^2 C^2} \frac{dC}{dt}] \Theta, \tag{19}
\]

which is a generalized Nyquist formula. One can see that in the general case the spectral density of the e.m.f. for slow processes depends on the frequency and is not always white noise.

Now let us come back to a point discussed in the beginning, namely to the quality factor of the oscillation system. As the time derivative can have different signs, the dispersion corrections in Eq. (16) may both decrease and increase the line width and therefore also the oscillation system quality factor. The independent variation of the reactive parameters \( L \) and \( C \) results in a shift of the circuit proper frequency. To avoid this frequency shift we should change the reactive parameters \( L \) and \( C \) as

\[
\frac{dC}{dt} = -\frac{C}{L} \frac{dL}{dt}, \tag{20}
\]

which follows from the condition of the stability of the circuit frequency: \( \omega_0 = (LC)^{-1/2} = \text{const} \). In this case Eq. (16) takes the form

\[
(J^2)_{\omega} = \frac{2\Theta[R - \frac{dL}{dt}(1 + \frac{1}{\omega_0^2 C^2})]}{(L\omega - 1/C\omega)^2 + [R - \frac{dL}{dt}(1 + \frac{1}{\omega_0^2 C^2})]^2}. \tag{21}
\]

Near the resonance point \( \omega = \omega_0 \)

\[
(J^2)_{\omega} = \frac{\Theta}{L(\omega - \omega_0)^2 + \gamma^2} + \frac{\Theta}{L(\omega + \omega_0)^2 + \gamma^2}, \tag{22}
\]

where the line width is given by

\[
\gamma = \frac{1}{2L}(R - 2\frac{dL}{dt}). \tag{23}
\]

We see from Eqs. (22), (23) that the correction is still symmetric with respect to the change of sign of \( \omega \), but the intensities and broadening are different from the stationary case. In the case of local equilibrium, the integral of the intensity over frequency remains the same as in the stationary case (Fig 1).

The quality factor becomes now

\[
Q = \frac{\omega}{2\gamma} = (\frac{L}{C})^{1/2} \frac{1}{R - 2\frac{dL}{dt}}. \tag{24}
\]

Note that the initial correlation is not present in the expressions for the line width (23) and the quality factor (23), these expressions being fully determined by the singularities of the resonant. Usually the quality factor increases as the inductance increases and the capacity decreases, but due to the nonstationary dispersion terms it can increase drastically.
The higher the initial quality factor of the system, the stronger the effect. Thus for a circuit proper frequency of 1 kHz and a quality factor =1000, the second term in Eq. (23) is comparable to the first one, when the reactive parameters $L$ and $C$ of the system vary by several tenths per second. As we consider the linear approximation, to avoid misunderstanding we assume that $R > 2 \frac{dL}{dt}$. Therefore, at finite time intervals one can increase drastically the quality factor by simultaneously increasing the inductance and decreasing the capacity. Similar situations can appear in other oscillating systems.

III. Conclusion

Using the momentum method and the time multiscale technique, we have generalized the Callen-Welton formula to systems with slowly varying parameters. The important conclusion of this analysis is to reveal that the spectral function of the fluctuations is determined not only by dissipation but also by the derivatives of the dispersion. The non-Joule dispersion contribution is characterized by a new non-local effect originating from an additional phase shift between the force and the response of the system. That phase shift results from the parametric control to the system. Finally, an electrical oscillation circuit is considered as a concrete example. In that system it is shown that the dispersive contributions strongly affect the quality factor. These results are applicable to other systems and are important for the understanding of various behaviors observed in different field of physics, communication, chemistry and biophysics.

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Charge carrier dynamics of strongly-correlated electrons in low-dimensional molecular metals studied by fluctuation spectroscopy

Jens Müller*

Institute of Physics, Goethe-University Frankfurt, 60438 Frankfurt (M), Germany

*j.mueller@physik.uni-frankfurt.de

In the past decade, we have utilized fluctuation spectroscopy as a new approach for studying the low-frequency dynamics of strongly correlated charge carriers in quasi-two-dimensional organic charge-transfer salts (BEDT-TTF)$_2$X [1,2]. These materials are considered as model systems to study the Mott metal insulator transition – a key phenomenon in modern condensed matter physics – and related phases in reduced dimensions.

In this talk, we review some basic aspects of electronic fluctuations in solids, and give a brief overview of selected problems where the analysis of $1/f$-type or random telegraph fluctuations and the corresponding slow dynamics provide a better understanding of the underlying physics of strongly-correlated electrons [3]. These examples are related to (i) an inhomogeneous current distribution due to phase separation and/or a percolative transition [4]; (ii) slow dynamics due to a glassy freezing either of structural degrees of freedom coupled to the electronic transport properties [5] or (iii) of the electrons themselves, e.g., when residing on a highly-frustrated crystal lattice, where slow and heterogeneous dynamics are key experimental properties for the vitrification process of a supercooled charge-liquid [6]. Another example is (iv), the near divergence and critical slowing down of charge carrier fluctuations at the finite-temperature critical endpoint of the Mott metal-insulator transition [7]. Here also, indications for a glassy freezing, and temporal and spatial correlated dynamics are found.

As an outlook, mapping out the region of ergodicity breaking and understanding the influence of disorder on the glassy dynamics will be an important realm of future studies, as well as the fluctuation properties deep in the Mott or charge-ordered insulating states providing a connection to relaxor or ordered ferroelectric states studied by dielectric spectroscopy.

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Lorentzian Noise Approach for 1D Transport Studies

Mykhaylo Petrychuk
Bioelectronics (ICS-8)
Forschungszentrum Jülich
52425 Juelich, Germany
m.petrychuk@gmail.com

Bor Zadorozhnyi
Bioelectronics (ICS-8)
Forschungszentrum Jülich
52425 Juelich, Germany
b-zadorozhnyi@fz-juelich.de

Yuri Kutovyi
Bioelectronics (ICS-8)
Forschungszentrum Jülich
52425 Juelich, Germany
y.kutovyi@fz-juelich.de

Heike Riel
IBM Research
Zurich, Säumerstrasse 4
8803 Rüschlikon, Switzerland
hei@zurich.ibm.com

Siegfried Karg
IBM Research
Zurich, Säumerstrasse 4
8803 Rüschlikon, Switzerland
fzk@zurich.ibm.com

Svetlana Vitusevich*
Bioelectronics (ICS-8)
Forschungszentrum Jülich
52425 Juelich, Germany
*svitusevich@fz-juelich.de

Abstract—Nanowire structures (NW) exhibiting one-dimensional (1D) transport properties are attracting increased interest within the scientific community due to their unique ability to improve carrier mobility and reveal new effects. This field of research provides an important guideline for the development of advanced devices. However, several issues related to exchange processes with traps have to be addressed before the full potential of 1D transport can be utilized at room temperature. We study the mechanisms of variability in InAs NW structures due to several traps with different characteristic time constants. We show that the Lorentzian noise component and the random telegraph signal noise can be effectively analyzed to confirm the formation of quantum transport in 20 nm diameter InAs NW structures.

Keywords—1D transport, Lorentzian noise, nanowire

I. INTRODUCTION

InAs nanowires (NW) represent promising key materials for registering quantum transport properties [1, 2], which can be observed even at room temperature. At the same time, there are several related issues and one in particular needs to be resolved before the unique features of one-dimensional (1D) transport can be utilized for the development of quantum technologies. The main challenge is variability effects which deteriorate the stable quantum operation of such unique NW structures. As shown in [3], the traps in InAs NWs impact transport properties. Slow states with relaxation times in the order of minutes or even hours have a significant effect on the stability of the electrical characteristics of the samples. Detailed studies of slow states in a dielectric layer [3] involve measuring the magnitude of the hysteresis at room temperature. It is noted that relaxation of slow states occurs involving centers with different time constants. At the same time, the estimated density of slow centers substantially exceeds the density of fast centers by more than one order of magnitude. However, operation in that quantum regime close to room temperature has not yet been reported in the literature.

In this work, we demonstrate that the quantum operation regime can be achieved for InAs NW structures although there are a lot of traps influencing the behavior of 1-V characteristics. In this respect, a powerful method of noise spectroscopy was used. We show that the analysis of random telegraph signal (RTS) noise and Lorentzian noise components can be used to study quantum conductivity in InAs NW structures with a diameter of 20 nm.

II. EXPERIMENTAL DETAILS

The InAs nanowires under study were grown using metal organic chemical vapor deposition (MOCVD) technology. Nanowire structures with a diameter of 20 nm were coated with a 5 nm Al2O3 layer and transferred to a Si substrate covered with HfO2 dielectric layer. Metallic contacts made of Ni were patterned in such a way as to obtain transmission line model structures (TLM). Such TLM structures allow us to study transport properties in nanowires of different lengths (Fig. 1). The silicon substrate was used as the back gate.

Fig. 1. An optical image of an InAs nanowire chip.

III. RESULTS AND DISCUSSION

We investigated variability effects and current fluctuations in I-V characteristics of the fabricated InAs NW structures of different lengths: 3.0 µm, 1.2 µm, and 0.6 µm. Typical output characteristics $I_d(V_{DS})$ of the 3 µm long NW sample measured at different drain source voltages in the range $V_{DS} = 0 – 50$ mV and at different temperatures are shown in Fig. 2a. Each $I_d(V_{DS})$ dependence was recorded for 6 s.

Both sets of characteristics, measured in the absence of a gate bias (Fig. 2a) and when a voltage $V_G = 2$ V is applied (Fig. 2b), demonstrate the deviation from linearity of the dependencies $I_d(V_{DS})$ when $V_{DS} > 10$ mV, i.e. an increase in the drain potential with respect to the source leads to a change in the charge state of the centers in the dielectric, which in turn leads to a decrease in the channel conductivity. Thus, not only the potential of the gate, but also the voltage across the channel of the InAs NW field structure increases the probability of free carrier trapping at the centers in the dielectric.
dependences of the sample with $L = 3\,\mu$m at two gate voltages (a) $V_G = 0$ and (b) $V_G = 2\,V$ as well as at different temperatures in the range from $T = 100\,K$ to $T = 297\,K$. The change step is $\Delta V_{DS} \approx 0.5\,mV$. The curves are smoothed by the Savizky-Golay method, a second-order polynomial, and the window length is 15 points. The dashed line indicates a linear relationship.

The nature of such captures can be traced by registering two typical dependences of the channel current on the $V_{DS}$ channel voltage (Fig. 3a) and on the gate bias (Fig. 3 b). As can be seen in Fig. 3b, the periodic application of voltage to the gate led to a gradual decrease in the channel current, first due to faster traps, and then due to slower ones. In this case, the step of decreasing the current, apparently caused by the capture of one electron, became an essential part of the total current. Changes in the current upon the application of $V_{DS}$ are mainly due to thermal fluctuations and are smaller than the current changes upon the application of the voltage $V_G$. In the first case, the main role is played by faster transitions with characteristic times in the order of seconds.

Remarkably, random telegraph signal (RTS) behavior was registered in time traces of the NW structure. It should be noted that, in contrast to previously considered disadvantages of RTS its characteristic time constant can be used to obtain enhanced sensitivity biosensors [4, 5]. Thus, RTS component studies represent a powerful method for the stochastic and dynamic characterization of single traps.

Fig. 2. $I_D(V_{DS})$ dependences of the sample with $L = 3\,\mu$m at two gate voltages (a) $V_G = 0$ and (b) $V_G = 2\,V$ as well as at different temperatures in the range from $T = 100\,K$ to $T = 297\,K$. The change step is $\Delta V_{DS} \approx 0.5\,mV$. The curves are smoothed by the Savizky-Golay method, a second-order polynomial, and the window length is 15 points. The dashed line indicates a linear relationship.

Fig. 3. Typical current-voltage (a) ($V_G = 1.75\,V$) and drain-gate (b) ($V_{DS} = 25\,mV$) characteristics of an InAs NW sample with length $L = 3\,\mu$m, measured successively with a period of (a) 6 s and (b) 12 s. The dashed lines show the estimated curves in the absence of a current change. The measurement temperature is shown in the figure.

Fig. 4 demonstrates typical time traces obtained for InAs NW FETs. Based on the results obtained with increasing gate voltage (Fig. 3b), it can be argued that the physical cause of the current change is the lowering of the potential barrier due to the trapping of electrons to centers in the surface dielectric when a voltage is applied both to the gate $V_G$ and to the channel, i.e. drain-source voltage $V_{DS}$. This result is in agreement with observations reported in [3].

In addition, we performed measurements of conductivity as a function of temperature at different $V_G$ and $V_{DS}$ voltages. Fig. 5a shows the results obtained for $V_{DS}$ ranging from 5 mV to 50 mV at gate voltage $V_G = 0$. Two sections of curves with different logarithmic slopes can be distinguished, which corresponds to two activation energies $\Delta E_1 = 0.20\,eV$ and $\Delta E_2 = 0.05\,eV$. With an increase in the voltage $V_G$, the first section expands practically over the entire range of measurement temperatures, whereas the second section almost completely disappears.
It could be assumed that at low temperatures shallow, and therefore faster, traps play a significant role in current changes and that with increasing temperature deeper (slower) traps prevail. For this reason, at low temperatures, the channel conduction time drift is insignificant and increases with increasing temperature. This is also due to the very “slow” centers, with time constants measured in hours, involved in the processes of changing the conductivity. The probability of capture/emission of electrons at slow centers increases with increasing temperature. Thus, the mechanism of activation of the centers appears to be rather complicated as both thermal activation and electro-activation are present.

The dependences of activation energy $\Delta E_1$ as a function of the gate bias $V_G$ (Fig.5b) are plotted using the results of the temperature dependences of the conduction channel $G(T)$ obtained for different gate voltages. The decrease of the energy $\Delta E_1$ with increasing bias $V_G$ supports the influence of the field effect on electron capture processes. Electron capture occurs with the participation of the field effect (Poole-Frenkel effect). The reduction in the height of the potential barrier $\Delta E_1$ occurs according to the law:

$$\Delta E_1(E) = \Delta E_1(0) - A \times \beta \times \sqrt{E} \quad (1)$$

where $E$ is the field at the InAs / HfO$_2$ interface, $\beta$ is the Poole-Franke coefficient, $A$ is the dimensionless coefficient considering the geometric and dielectric parameters of the gate stack. The dashed line in Fig. 5b is plotted in accordance with Eq. (1) considering the relation $E - V_G$. The good agreement between the experimental data and the theoretical dependence confirms the significant effect of the electric field at the interface on changes in the conductivity of InAs NW.

The noise spectra measured at different temperatures allowed us to obtain an Arrhenius plot and therefore to calculate the activation energies of the active traps (Fig. 6). As can be seen, similar traps (see Fig. 5a) with an energy of 50 meV can be found using Lorentzian noise components in the spectra of InAs NW samples.

We revealed an important effect by analyzing Lorentzian noise components (typical noise spectra are shown in Fig.7a) in the sample with a length of 600 nm.

The low-frequency Lorentzian noise plateau amplitudes, derived from the noise spectra, reflect the quantized behavior as a function of gate voltage (Fig. 7b). Moreover, similar behavior is also found in the time constant of capture-emission processes as a function of gate voltage thus confirming the characteristic synchronous patterns at room temperature.
This reflects the fact that electron exchange dynamics between the quantized levels can be analyzed even at room temperature using noise spectroscopy. The quantization effect becomes stronger with decreasing temperature, thus confirming that noise spectroscopy is a powerful advanced method for studying the formation of 1D quantum conductivity in nanowire structures.

IV. CONCLUSIONS

We investigated the transport and noise properties of InAs nanowire (NW) structures, currently recognized as a key material with potential for 1D transport at room temperature. The fine mechanisms for changing the conductivity of InAs NWs were investigated. It is shown that the application of a positive voltage to the drain-source contacts and to the substrate, which acts as a back gate, leads to a progressive capture of electrons to the centers in the gate dielectric at the InAs / HfO₂ interface. The time dependence of the degree of current change is non-exponential, which is determined by the participation in this process of centers with a wide distribution of time constants. Evidence is provided of conductivity jumps caused by the trapping of single electrons, which significantly changes the conductance of NWs. The physical mechanism that accelerates the capture of electrons at the centers by the application of an electric field is the Poole-Frenkel effect, which has been confirmed experimentally. Our results demonstrate that the Lorentzian noise component in noise spectra and random telegraph signal noise represents a powerful method for 1D transport studies in nanowire structures.

REFERENCES

Prehistory probability distribution of ionic transitions through a graphene nanopore

C. Guardiani∗, M.L. Barabash∗, W.A.T. Gibby∗, D.G. Luchinsky∗† I.A. Khovanov‡ and P.V.E. McClintock∗
∗Department of Physics
Lancaster University, Lancaster, UK, LA1 4YB
Email:c.guardiani@lancaster.ac.uk
†SGT Inc., Ames Research Center, Moffet Field, CA, 94035, USA
‡School of Engineering University of Warwick, Coventry, CV4 7AL, UK

Abstract—We analyze selective ionic conduction through an artificial nanopore in a single graphene sheet, using molecular dynamics simulations and the prehistory probability distribution. We assess position-dependent changes in the number and orientation of water molecules in the first and second hydration shells of the ion as it crosses the nanopore. We reveal coupling between an ionic double layer near the sheet to the statistical properties of the hydration shells.

Index Terms—graphene nanopore, ion permeation, prehistory probability distribution, molecular dynamics

I. Introduction

Ionic transport through graphene nanopores is a rapidly expanding field encompassing both the nano- and biophysics communities. Being the smallest ion-selective devices, they find a wide range of applications including e.g. the desalination of water [1] and biosensing [2]. Analysis of permeation through them also sheds new light on the function of biological ion channels [3].

There are several issues that render prediction of the current in this system a challenging task, including: (i) formation of an ionic double layer near the graphene sheet, significantly altering the spatial distribution of ions in the bulk electrolyte [4]; (ii) the existence of fixed charge on the nanopore walls, modifying the structure of the double layer [5]; (iii) spatial dependence of the number [6] and orientation of water molecules in the hydration shells of the ions in the pore; (iv) dependence of the effective dielectric constant [7], εw and excess chemical potentials μi of the ions [8] on their location within the pore; and (v) the position-dependent diffusivity [9] of ions in the proximity of the pore.

We note that each of the problems listed above is the subject of extensive independent research. The complexity of the system relates to the fact that the current through the pore, the statistics of the hydration shells, and the properties of the electrical double layer are mutually interrelated in a nontrivial manner, cf. [10]. These mutual interactions are often neglected to simplify the analysis, e.g. the first hydration shell is often considered to be intact near a charged surface [5] even though it is known that dehydration and saturation of the water orientation have significant effects on the local [11] dielectric permittivity, the potential of the mean force [12], and, therefore, on the current through the pore.

In this work we provide atomistic insight into the interplay between dehydration, water polarization in the hydration shells, and the ionic double layer induced by the electrostatic field near the graphene surface. Statistical analysis is facilitated by introducing the prehistory probability distribution of ionic trajectories crossing the pore.

The paper is organized as follows. First, we introduce the model in Sec. II. Next, we discuss the structure of the electrical double layer (EDL), and discuss the prehistory density of the permeating ions, in Secs. III and Sec. IV respectively. The dehydration and water orientation in the first two shells of potassium ions crossing the pore are discussed in Sec. V. Finally, conclusions are drawn in Sec. VI.

Fig. 1. VMD [13] image of the graphene nanopore surrounded by electrolyte solutions. Carbon atoms are shown in dark grey. Water molecules are explicitly resolved, but are shown here as a transparent glassy medium except for the 1st hydration shell of the K⁺ ion (blue ball) crossing the pore. Chloride ions are shown in green. Charged carbon ions around the rim of the pore are shown in cyan. The orientation of one water molecule in the 1-st shell of the K⁺ ion is illustrated in the inset.
II. Model
The model considered is shown in Fig. 1. It consists of a single-layer graphene sheet separating two electrolyte solutions of one molar KCl on both sides. The pore through the middle of the graphene sheet has diameter $\sim 8\text{Å}$ measured as the center-to-center separation of carbon atoms on opposite sides of the rim. Note that ionic radius of carbon atoms is $0.7\text{Å}$ and the closest distance between $K$ and $C$ atoms is expected to be $\geq 240\text{pm}$ (sum of van-der-Waals radii). So the actual radius of the pore seen by potassium ions is less than $3\text{Å}$ as was confirmed by our simulations. Electric fields of either $0.006$ or $0.01\text{V/Å}$ (potential drops across the cell of $0.3$ or $0.5\text{V}$) were applied in the direction normal to the graphene sheet. Two values of the fixed charge $Q_f$ at the rim were considered: $0.75$ and $2$ (in units of the electron charge $e$).

The theory of the ion current through such nanopore can be formulated in terms of the density functional [14], [15]

$$ F = \int dr \left[ \varepsilon_0 \varepsilon_r \left( \frac{i}{2} \nabla \psi \right)^2 + \varepsilon_r \psi (n_+ - n_-) - \mu_+ n_+ + \mu_- n_- - \mu_\theta n_\theta - T s \right] $$

where $\varepsilon_0$ and $\varepsilon_r$ are the vacuum and electrolyte permittivities, $n_i (i = +, -, \theta)$ are the number densities of positive ($K^+$) and negative ($\text{Cl}^-$) ions and water molecules, $\mu_i (i = +, -, \theta)$ are the chemical potentials of ions and water molecules, $T$ is the temperature and $s$ is the entropy density, respectively. Here the local electrostatic potential $\psi$ is found by solving Poisson’s equation taking explicit account of the applied potential, the fixed charge on the pore rim, the ionic distribution in the EDL, and the orientation of the water molecules in the bulk electrolyte [5].

Close to the charged nanopore, $\varepsilon_r$ and $\mu_\theta$ may each become a function of ionic position as well as of the EDL structure and the pore charge. One important mechanism of such coupling is the variation in the number and orientation of water molecules in the 1st and 2nd hydration shells of the permeating ion, in the presence of the EDL for nonzero $Q_f$. In turn, this variation affects the height of the potential barrier separating the left ($z > 0$) and right electrolytes [6], [12] which is what controls ion current through the pore.

To provide insight into this coupling we perform molecular dynamics (MD) simulations of the system in Fig. 1. The cell size of the model is $37 \times 37 \times 50\text{ Å}$, the number of water molecules is 1871, and the number of ions of each type is 36. The simulation time was $\sim 1\text{µs}$ in each case, the time step was 1 fs, and the NPT ensemble was used in the NAMD package [16]. The atomic trajectories were saved every 1 ps.

III. Electrical double layer
As discussed in the introduction a key feature of this system is the formation of the ion double layer near the graphene sheet, induced by the electric field and modified by the fixed charge of the pore. An example of the layers of potassium ions is shown in Fig. 2 for the case $Q_f = 0.75e$ and $\Delta V = 0.5\text{V}$. The 2D PDF corresponds to the distribution of potassium ions in the layer $|y| < 8\text{Å}$.

It can be seen from the figure that the layers have plane symmetry away from the charge pore. Near the pore the plane symmetry is broken and the concentration of $K^+$ ions has a well pronounced peak.

On the back plane of the figure the local structure of the EDL is shown as 1D distributions of the potassium and chloride ions in a cylinder aligned with the pore axis. The cylinder radius $R_2 \approx 6\text{Å}$ corresponds to the outer radius of the 2nd hydration shell. It can be seen from the figure that the electrolyte solutions are charged on both sides near the pore. The charge in this case can be estimated as $\sim 0.26e$ for the left ($z > 0$) and $\sim -0.05e$ for the right-hand baths. The structure and charge of the EDL is adjusted self-consistently to the changes in applied voltage and fixed charge on the rim. We therefore conjecture that the number of water molecules and their orientation in the 1st and 2nd hydration shells of $K^+$ ion crossing the pore are affected by the EDL. The coupling between the EDL, the charge of the rim, and the structure of hydration shells is nontrivial.

IV. Prehistory probability distribution
To reveal this coupling we employ a method of analysis of the permeating ion trajectories based on the prehistory probability distribution function (PPDF) [17], [18] extended to non-equilibrium systems [19]. To build the PPDF we note that, for $Q_f \leq 2e$, crossing events are well-localized in time – it takes on average less than 1 ps for potassium ion to cross the plane of the graphene sheet at $z = 0\text{Å}$. We also note that the crossings are sufficiently rare to be considered as independent events: the transition from the left ($z > 0$) to the right bath can be considered as rare escape events over a potential barrier.
with the time interval between them ranging between $\sim 14$ ns ($Q_f = 0.75e$) and 0.4 ns ($Q_f = 2e$).

It therefore becomes possible to collect the escape trajectories of potassium ions and superimpose them using the moment of the ion’s transition through the pore as a time marker. The result of such superposition for $Q_f = 2e$ is illustrated by plotting three superimposed escape trajectories at the top plane of Fig. 3. It can be seen from the figure that the transition events are indeed easily recognised sharp features of the trajectories that can be used for their efficient superposition.

To build the PPDF escape, trajectories were extended both backwards in time (hence the name “prehistory” PDF) and forwards (which for convenience we refer to as the “posthistory” PDF) from the marker by 200 ps. The resultant distribution of superimposed trajectories is shown in Fig. 3. The 1D distributions of the potassium and chloride ions in a cylinder aligned with the pore axis, shown here for $Q_f = 2e$ in the back plane, is similar to that shown in Fig. 2.

It can be seen from the figure that the local structure of the EDL has significantly changed its shape as compared to the case $Q_f = 0.75e$. We now observe two sharp peaks of potassium ions on both sides of the pore due to the strong negative charge of the pore rim. The charge of these layers is estimated as $\sim 0.42e$ for the left ($z > 0$) and $\sim 0.45e$ right baths correspondingly.

It can also be seen from the figure the PPDF has sharp peaks for permeation trajectories approaching and leaving the pore. These peaks indicate that $K^+$ ions accumulate and dwell (for $\sim 100$ ps) in the local EDL before and after escape event. This observation confirm the conjecture (see Sec. III) that the hydration shells of escaping ions can be affected by the structure of the EDL, as well as by the charge of the rim and the applied voltage. The details of these correlations can now be quantified by analysing the measured prehistory probability distribution.

V. Dehydration and water polarization in the pore

To quantify these correlations we build the most probable escape path [17]–[19] by averaging the $z$-coordinate along the PPDF. These paths are shown in Fig. 4 by black solid and thin dashed lines for $Q_f = 2e$ and 0.75e respectively. The changes in the local structure of the EDL discussed above for $Q_f = 2e$ reveal themselves as the change in slope of the mean path immediately after a permeation event and the shift of the mean path towards the graphene sheet ($z = 0$). The properties of the hydration shells can now be analyzed along the mean path.

First, we notice that the numbers of water molecules in the 1st ($N_1$) and 2nd ($N_2$) hydration shells of $K^+$ ions (shown by red and blue lines respectively in the figure) alter during the transition, as compared to their bulk values. Thus $N_2$ is reduced from 20 (in the bulk) to $\sim 12$ (in the pore) for $Q_f = 0.75e$ and to $\sim 14$ for $Q_f = 2e$. Similarly, $N_1$ is reduced from 6 to $\sim 3.8$ in the pore for both values of $Q_f$. This dehydration plays a major role in formation of the potential barrier impeding permeation events, cf [6], [12].

The effect of the local EDL structure on the number of water molecules in the hydration shells is most clearly seen in the relaxation of these numbers back to their bulk values immediately after a transition event. For $Q_f = 0.75e$ the relaxation time is shorter than 50 ps for both shells. For $Q_f = 2e$ the relaxation time is increased to $\sim 70$ ps for the 1st shell and to $\sim 160$ ps for the 2nd shell.

Next, we note the changes in the orientation of water molecules in the 1st ($\alpha_1$, see Fig. 1) and the 2nd ($\alpha_2$) hydration shells of $K^+$ ions (shown by magenta and green lines respectively in the figure) during the transition
events. Thus $\alpha_1$ is reduced from 0.22 in (units of $\pi$) to 0.15 for $Q_f = 0.75e$. One would expect that the value of $\alpha_1$ will be further reduced by increasing the $Q_f$ from 0.75e to 2e. However counter-intuitively, we observe that the value of $\alpha_1$ in the pore is 0.2 and remains nearly the same as in the bulk. The relaxation time of $\alpha_1$, towards its bulk value, after the transition event, is increased (cf. $N_1$ above), from 25 ps for $Q_f = 0.75e$, to ~70 ps.

The most significant EDL-induced changes are observed in the orientation of the water molecules in the 2nd shell. In the bulk, orientation of the water dipoles in this shell is almost tangential to the spherical surface passing through the water and centered at the ion with $\alpha_2 = 0.44$ (in $\pi$ units). For $Q_f = 0.75e$ the dipoles within the pore turn slightly towards the ion crossing the pore $\alpha_2 = 0.4$. For $Q_f = 2e$, however the changes in the orientation are reversed and the dipoles turn away from the ion with $\alpha_2 = 0.51$. This flipping can be attributed to the fact that the 2nd hydration shell extends all the way to the local EDL of potassium ions, and the accumulation of these ions near the pore forces the water dipoles to turn away from the ion in the pore.

VI. Conclusions

In conclusion, we have investigated the distribution of trajectories of potassium ions crossing a nano-pore in a single graphene sheet. We observed correlations between the number and orientation of water molecules in the 1st and 2nd hydration shells of these ions and the structure of the EDL formed near the pore, due to applied electrostatic potential and the charged rim of the pore.

In particular, it was found that the shape of the most probable escape trajectory and the relaxation time towards bulk values of the number of molecules in both shells, and of orientation of waters in the 1st shell, are significantly affected by the changes in the EDL induced by increasing the charge of the pore from 0.75e to 2e. The same changes in the EDL structure force the orientation of the water dipoles in the 2nd hydration shell to flip from being oriented towards the pore center at 0.75e to orientation away from the pore center at 2e.

The observed features shed a new light on the influence of the EDL on the spatial dependence of the excess chemical potential and the effective dielectric constant seen by ions transiting the pore, and will help in the development of self-consistent density functional theories of ion currents through nanopores.

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References

Resistivity Characteristics and Noise Spectroscopy of Composites with Carbon Fiber Felts

Marina Tretjak  
Institute of Applied Electrodynamics and Telecommunications  
Vilnius University  
martina.tretjak@ff.vu.lt

Sandra Pralgauskaite  
Institute of Applied Electrodynamics and Telecommunications  
Vilnius University  
sandra.pralgauskaite@ff.vu.lt

Jonas Matukas  
Institute of Applied Electrodynamics and Telecommunications  
Vilnius University  
jonas.matukas@ff.vu.lt

Ieva Kranauskaitė  
Institute of Applied Electrodynamics and Telecommunications  
Vilnius University  
ieva.kranauskaite@ff.vu.lt

Jan Macutkevič  
Institute of Applied Electrodynamics and Telecommunications  
Vilnius University  
jan.macutkevic@ff.vu.lt

Jūras Banys  
Institute of Applied Electrodynamics and Telecommunications  
Vilnius University  
juras.banys@ff.vu.lt

Vanessa Fierro  
Institut Jean Lamour, UMR CNRS  
Univ Lorraine, EPF Saclay, CNRS, Epinal, France  
vanessa.fierro@univ-lorraine.fr

Alain Celzard  
Institut Jean Lamour, UMR CNRS  
Université de Lorraine, CNRS, Epinal, France  
alain.celzard@univ-lorraine.fr

Blagoy Karakashov  
Institut Jean Lamour, UMR CNRS  
Université de Lorraine, CNRS, Epinal, France  
blagoy.karakashov@univ-lorraine.fr

Abstract—Resistivity and low frequency (from 10 Hz to 20 kHz) noise characteristics of composites materials with carbon fiber felts of two types: PAN (polyacrylonitrile) and Rayon (regenerated cellulose), have been carried out. Measurements were performed in the temperature range from 73 K to 380 K. Resistivity of the investigated materials is a constant at lower voltage and starts to decrease above (0.1-1) V due to increased charge carriers tunneling. The spectrum of low-frequency noise consists of 1/fα type components over the entire range of measured frequencies and the noise spectral density is proportional to the voltage square. The observed fluctuations in the investigated materials are resistance ones. The resistivity and low frequency noise intensity of the investigated composites is almost constant in temperature range from 75 K to 250 K, their increase above 250 K is caused by the matrix expansion and decrease above 360 K is due to an onset of the electrical conductivity in the matrix. In the (307-332) K temperature range a kink in resistivity and noise characteristics was observed. And it is determined by reduction of carbon fiber dimensions. This shrinkage of carbon fibers does not introduce additional noise origins in the investigated materials.

Keywords—bisphenol A, carbon felt, composite, noise, resistivity

I. INTRODUCTION

Carbon-epoxy composites are widely used in various modern sophisticated technologies: as functional layers of electronic devices, in the aviation industry as a solid protective material, in robotic structures and automotive components, etc. [1-3]. Composites with carbon fibers are more wanted than other high-performance fiber composites because of their general stability of characteristics in the most difficult operating conditions: mechanical load, humidity and corrosive environment, radiation, elevated temperature, thermal cycles [4-6]. Materials with carbon fibers are widespread in fabrication of structures with special mechanical characteristics [3, 7]. Carbon fiber reinforced composites exhibit a very high specific strength and elastic modulus, and their weakess lies in their sensitivity to mechanical damage. To achieve the high strength, the composite material is heated and its structure and properties are obtained in the field of mechanical stress. When composites are subjected to mechanical stress, an interlayer stratification is observed [5]. Therefore, when carbon fibers or polymer composite materials based on them are heated, a decrease in linear dimensions of the fibers is observed in the direction of reinforcement [7].

An important issue of the carbon composites employment in electronic devices is charge carrier transport mechanisms in such materials, their dependencies on type of the carbon filler, filler density, dimensions, distribution, orientation etc. In most composites, where the matrix is a dielectric substance and the filler is conductive, two types of conductivity can be expected: charge carriers movement inside the conductor filler and charge carrier tunneling through the dielectric matrix between the filler particles [8, 9]. One of the most sensitive methods to study charge carrier transport and related physical processes in various materials is low frequency noise spectroscopy [10-13]. Nevertheless, there are not many papers on the low frequency noise investigation in carbon composite materials. The spectral analysis of low frequency noise is important as fixing noise spectrum type and intensity can determine the prospect of using such materials in electronic and sensory applications, since noise is a limiting factor for signal detection.

To study a composite with new proportions of fillers and their manufacturing techniques, it is necessary to make many different measurements. This paper looks at the analysis of the low-frequency noise and resistivity characteristics of composite materials with carbon fiber felts, which help to clarify the charge carrier transport mechanisms and their variation during structural changes (due to temperature changes) of the composite.

II. MATERIALS

The investigated materials are composites with carbon fiber felts. The binding matrix is bisphenol A epoxy resin. The used graphitized soft carbon felts are anistotropic materials and comprise of randomly laid/dispersed fiber layers, consolidated by needle-punching. The SEM micrographs of the materials are presented in Fig. 1 (a) and (b). The sample’s x- and y-directions are fully equivalent and define the “in-plane” direction, i.e., the bedding plane of the fibers within which no...
preferential orientation exists. The orthogonal direction, $z$, is the “out-of-plane” direction of the material.

The results presented in the paper were carried out for materials with two different types of carbon fibers: Rayon (regenerated cellulose) and PAN (polyacrylonitrile). The cross sections of the both types of fibers can be seen in the SEM micrographs in Fig. 1 (c) and (d). Deviations from circular cross-section were much more obvious in the case of Rayon-derived carbon fibers than for PAN-derived ones.

Samples which have the same filler were cut out from one piece of the fabricated material. The difference between samples “PAN II” and “PAN X” and between “Rayon II” and “Rayon X” is in the contact plane: the “II” samples have contacts at the $xy$ plane while the “X” ones - at the $xz$ plane. The fiber cross-section diameter, bulk density, skeletal density and overall porosity are presented in Table 1.

### III. MEASUREMENT TECHNIC

The low frequency (from 10 Hz to 20 kHz) noise and resistivity characteristics were measured at room temperature and in temperature range from 78 K to 380 K. The noise measurements were carried out in a specially shielded laboratory room (Faraday cage) to avoid the interference from electrical network and communication systems.

The measured noise signal was processed by low-noise amplifier, filter system, and analogue digital converter (the measuring scheme is shown in Fig. 2). Noise signal spectrum was obtained by the fast Fourier transform, and voltage noise spectral density, $S_U$, was evaluated by comparing with the thermal noise of the standard resistor, which was at least 100 times larger comparing to the resistance of the sample under test. The equation for $S_U$ is:

$$S_U = \frac{\overline{U^2} - \overline{U_0^2}}{U_0^2} 4kT_0 R_{load};$$

where $\overline{U^2}$, $\overline{U_0^2}$ and $\overline{U_0^2}$ are the sample (and the measuring system), the measuring system, and the standard (load) resistor (and the measuring system), respectively, noise variances in the narrow frequency band $\Delta f$; $T_0$ is the absolute temperature of the standard resistor, and $k$ is the Boltzmann constant.

The sample resistance was measured by the semiconductor device parameter analyzer “Keysight Technologies B1500A”.

### IV. RESULTS AND DISCUSSION

The resistivity characteristics were measured not exceeding 100 mA current in order to avoid a breakdown of the sample. The resistivity dependencies on voltage at room temperature are presented in Fig. 3. The resistivity of the investigated materials does not vary with voltage at low its values. Above the certain voltage (approximately (0.1-1) V), a deviation from the Ohmic resistance is observed (Fig. 3). Depending on the sample resistivity, the variation in the voltage of the start of the nonlinear characteristic is not large: the slope for the investigated materials is 0.985. Two factors can be indicated as origin of this non-linear current-voltage characteristic: in one case, the conductive particles may be non-linear in nature [14], in the other case, the macroscopic conductivity becomes not Ohmic due to the possible appearance of additional conductive channels (e.g., charge carriers tunneling through the dielectric matrix) that arise at higher electrical field [15, 16].

The resistivity of the samples depends on the measurement direction (in relation to the carbon fibers direction) as well as on the diameter of the carbon fiber. Carbon fibers roughly can be treated as having metallic conductivity. PAN fibers are
larger in diameter than Rajon ones. Hence, composites with PAN fibers filling are better conductors and the obtained results confirm this (Figs. 3 and 4). It was also observed that resistance of the samples with contacts applied along the carbon fibers is larger than for samples with contacts in the perpendicular direction to the fibers. This is caused by the fact that in the composite sample resistance for current flowing along the conductive fiber is smaller comparing to the case of charge carrier tunneling from fiber to fiber through a non-conducting matrix (what takes place when current flows in the perpendicular direction to the fibers).

Temperature characteristics of resistivity from 75 K to 380 K are presented in Fig. 4. The resistance from 75 K to 225 K varies very slightly with temperature. The resistivity decreases with temperature for Rayon samples below 250 K due to the electron tunneling, while for PAN ones the resistivity is much smaller and electron tunneling is less important. Starting from 275 K the resistance grows with temperature more steeply. Above 360 K a decrease of resistance with temperature increase is observed. Such resistance dependence on temperature is explained by the expansion of the matrix (resistance growth in ((275-360) K)) and by the appearance of electrical conductivity in the matrix (resistance decrease above 360 K) [17].

Temperature characteristics of the investigated materials have a kink in the temperature range from 307 K to 332 K (Figs. 4 and 5). The appearance of this kink can be caused by two origins: decrease in the linear dimensions of the carbon fiber in the reinforcement direction of the composite and the glass transition of the epoxy matrix of the sample. In the first case, reduced size of the conductive particles in a non-conductive matrix creates an increase of resistance. When repeating measuring cycles the carbon fibers lose their ability to shrink [7]. As it is shown for sample PAN X in Figs. 4 and 5, the kink maximum decreases after the repeating measurements. The glass transition can have a small impact to the composites resistivity only as a softening of the matrix, what creates a better environment for the carbon fiber shrinkage, but this should not have a significant effect on the resistance in temperature range from 307 K to 332 K.

Measurements of the low-frequency noise spectra have showed that investigated materials have a 1/f\(^{\alpha}\) type spectrum in all investigated temperature range (Figs. 6 and 7). As well known, there are several theories explaining the nature of 1/f\(^{\alpha}\) noise, however, for composite materials the most suitable one is based on the superposition of many processes with widely distributed characteristic times that randomly change number of free charge carriers in the sample - superposition of many charge carrier capture and release processes. It was observed that at room temperature noise spectra are mostly 1/f type in all investigated frequency range – the slope is almost constant and does not depend on...
voltage (Fig. 6). While at lower temperature approaching the lower frequencies $1/f^\alpha$ type spectrum slightly changes to $1/f^\alpha$ form, where $\alpha \neq 1$ (Fig. 7). This indicates that at lower temperature the deeper charge carrier capture centers are detected at lower frequency.

The noise spectral density is proportional to the square of voltage (Fig. 8). And the voltage noise spectral density dependency on temperature roughly repeats the resistivity characteristic (Fig. 5). Such noise spectral density features are characteristic for resistance fluctuation. Noise spectral density proportionality to voltage square in all investigated voltage range shows that tunneling that’s probability increases with voltage increase is not significant, nevertheless, small decrease of resistivity was observed above (0.1-1) $V$ (Fig. 3).

Therefore, the dominant charge carrier transport mechanism is charge carriers generation and recombination processes in the carbon fibers.

![Fig. 8. Voltage noise spectral density dependencies on voltage of PAN and Rayon composites at room temperature at 968 Hz frequency.](image)

In temperature range (307-332) K, where the kink in resistivity and noise intensity was observed (Figs. 4 and 5). noise spectra stay $1/f$ type, what shows that physical processes that cause this kink in temperature characteristics do not introduce new noise origins. Noise characteristics compliance with resistivity dependencies indicate that in the investigated samples resistance fluctuation due to the fluctuation of number of free charge carriers is the main noise origin.

V. CONCLUSIONS

Low frequency noise and resistivity characteristics of composite materials: bisphenol A epoxy resin reinforced by the carbon fiber felts (PAN and Rayon), have been investigated in a wide temperature range. Low frequency voltage fluctuations in the investigated materials are $1/f^\alpha$ type and are caused by the resistance fluctuations due to random charge carrier capture and release processes in centers inside the carbon fibers. Tunneling through the dielectric matrix is small and have no effect on noise characteristics.

The samples with contacts deposited in perpendicular direction to the fibers are more conductive and demonstrate lower $1/f^\alpha$ type noise comparing with samples with contacts applied along the carbon fibers.

An observed kink in the resistivity and noise characteristics in (307-332) K temperature range is caused by the reduction of carbon fiber dimensions. This shrinkage of carbon fibers does not introduce additional noise origins in the investigated materials.

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Doubly-clamped pre-stressed silicon nitride string resonators excel as high Q nanomechanical systems enabling room temperature quality factors of several 100,000 in the 10 MHz eigenfrequency range. Dielectric transduction ideally complements the silicon nitride strings, providing an all-electrical control scheme while retaining the large mechanical quality factor [1-3]. It is mediated by an inhomogeneous electric field created between adjacent electrodes. The resulting gradient field provides an integrated platform for actuation, displacement detection, frequency tuning as well as strong mode coupling. Dielectrically controlled silicon nitride strings are an ideal testbed to explore a variety of dynamical phenomena ranging from multimode coupling to coherent control. Here I will focus on the nonlinear dynamics of a strongly driven string. The response of the string is described by the cubic nonlinearity of the well-established Duffing model. As a result of the large mechanical quality factor, driving the string results in classical squeezing of its thermal fluctuations as theoretically predicted [4]. The power spectrum of the driven resonator reveals two well-resolved satellite peaks, the areas of which encode the squeezing ratio [5], which can thus directly be extracted from the power spectrum without the need to perform a homodyne measurement. Further insights into noise-induced nonlinear dynamics include nonlinear switching between the two stable solutions of the Duffing resonator.

Keywords—nanomechanical systems, nanoelectromechanics, Duffing model, thermomechanical fluctuations, thermal squeezing, thermally induced switching

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Analysis of the ultimate noise performance of a mesoscopic cavity magnetic sensor

Massimo Macucci, Paolo Marconcini
Dipartimento di Ingegneria dell’Informazione - Università di Pisa
via Girolamo Caruso 16, 56122 Pisa, Italy
email address: macucci@mercurio.iet.unipi.it

Index Terms—1/f noise, magnetic sensors, III-V semiconductor materials

I. ABSTRACT
Transport through a mesoscopic cavity divided into two identical dots by a tunnel barrier is characterized by constructive interference among symmetric paths, which has the effect to enhance the transmission through the cavity itself. Since this symmetry can be broken by a magnetic field, this device can be used as a magnetic field sensor. We first discuss a tradeoff for the definition of the most convenient geometrical dimensions and then, with the inclusion of the variance due to electrical noise (specifically flicker noise) we find the maximum sensitivity that can be achieved with this device.

II. INTRODUCTION
A particular conductance enhancement effect that we discovered a few years ago [1] can in principle be applied to the implementation of magnetic field sensors. The effect is somewhat counterintuitive and consists in the strong enhancement of conductance through a barrier inserted in a 2-dimensional electron waveguide when two symmetric constrictions are added around the barrier [2] (see Fig. 1). While a first guess would be that the addition of the constrictions lowers the conductance, it can indeed increase it by more than an order of magnitude, as long as such constrictions are located exactly at the same distance on the two sides of the barrier. This conductance enhancement effect is the result of the constructive interference of symmetric paths. Thus it is sensitive to phase coherence, thereby setting a limit to the maximum size of the cavity for which the enhancement can be observed. Considering that many paths undergo a number of reflections within the cavity, the mean free path must be much longer than the cavity. Considering a mean free path of about 160 µm (corresponding to the highest mobility achieved for a 2-dimensional electron gas of $2 \times 10^7$ cm²/(V s)) the maximum cavity length for which a significant conductance enhancement effect is still visible is of the order of 10 µm (allowing about 8 complete roundtrips in the cavity). Magnetic field breaks the symmetry of the paths [2] and therefore quenches the conductance enhancement effect: thus detection of a magnetic field is possible by looking at the conductance through a geometrically symmetric device. The conductance will have a maximum for zero magnetic field and will decrease as the magnetic field is increased. The achievable sensitivity is limited by the signal-to-noise ratio: here we perform an evaluation of the different noise contributions (thermal noise, shot noise, flicker noise) and determine the minimum value of the magnetic field that can be measured with a given accuracy.

The physical implementation of the device that we consider is based on a 2-dimensional electron gas in a GaAs/AlGaAs heterostructure, because of its high mobility: as already mentioned, it is essential to achieve the largest possible mobility since this allows the conductance enhancement effect to survive in the relatively large cavities that are needed to maximize the sensitivity. Indeed, as we will show in the following, the response of the device is substantially proportional to the magnetic flux threading the cavity, which implies that a tradeoff must be reached between two opposing requirements: the cavity should be large to thread more magnetic flux while it should be small to preserve phase coherence.

The cavity walls, as well as the barrier, can be defined with metal gates, which deplete the 2-dimensional electron gas outside the cavity and create the potential barrier. Such an implementation of the cavity has an important advantage with respect, for example, to a cavity obtained by means of etching: unavoidable geometrical asymmetries can, to a good extent, be compensated for by properly tuning the gate bias voltages [3]. Furthermore, also the height of the barrier can be controlled by tuning the voltage applied to the corresponding gate.

For all of the simulations that we will be presenting, we will assume a Fermi level of 10 meV, which is typical for the 2-dimensional electron gas in a GaAs/AlGaAs heterostructure.

III. CAVITY GEOMETRY AND NUMERICAL TECHNIQUE
As shown in Fig. 1, the cavities that we will be considering are rectangular in shape, defined by means of two constrictions. The choice of a rectangular shape and of hard walls
leads to a significant reduction of the computational burden, while not appreciably affecting the results.

Also the constrictions are abrupt and with hard walls, and their extension in the longitudinal direction is not relevant, as long as it is large enough to make coupling via evanescent modes negligible.

The barrier is assumed to be uniform in the transverse direction and to have a rectangular cross-section in the longitudinal direction.

We compute the conductance through the cavity as a function of the magnetic field, keeping the Fermi level constant at the previously mentioned value of 10 meV. Calculations are performed at zero temperature within the Landauer-Büttiker approach to transport in mesoscopic systems. The evaluation of conductance is thus turned into the solution of a scattering problem across the structure. Considering that a bias of a few millivolts between the input and output leads of the cavity will be needed to reliably measure the device conductance, the transmission calculation is performed by averaging a few tens of energy values over an interval of a few millielectronvolts. This smooths out the transmission curves, which would otherwise present some rapid oscillations, as a result of the very large number of modes propagating in the cavity (of the order of 180).

The transmission is computed with a recursive Green’s function approach [4]–[6], operating in real space in the longitudinal direction and in the mode space in the transverse direction. The cavity is subdivided into a series of slices, within each of which the potential can be assumed to be constant. In the absence of magnetic field, just 5 sections plus the input and output leads would be sufficient (2 for the constrictions, 2 for the halves of the cavity and 1 for the barrier), but when magnetic field is present we need a much larger number of slices. This is because we choose a gauge in which the only non-vanishing component of the vector potential is along the transverse direction [7]: on the one hand this makes the calculation of the transverse eigenfunctions much easier than with different choices of gauge (in this case the transverse eigenfunctions are obtained multiplying the ones for no magnetic field by a proper phase factor); on the other hand this approach requires that each slice is threaded by a flux less than a flux quantum, and therefore we had to use up to a few hundred slices. The magnetic field is assumed to be applied to the constrictions, the cavity, and part of the leads, while it is set to zero at the lead locations between which transmission matrices are computed, in order to make their computation easier. Between such lead locations and the constrictions the magnetic field is ramped linearly.

IV. CHOICE OF CAVITY PARAMETERS

As discussed in the introduction, the choice of the cavity size is a result of a trade-off between the requirement of threading the largest possible magnetic flux and that of preserving phase coherence even for paths that undergo a relatively large number of reflections. In Fig. 2 we report the conductance through cavities 4 \( \mu \text{m} \) wide and with a length of 4 \( \mu \text{m} \) (blue curve), 8 \( \mu \text{m} \) (red curve), and 16 \( \mu \text{m} \) (green curve) as a function of magnetic field. The constrictions are assumed to be symmetric, 340 nm wide and 400 nm long, while the barrier is 12 nm thick and with a height of 45 meV. It is apparent that increasing the length of the cavity the slope of the transmission vs. magnetic field increases, but it is not recommendable to go beyond 8 \( \mu \text{m} \), even at very low temperatures, in order to prevent excessive decoherence. We then try varying the width of the barrier, keeping, for the rest, the same parameters as in the previous calculation. In Fig. 3(a) we report the computed transmission vs. magnetic field for a 4 \( \mu \text{m} \times 8 \mu \text{m} \) cavity for different barrier thicknesses: 8 nm (blue curve), 12 nm (red curve) and 16 nm (green curve). It is apparent that a thicker barrier leads to a reduced slope, but, at the same time, the conductance enhancement ratio for the 8 nm barrier is decreased, therefore for our evaluations we choose a 12 nm thick barrier.

We have further explored the parameter space looking at the behavior of the conductance as a function of magnetic field for different values of the constriction width. Results are reported in Fig. 4: we notice that the largest conductance enhancement is observed for a width of 340 nm, while for smaller (200 nm) or larger (1000 nm) widths the conductance drops. Further increasing or decreasing the width of the constrictions a strong reduction of the conductance is observed. On the basis of the results shown so far, we have decided to take into consideration, for our analysis, a cavity that is 4 \( \mu \text{m} \) wide, 8 \( \mu \text{m} \) long, with 340 nm constrictions, a 12 nm thick and 45 meV high barrier.
V. Signal to Noise ratio and sensitivity

We take into consideration thermal, shot, and flicker noise. Thermal noise has a power spectral density $S_{th} = 4kT R_{dev}$ (which depends only on temperature and on the value of the device resistance $R_{dev}$). Since the output signal consists of the current flowing for a given applied bias voltage, it can be increased proportionally to the bias voltage, while thermal noise is not affected by the bias voltage, unless a value causing non-negligible self-heating is reached. Thus thermal noise does not represent an actual limiting factor for sensitivity.

Let us then consider the contribution of shot noise: the noise power spectral density $S_{sh} = 2qI$ increases linearly with the bias current, while the signal power increases quadratically. Thus the signal-to-noise ratio associated with shot noise increases as the bias voltage is increased.

Finally, we consider the contribution from flicker noise: the flicker noise power spectral density increases quadratically with the bias current (according to the Hooge relationship: $S_{fl} = \alpha H I^2/(Nf)$, where $\alpha H$ is the Hooge constant, $I$ the bias current and $N$ the number of charge carriers). As a result, the flicker noise power spectral density has the same, quadratic, dependence on the applied voltage, and therefore the signal-to-noise ratio cannot be improved by increasing the bias voltage. Thus flicker noise represents the fundamental limitation to the achievable sensitivity. In particular, it has to be pointed out that, contrary to what can be sometimes found in the literature, using an AC bias does not help, because in such a case the $1/f$ spectrum will just be translated around the bias frequency. At zero frequency (or around the frequency of the AC bias) the spectrum diverges, but this does not involve that the noise power will become infinite, because the observation time is finite. A finite observation time implies that the contribution from the lower frequency components is limited, and therefore a finite variance on the measured value is achieved. The standard deviation (i.e. the square root of such a variance) can be evaluated with the approach due to Allan [8], [9], which, in the case of flicker noise with a power spectral density $S_{fl} = \theta/f$ is given by

$$\sigma = \sqrt{1.386 \times \theta}$$

We notice that the slope of the transmission curve vanishes when approaching the origin, which leads to a loss of sensitivity for very small values of the magnetic field. This can be circumvented by applying a small constant magnetic field bias of about 100 $\mu$T, in such a way as to operate in the region with the largest derivative.

Unfortunately, it has not been possible to find in the literature a direct measurement of the flicker noise power spectral density for a device such as the one we have presented or just for a mesoscopic cavity. However, we can perform some estimate of the low-frequency noise in this structure. First of all, we expect, based on Hooge’s relationship, that the most relevant contribution to flicker noise will come from the regions with the smallest number of carriers, therefore from the constrictions. To obtain an estimate of the conduction noise we can either compute the number of carriers therein and use Hooge’s formula, with a value for $\alpha H$ taken from the literature for a 2-dimensional electron gas in a GaAs/AlGaAs heterostructure, or collect from the literature the flicker noise data measured on quantum point contacts obtained for GaAs/AlGaAs heterostructures. In a previous study [10] we used the former approach, but we believe the latter to be more reliable, because a value of the Hooge coefficient obtained in rather different experimental conditions cannot be applied with confidence to our problem. From Ref. [11] we find that $S_{fl}/I^2 = 9 \times 10^{-22} \text{A}^2/\text{Hz}$ at 100 Hz. Thus, for a current of, for example, 0.3 $\mu$A the spectrum of the flicker noise power spectral density should be $S_{fl} = 9 \times 10^{-22} \text{A}^2/\text{Hz}$.

In order to compute the resulting overall noise, we can represent the device as the series of two noisy resistors, representing the two constrictions, separated by a resistor representing the mesoscopic cavity (see Fig. 5). Notice that since the cavity contains a much greater number of carriers than the constrictions, according to Hooge’s formula we can neglect its noise contribution and represent it with a noiseless resistance. If the left and right contacts of the sensor are connected through an external conductor, the noise in this conductor will result from the superposition of the effects of the flicker noise generators (assumed to be uncorrelated).
corresponding to the two constrictions (with power spectral density $S_{1_L}$ and $S_{1_R}$, respectively). Therefore, the noise power spectral density in the outer conductor $S_{\text{out}}$ will be equal to

$$S_{\text{out}} = S_{1_L} \left( \frac{R_L^2}{R_L + R_C + R_H} \right)^2 + S_{1_R} \left( \frac{R_R^2}{R_L + R_C + R_H} \right)^2. \quad (2)$$

If the constrictions are identical, we can consider $R_L = R_R = R$ and $S_{1_L} = S_{1_R} = S_\theta$. A worst case estimate can be obtained neglecting $R_C$: in this case, we have that $S_{\text{out}} = S_\theta / 2$, which in our case (with a current of 0.3 µA) means $S_{\text{out}} = \theta / f = 4.5 \times 10^{-22} \text{A}^2 / \text{Hz}$. Using Allan’s approach, this corresponds to a standard deviation $\sigma = \sqrt{1/3} \times 0.3 = 25 \text{ pA}$.

As we have previously anticipated, it is preferable to operate the device with a small constant magnetic field bias of about 100 µT, in order to be in condition for which the transmission curve as a function of magnetic field has the largest derivative $\Delta G/\Delta B \approx -1.25 \Omega^{-1} / \text{T}$, from Fig. 4.

For this value of the magnetic field we have a conductance $G_0 = 0.15 \text{mF}^{-1}$, which means that, in order to obtain the considered bias current $I_0 = 0.3 \text{ µA}$, a bias voltage $V_0 = I_0 / G_0 = 2 \text{ mV}$ has to be applied to the sensor.

With the addition of noise, the measured current values will be scattered, but will mainly fall within an interval of ±3σ, i.e. in our case 75 pA, around the expected value. Therefore, an accuracy of 10% in the measurement will be obtained only for $\Delta I > 10.75 \text{ pA} = 0.75 \text{ nA}$. The current variation $\Delta I$ is related to the conductance variation $\Delta G$ and thus to the magnetic field variation $\Delta B$ that has generated it by the following relation:

$$\Delta I = V_0 \Delta G = V_0 \frac{\Delta G}{\Delta B} \Delta B. \quad (3)$$

Substituting the values of $\Delta I, V_0$ and $\Delta G/\Delta B$, we obtain a value $\Delta B \approx 300 \text{ nT}$, which therefore represents an estimate of the sensitivity of the sensor.

VI. Conclusion

We have estimated the maximum sensitivity achievable with a magnetic field sensor consisting of a mesoscopic cavity with a potential barrier in the middle. The constructive interference among paths, which, in the presence of phase coherence, dominates transmission in this geometrically symmetric device are is progressively washed out by the presence of an orthogonal magnetic field. We have first focused on the choice of a geometry suitable for the best detection sensitivity, by reaching a tradeoff between the requirement of collecting the largest possible magnetic flux and the requirement of preserving phase coherence for the paths that perform up to about 15 bounces within the cavity. We have then made an estimate of the power spectral density of flicker noise in the considered device, after reaching the conclusion that it represents the main limitation to the achievable sensitivity. Using an approach based on Allan’s variance we have finally evaluated a maximum possible sensitivity of the order of 300 nT.

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An Itô-Stratonovich dilemma-free treatment for nonlinear oscillators with colored noise

Michele Bonnin, and Fabrizio Bonani
Department of Electronics and Telecommunications
Politecnico di Torino, Turin, Italy

Fabio L. Traversa
MemComputing Inc.
San Diego, CA, USA

Abstract—We present here an innovative treatment of nonlinear oscillators subject to colored noise sources, namely Lorentzian fluctuations. Exploiting averaging techniques, the full system is transformed into an equivalent oscillator with white noise sources in the limit of small noise correlation time, thus avoiding the Itô-Stratonovich ambiguity. The equivalent noisy oscillator is then transformed into a phase-amplitude model, thus simplifying phase noise analysis. The soundness of the approach is demonstrated for a 2 dimensional example.

Index Terms—Oscillator noise, phase noise, colored noise, stochastic averaging, phase models

I. INTRODUCTION

Phase noise, in the analog world, and its time-domain counterpart time-jitter (more commonly used for digital applications) are essential figures of merit in assessing the quality of oscillators. In biological and neural systems, phase noise plays a fundamental role explaining synchronization/desynchronization processes which, in turn, influence neural information processing. In electronic systems, phase noise is responsible for errors in bit transmission rate, modulation and demodulation of data. As a consequence the study of oscillator phase noise plays a major role.

The physics and, as a consequence, the modeling of oscillators subject to white Gaussian noise is rather well understood. However, real world random fluctuations are characterized by a finite, non null time-correlation, i.e. the power spectral density (PSD) is often characterized by a larger value at low frequency. A more realistic description of such a perturbation is represented by an exponentially correlated process, known as colored (Lorentzian) noise.

In this contribution we present a novel approach for phase noise analysis in nonlinear oscillators subject to colored noise. The model we study can be decomposed into two parts: the oscillator dynamics and the stochastic process modeling noise. We consider oscillators of a generic order \( N \), subject to colored noise that can be either modulated (multiplicative) or un-modulated (additive). We do not impose any \textit{a priori} restriction on the noise intensity. The colored noise is modeled as an Ornstein-Uhlenbeck process (OUP), where the OUP is expressed as the solution of a linear stochastic differential equation (SDE) with an un-modulated (additive) white Gaussian noise. OUP has exponentially decaying expectation value and correlation, and it is characterized by a Lorentz (Cauchy) distribution. The only assumption we use is that the noise correlation time is small, although not vanishingly small (in the zero correlation time limit, OUP reduces to white noise).

Using a method proposed in [1], [2] the oscillator with colored noise is first transformed into an equivalent system subject to white Gaussian noise. In other words, the \( N + 1 \) dimensional, coupled SDEs describing oscillator dynamics and OUP are reduced to an \( N \) dimensional SDE describing the evolution of an equivalent nonlinear oscillator subject to white Gaussian noise. The solution of the reduced oscillator model converges weakly to the solution of full system, in the sense that all of the statistical properties of the exact (strong) solution are retained by the reduced system: As a consequence, for most practical applications this information is quite adequate. The advantage in considering the reduced system is twofold. First, not only numerical simulations are greatly simplified, because the OUP SDE doesn’t need to be solved, but also expected quantities and other useful information can be found by direct computation on the reduced system applying stochastic calculus. Second, the transformation resolves the Itô-Stratonovich dilemma [3]: even if the original system (oscillator + OUP noise modeling) has unmodulated (additive) noise, any order reduction and/or nonlinear coordinate transformation introduces noise modulation, making the noise source multiplicative [4], [5]. This implies that the resulting SDE yields different results depending on whether it is interpreted as a Stratonovich or an Itô SDE.

Exploiting Floquet theory [6], the reduced system with white noise is then transformed into a phase-amplitude model [7]. The phase variable describes a random walk process along a direction tangent to the limit cycle of the unperturbed oscillator, while the amplitude describes motion transversal to the cycle. The main findings that we provide are twofold: 1) the transformation into an equivalent system with white Gaussian noise highlights the effect of finite noise correlation time; 2) the transformation to amplitude and phase equations greatly simplifies phase noise analysis, and represents the ideal starting point to derive further simplified phase models.

II. INFLUENCE OF COLORED NOISE

We consider a nonlinear oscillator subject to colored noise modeled as an Ornstein-Uhlenbeck process. Adopting standard notation for stochastic differential equations, we write the
system in the form
\begin{align}
\dot{x}_t &= [a(x_t) + B(x_t)\eta_t] \, dt \quad (1a) \\
\tau d\eta_t &= -\eta_t \, dt + DdW_t \quad (1b)
\end{align}
where $x_t : \mathbb{R} \mapsto \mathbb{R}^n$ denotes the state of the system, $a : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a smooth vector field that defines the system internal dynamics, $B : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a smooth vector valued function representing noise modulation, $\eta_t : \mathbb{R} \mapsto \mathbb{R}$ is a scalar function describing the source fluctuations, both internal and external, and $W_t$ denotes a Wiener process, i.e. the integral of a white noise. Finally, parameters $\tau$ and $D$ represent the noise correlation time and diffusion coefficient, respectively. It is worth noting that equation (1) describes a diffusion process with unmodulated white Gaussian noise $dW_t$. In the limit of uncorrelated noise $\tau \to 0$, (1) reduces to a SDE describing a nonlinear oscillator subject to white Gaussian noise
\begin{equation}
\dot{x}_t = a(x_t) \, dt + DB(x_t) \, dW_t \quad (2)
\end{equation}
However the problem arises about the correct interpretation of (2) (Stratonovich, Itô or others), because white Gaussian noise is now multiplied by the modulating function $B(x)$. The problem can be tackled by applying the procedure described in [1], [2]. Using that method, it can be shown that in the limit of short correlation time $\tau$, the solutions of (1) converge in probability to the solutions of the Stratonovich SDE
\begin{equation}
\dot{x}_t = a(x_t) \, dt + DB(x_t) \circ dW_t \quad (3)
\end{equation}
or to the solutions of the equivalent Itô SDE
\begin{equation}
\dot{x}_t = \left[ a(x_t) + \frac{D^2}{2} \frac{\partial B(x_t)}{\partial x} - B(x_t) \right] \, dt + D \, BB(x_t) \, dW_t \quad (4)
\end{equation}
where, adopting standard the notation, we used symbol $\circ$ to denote Stratonovich stochastic integration. A short explanation is required. Convergence in probability, or weak convergence, means that the solutions of two equation for a specification realization of the stochastic process differ in details, but they have the same statistical properties. From the point of view of practical applications, a weak solution is all is needed, because the user is typically interested in expected quantities. Equivalence of (3) and (4) means that the two equations, although being solved using two different definitions of stochastic integration, have exactly the same solution. As a consequence, it is just a matter of personal taste and practical convenience which interpretation should be preferred. Here, we prefer Itô interpretation and thus we shall consider (4). This choice will simplify the calculation of expected quantities and numerical simulations.

Equation (4) casts light on the effect of colored noise with respect to white noise. The last term in the equation, $DB(x_t) \, dW_t$, represents a diffusion contribution. By contrast the term $(D^2/2) \partial B(x_t) / \partial x \cdot B(x_t) \, dt$ represents an additional drift, that can be ascribed to the finite, non null noise correlation time.

III. Phase dynamics

The concept of phase is one of the most important in the theory of oscillators. While it seems trivial to define the concept of phase for linear second order oscillators, great attention must be paid to extend the same concept for higher order, nonlinear systems. The most general and fruitful way to define the phase of a nonlinear oscillator is based on the concept of asymptotic phase, or the phase function.

In the absence of perturbations, a nonlinear oscillator exhibits a $T$-periodic solution, represented by a limit cycle $x_i(t)$ in its phase space. In order to define the phase of a point $x_i$ in the basin of attraction of the limit cycle, let us introduce a phase function $\phi : \mathbb{R}^n \to [0, 2\pi)$. We define a reference initial condition $x_0 \in x_i(t)$, and we assign $\phi(x_0) = 0$. Let $\tilde{x}(t, x_0)$ denote the trajectory of the unperturbed system at time $t$ with initial condition at $x_0$. Obviously, $\tilde{x}(t, x_0)$ solves $\dot{x}(t) = a(x(t))$ and $\phi(0, x_0) = x_0$. The phase of the trajectory $\phi(t, x_0)$ is $\phi(\tilde{x}(t, x_0)) = 2\pi t/T = \omega_0 t$, where $\omega_0$ is the oscillator free running angular frequency. In other words, the phase function represents a re-parametrization of the limit cycle. The concept of phase can be extended to the basin of attraction of the limit cycle $B(x_i)$, introducing isochrons. Isochrons are $(n−1)$-dimensional manifolds (surfaces) transverse to the limit cycle, representing the set of initial conditions $x_0 \in B(x_i)$ such that the trajectories leaving from $x_i$ meet asymptotically on the limit cycle $x_i(t)$. Mathematically, the isochron transversal to the limit cycle at $x_0$ is
\begin{equation}
I_{x_0} = \left\{ x_0 \in B(x_i) \mid \lim_{t \to +\infty} ||\phi(t, x_i) - \phi(t, x_0)|| = 0 \right\}
\end{equation}
where $|| \cdot ||$ denotes the Euclidean distance. The phase of points in $B(x_i)$ remains defined if we assign the same phase to all points belonging to the same isochron. Thus, given a point on the limit cycle $x_0 \in x_i$ and set of initial conditions on the isochron based at $x_i$, $\{ x_0 \} \in I_{x_0}$, the phase of the trajectories starting at $x_i$ are $\phi(\tilde{x}(t, x_0)) = \omega t + \phi(x_i)$. In analogy with the flow box theorem, the phase function $\phi$ is the diffeomorphism that realizes a rectification of trajectories in the basin of attraction of the limit cycle, mapping solutions of $\dot{x}(t) = a(\tilde{x}(t))$ onto those of $\dot{\tilde{x}} = \omega_0 t + \phi_0$, and isochrons are the level sets of the phase function.

To study the influence of random perturbations we consider a vector basis $\{ u_1(t), \ldots, u_n(t) \}$, where $u_i(t)$ is chosen as the unit vector tangent to the limit cycle at any $t$
\begin{equation}
u_1(t) = \left[ a(x_0(t)) \right] / ||a(x_0(t))||
\end{equation}
while the remaining $n−1$ vectors $u_2(t), \ldots, u_n(t)$ are chosen as the Floquet vectors (apart from the limit cycle tangent $u_1(t)$) of the linearized variational equation [5]–[7]
\begin{equation}
\frac{d\tilde{x}(t)}{dt} = J(t) \tilde{x}(t)
\end{equation}
Define the matrix $U(t) = [u_1(t), \ldots, u_n(t)]$, and the reciprocal vectors $v_1^T(t), \ldots, v_n^T(t)$ as the rows of the inverse
matrix $V(t) = U^{-1}(t)$. Thus $\{v_1(t), \ldots, v_n(t)\}$ is also a basis for $\mathbb{R}^n$ and the vectors satisfy the bi-orthogonality condition $v_i^T u_j = u_j^T v_i = \delta_{ij}$. Finally, introduce matrices $Y(t) = \begin{bmatrix} u_2(t), \ldots, u_n(t) \end{bmatrix}$, $Z(t) = \begin{bmatrix} v_2(t), \ldots, v_n(t) \end{bmatrix}$, and the modulus of the vector field evaluated on the limit cycle, $r(t) = ||a(x(t))||$.

We look for a solution of (4) in the form

$$x_t = x_\theta(t) + Y(\theta_t)R_t$$

where $x_\theta(t)$ represents the projection of the stochastic process $x_t$ onto the limit cycle, evaluated at an unknown time instant $\theta_t$. The second component $Y(\theta_t)R_t$ represents an unknown distance between the solution and the limit cycle, measured along the directions spanned by the vectors $v_2, \ldots, v_n$ at the random time $\theta_t$. Following the procedure given in [5, Theorem 3.1] and [7, Theorem 1], the following Itô SDEs for the stochastic processes $\theta : \mathbb{R} \mapsto \mathbb{R}$ and $R : \mathbb{R} \mapsto \mathbb{R}^{n-1}$ are found

$$d\theta = [1 + a_\theta(\theta, R_t) + \hat{a}_\theta(\theta, R_t) + b_\theta(\theta, R_t)] dt + B_\theta(\theta, R_t) dW_t$$

$$dR = [L(\theta, R) + a_R(\theta, R_t) + \hat{a}_R(\theta, R_t) + b_R(\theta, R_t)] dt + B_R(\theta, R_t) dW_t$$

where (the ‘$\cdot$’ sign denotes the derivative with respect to $\theta$)

$$a_\theta(\theta, R) = \kappa v_1^T[a(x_t + YR) - a(x_t) - Y^T R]$$

$$\hat{a}_\theta(\theta, R) = -\kappa v_1^T \left[ Y^T BR(\theta, R) B_\theta(\theta, R) + \frac{1}{2} B_\theta^2(\theta, R)(x''_t + Y''R) \right]$$

$$b_\theta(\theta, R) = \frac{D^2}{2} \kappa v_1^T \frac{\partial B(x_t + YR)}{\partial x} B(x_t + YR)$$

$$B_\theta(\theta, R) = D \kappa v_1^T B(x_t + YR)$$

$$L(\theta) = -Z^T Y^T$$

$$a_R(\theta, R) = Z^T \left[a(x_t + YR) - Y^T R a_\theta(\theta, R)\right]$$

$$\hat{a}_R(\theta, R) = -Z^T \left[Y^T \hat{a}_\theta(\theta, R) + Y^T BR(\theta, R) B_\theta(\theta, R)\right] + \frac{1}{2} B_\theta^2(\theta, R)(x''_t + Y''R)$$

$$b_R(\theta, R) = -Z^T Y^T R B_\theta(\theta, R) + \frac{D^2}{2} Z^T \frac{\partial B(x_t + YR)}{\partial x} B(x_t + YR)$$

$$B_R(\theta, R) = -Z^T Y^T R B_\theta(\theta, R) + D Z^T B(x_t + YR)$$

and

$$\kappa = \frac{1}{\mu \tau}$$

In the neighborhood of the limit cycle, the phase $\theta$ is a local first order approximation of the phase function $\phi$ defined through isochrons [7, 8].

Analysis of the SDEs (9) and (10) shows that the phase noise problem in a nonlinear oscillator is a drift-diffusion process. Function $B_\theta$ is the diffusion coefficient. Apart from the constant $D$, (14) is not different from the one found for the case of white Gaussian noise [7], [8]. Therefore we conclude that, as far as phase diffusion is concerned, colored noise does not play a different role from a white noise of the same intensity. The situation is significantly different for phase drift. Comparing with the results of [7], [8], SDE (9) shows the additional drift coefficient $b_\theta$. This term represents a further drift effect, not present for the case of white noise, that can be ascribed to the finite noise correlation time.

### IV. Example

As an example we study a second order oscillator subject to colored noise, described by the SDE

$$dx_1 = \left[ \mu \left( x_1 - \frac{x_1^3}{3} - x_2 \right) + x_1 y \right] dt$$

$$dx_2 = \left[ \frac{1}{\mu} x_2 + x_2 y \right] dt$$

where $\mu$ is a real valued parameter. After transformation into the equivalent system with white Gaussian noise we obtain

$$dx_1 = \left[ \mu \left( x_1 - \frac{x_1^3}{3} - x_2 \right) + \frac{D^2}{2} x_2 \right] dt + Dx_2 dW_t$$

$$dx_2 = \left[ \frac{1}{\mu} x_1 + \frac{D^2}{2} x_2 \right] dt + D x_2 dW_t$$

Figure 1 shows the probability density function (PDF) $p(x_1, x_2)$ for different values of the correlation time $\tau$. The PDF is obtained through integration of (21) and (22) exploiting the Euler-Maruyama numerical integration scheme with a time integration step $\Delta t = 3 \times 10^{-6}$. The probability to find the system in state $x_1 + dx_1, x_2 + dx_2$ is evaluated as the fraction of time spent in that interval, normalized to the total simulation length. As the correlation time decreases, the PDF approaches that of the equivalent system with white Gaussian noise.

Figure 2 (a) shows the detailed phase portrait for the nonlinear oscillator (21) in absence of noise. The thick blue line is the limit cycle, blue and red arrows are the tangent and the transversal Floquet vectors $u_1(t)$ and $u_2(t)$, respectively. Floquet vectors are computed with the help of the analytical formulas given in [9], however for higher order systems specialized numerical algorithms are available [10]–[12]. Thin black lines are some of the isochrons, computed using the algorithm given in [13]. Figure 2 (b) shows the Jacobian of the coordinate transformation (8) for the nonlinear oscillator under investigation. Thick black lines identify where the determinant is null, and thus the transformation is not invertible. Clearly the Jacobian is regular for large values of the amplitude deviation.
Exploiting stochastic averaging, we have obtained an Itô-Stratonovich dilemma free formulation for the noisy behavior of nonlinear oscillators subject to colored fluctuation with small noise correlation time. Transformation to amplitude and phase description is also presented, along with an example of application that enables to verify the correctness of the approach.

**V. CONCLUSIONS**

Exploiting stochastic averaging, we have obtained an Itô-Stratonovich dilemma free formulation for the noisy behavior of nonlinear oscillators subject to colored fluctuation with small noise correlation time. Transformation to amplitude and phase description is also presented, along with an example of application that enables to verify the correctness of the approach.

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Design and realization of ultra-low noise cryoHEMTs for cryogenic readout electronics

Y Jin, Q Dong, L Couraud, A Cavanna, U Gennser, C Ulysse, E Cambril

C2N, CNRS, Univ. Paris-Sud, Univ. Paris-Saclay, 10 bd Thomas Gobert, France

Based on a long-term investigation of material growth and device process, the cryoHEMTs (Cryogenic High Electrons Mobility Transistors) made at the C2N (formerly LPN) are now in the process to fill the gap of the FET for high impedance, low-power and low-frequency deep cryogenic readout electronics. Different input capacitance cryoHEMTs have been fabricated and characterized, and ultra-low noise voltage and noise current have been obtained. Our cryoHEMTs at 4.2 K or lower facilitate the following accomplishments: reaching unprecedented low noise current of aA/Hz$^{1/2}$ and low noise voltage of sub-nV/Hz$^{1/2}$; attaining unrivaled readout rates and improve the Signal-to-Noise Ratio; realizing novel experimental observations. Their implementation has already resulted in the publications of 1 Science, 2 Nature Physics, 3 Nature Communications, 1 Nano Letters (http://phynano.c2n.universite-paris-saclay.fr/en/teams/nanofet/). Details in device investigation, physical understanding, comparison with conventional readout electronics and applications will be presented.
Shot noise and squeezing in the conduction channel of a Field Effect Transistor at ultra-low temperature

Anthoni Manseau  
Université de Sherbrooke  
Dépt. de Physique, Institut Quantique  
Sherbrooke, QC, Canada  
anthoni.manseau@usherbrooke.ca

Édouard Pinsolle  
Université de Sherbrooke  
Dépt. de Physique, Institut Quantique  
Sherbrooke, QC, Canada  
edouard.pinsolle@usherbrooke.ca

Christian Lupien  
Université de Sherbrooke  
Dépt. de Physique, Institut Quantique  
Sherbrooke, QC, Canada  
christian.lupien@usherbrooke.ca

Bertrand Reulet  
Université de Sherbrooke  
Dépt. de Physique, Institut Quantique  
Sherbrooke, QC, Canada  
bertrand.reulet@usherbrooke.ca

Abstract—We present two measurements of shot noise generated by the drain-source channel of a field effect transistor placed at ultra-low temperature. In a first experiment performed on a high impedance channel, we demonstrate classical, low frequency shot noise. In a second one performed in the microwave regime on a $50\,\Omega$ channel, we observe quantum shot noise as well as vacuum squeezing. We also discuss the possibility of using another mechanism, the high frequency modulation of the gate voltage, to squeeze vacuum fluctuations.

Index Terms—Shot noise, cryogenic, FET, quantum noise, squeezing

Measurements of electronic noise at ultra-low temperature is often performed on devices that are specifically designed for the experiment. In particular, since inelastic processes have the tendency to kill shot noise, the latter is usually studied in small structures such as tunnel junctions [1], quantum point contacts [2], hybrid superconductor / normal metal junctions [3], nanowires [4], chaotic cavities [5], etc. Much less is known on the noise in more conventional structures such as field effect- or bipolar transistors at ultra-low temperature and very low bias. However, elastic transport can be achieved in such structures provided they are small enough and put at low enough temperature, so usual phenomena associated with noise in mesoscopic devices should be visible, like photo-assisted noise or quantum noise.

We present two measurements of the shot noise generated by the conducting channel of a commercial FET (pHEMT ATF-35143 from Avago Technologies) placed at ultra-low temperature $T = 30\,\text{mK}$. We explore two very different regimes of drain-source channel with high impedance $R_{DS} = 10\,\text{k}\Omega$, and channel with low impedance $R_{DS} = 50\,\Omega$. The impedance of the channel is tuned by applying a voltage between the gate and the source of the FET. In the first case we perform a measurement at relatively low frequency ($f < 1\,\text{MHz}$), which corresponds to the classical regime. In the second case where the channel is matched to microwave circuitry, we observe quantum noise measured at high frequency $f = 7.2\,\text{GHz}$. By exciting the FET with a microwave tone at 14.4 GHz, we observe photo-assisted noise. Then, with phase-sensitive detection we demonstrate that the channel can squeeze vacuum fluctuations. Finally, we propose another way to generate vacuum squeezing by modulating the resistance of the channel at high frequency.

I. LOW FREQUENCY SHOT NOISE OF HIGH IMPEDANCE CHANNEL

A. Experimental setup

![Experimental setup for the measurement of low frequency noise](image)

In our first experiment we investigate the noise generated by the channel of the FET of resistance $R_{DS} = 10\,\text{k}\Omega$. This is achieved by imposing a gain-source voltage $V_{GS} \approx -0.36\,\text{V}$. The measurement is based on low frequency techniques, with a cryogenic tank circuit and room temperature voltage amplifier. The experimental setup is presented in Fig. 1. A voltage source at room temperature followed by a cryogenic low-pass filter allows to impose $V_{GS}$. A voltage source followed by a $1\,\text{M}\Omega$...
A resistor placed at low temperature allows to impose the drain-source current $I_{DS}$. A detection of noise at low frequency would be limited one one end by the $1/f$ noise of the sample and/or the amplifier, and on the other end by the low-pass filter made of the resistance of the channel in parallel with the capacitance $C$ of the several meter long cables in parallel with the input capacitance of the amplifier. To circumvent this problem we short the sample by an inductor $L \approx 70 \mu H$ and work at the resonant frequency $f = 388$kHz of the LC circuit (a capacitor in series with $L$ is necessary to block the dc current injected to bias the channel). The inductor is placed at the lowest temperature to avoid its Johnson noise. The total capacitance in parallel with the sample is $C = 2.3 nF$ and we obtain a bandwidth of $\sim 20$kHz for noise measurements. We record noise spectra after amplification by a high impedance voltage amplifier placed at room temperature, for various drain-source currents and gate-source voltages. The shape of these spectra as a function of frequency depends on the LC circuit but also on the frequency-dependent gain, input impedance and voltage/current noise spectral density of the amplifier (see an example of spectrum in the inset of Fig. 2).

We have characterized the frequency dependence of the gain and impedances independently. We use the fact that the noise of the amplifier is independent of the current in the sample to extract their values from the noise spectra.

**B. Results**

![Fig. 2. Rescaled low frequency noise $S/I$ of the conducting channel of the FET as a function of the dc bias current.](image)

The height corresponds to $2.5 \times 10^{-17} V^2/Hz$. From the width of the noise spectra we deduce the value of the resistance $R_{DS}$. This avoids extra cables and loxcin amplifier which would modify the impedance involved in the noise measurement. We observe that it depends on $I_{DS}$. Thus, the current noise of the amplifier flowing in the sample leads to a parasitic contribution to the current dependence of the total noise. As a matter of fact, the total voltage noise detected is given by:

$$\langle \delta V^2 \rangle = \langle \delta V_{DS}^2 \rangle + [Z_{eff}]^2 (\langle \delta I^2 \rangle + \langle \delta I_{eff}^2 \rangle)$$ (1)

where $\langle \delta I^2 \rangle$ is the current noise of the sample, which we seek to measure, and $\langle \delta V_{DS}^2 \rangle$, $\langle \delta I_{eff}^2 \rangle$ are respectively the voltage and current noise of the amplifier. $Z_{eff}$ represents the impedance of the sample in parallel with the rest of the circuit. Thus if the sample resistance depends on the bias current, so does $Z_{eff}$ and the measured voltage noise contains a parasitic bias-dependent term proportional to the current noise of the amplifier. In order to remove this effect we have repeated the measurements for many values of the gate-source voltage and combined the data that correspond to a constant $R_{DS}$.

We show the result in Fig. 2. Different points on the curve correspond to different values of $V_{GS}$ such that $R_{DS}$ is kept constant at 10kΩ. We clearly observe that the current noise spectral density $S$ of the sample obeys the usual law $S = F e/|I|$ for $|I| < 0.4 \mu A$, with $e$ the electron charge. We obtain a Fano factor of $F = 0.31$. This is close to that of a diffusive wire, indicating that transport in the channel is probably close to diffusive [4]. No thermal rounding is apparent at low voltage. We expect the cross-over between thermal- and shot noise to occur at a voltage of order $k_B T/e$ which corresponds to a current of $\sim 0.26 nA$. At very low current we observe a jump in the noise, which is also associated with a strong dependence of $R_{DS}$ vs. $I_{DS}$. We do not know the origin of this effect, which looks like trapping of the charge carriers at low electric field.

**II. HIGH FREQUENCY SHOT NOISE OF LOW IMPEDANCE CHANNEL**

**A. Experimental setup**

Our second experiment is a phase-sensitive detection of high frequency quantum shot noise (at $f \approx 7.2$GHz) with a low impedance channel ($R_{DS} = 50 \Omega$). This experiment is based on the observation of squeezing of shot noise in a tunnel junction [6]. The experimental setup is presented in Fig. 3. A dc voltage source followed by a cryogenic low-pass filter imposes $V_{GS} \approx -0.19$V to set the channel resistance to 50Ω. Another dc voltage source followed by 1MΩ resistors forces a current $I_{DS}$ into the FET. A microwave triplexer is used to separate the signals of the FET into three frequency bands. The 0-4 GHz band is used for the dc bias (with an additional 1.9MHz low-pass filter). The 8-18 GHz band is connected, through many cryogenic attenuators, to a microwave source. This source allows to excite the drain of the FET with an ac voltage at 14.4 GHz. The 4-8 GHz band is used to send the noise emitted by the sample into a cryogenic amplifier (placed at 3K) after crossing two circulators, which prevent the noise emitted by the amplifier to impinge on the sample and heat its electrons. After cryogenic and room temperature amplification, band-pass filtering around 7.2 GHz, the noise generated by the sample is downconverted to low frequency by an IQ mixer with a 7.2 GHz LO phase-locked with the 14.4 GHz excitation. The mixer provides the amplitudes of...
the signal that is in-phase (\( X \)) and out-of-phase (\( P \)) with its reference. Two power detectors are then used to measure the power of the two quadratures \( I \) and \( Q \), i.e. \( \Delta X^2 \) and \( \Delta P^2 \), integrated over a bandwidth of 200 MHz.

**B. Results**

The results are plotted in Fig. 4 as a function of the dc bias of the channel. Black symbols correspond to \( \Delta X^2 \approx \Delta P^2 \) in the absence of photo-excitation. This is the noise generated by the sample due to the dc voltage. The plateau at \( V_{DS} \) correspond to vacuum fluctuations, since no photon of energy \( hf \) can be emitted for voltages smaller than \( hf/e \). The shaded region corresponds to noise being below that of vacuum fluctuations. The rounding of the noise at \( eV_{DS} \approx hf \) is a direct measure of the electron temperature. By fitting our data with theory we find an electron temperature of \( T_e = 31\text{mK} \).

We clearly observe that the noise increases only when the bias voltage is greater than \( hf/e \), demonstrating that transport is elastic. The Fano factor \( F = 0.21 \) is significantly smaller than the one we observed for a more resistive channel, which probably indicates that transport is closer to the ballistic regime at low resistance. Our observation of a plateau for voltage smaller than \( hf/e \) shows that the FET generates quantum noise. In the presence of photo-excitation but when detection and excitation are not phase-locked (green triangles), \( \Delta X^2 \approx \Delta P^2 \) corresponds to photo-assisted noise as usually measured, which is above vacuum fluctuations. This measurement provides us with a way to calibrate the ac voltage experienced by the sample. The observation of photo-assisted noise is another proof that the noise generated by the channel of the FET is of quantum origin. Restoration of the phase lock between the two microwave generators allows to separate \( \Delta X^2 \) (blue circles) from \( \Delta P^2 \) (red squares). Clearly there is a voltage range in which \( \Delta X^2 \) or \( \Delta P^2 \) is smaller than its value in the absence of excitation, i.e. smaller than vacuum fluctuations: this is vacuum squeezing. The average \( \Delta X^2 + \Delta P^2 \)/2 is the photo-assisted noise (green triangles).

Dashed lines of Fig. 4 represent theoretical expectations. The measured photo-assisted noise is slightly above its expectation which results in vacuum squeezing being smaller than expected. We observe a squeezing by \( \approx 2\% \). A possible explanation for this is a slight non-linearity of the sample. The inset of Fig. 4 shows the resistance \( R_{DS} \) as a function of the voltage bias. \( R_{DS} \) varies between 47 and 53\( \Omega \) when the bias voltage is varied between \( \pm 80\mu \text{V} \). The observed dip disappears at higher temperature. Further investigations are necessary to understand the possible effect of this non-linearity.

The amount of squeezing we observe is rather modest. However, it is in principle very broadband, and should run from 0 to 14.4 GHz [7]. In comparison, a measurement of such broadband two-mode squeezing using Josephson junction has been reported recently [8]. In this study the squeezing was of the same order as with our method. Thus noise modulation can achieve a degree of squeezing that is similar to that of a Josephson junction in the absence of a resonator. The resonator, by forcing the signal to experience many times the non-linearity of the junction, leads to higher degree of squeezing at the cost of reduced bandwidth. In contrast, noise modulation as we do it here would not be enhanced by a
resonator.

III. CHANNEL RESISTANCE MODULATION

The generation of squeezing in the high frequency experiment is based on the idea that shot noise can be modulated at frequency 2f even when eV < hf, i.e. in the regime where the noise in the absence of photo-excitation is that of vacuum. Here we want to explore the same idea when replacing shot noise by thermal one: thermal noise of a resistor is proportional to the conductance of the sample, and given by vacuum fluctuations for h/2k_B T. Can one generate vacuum squeezing by modulating a resistor at high frequency? It has been indeed predicted that modulating the damping rate of a resonator can lead to perfect squeezing. A time-dependent resistor could be realized with a FET by applying a time-dependent gate voltage V_{GS}(t) in the absence of bias on the drain-source channel, i.e. V_{DS} = 0. Below we make a simple calculation based on a semi-classical picture of noise.

We consider the simple circuit of Fig. 5. The signal ξ(t) detected by the amplifier is the sum of the noise ξ_ε(t) emitted by the modulated resistance Z_r(t) and that. ξ_θ(t) emitted by the load of the circulator Z_0 that is reflected on Z_r with a time-dependent reflection coefficient Γ(t) = [Z_r(t) - Z_0]/[Z_r(t) + Z_0]:

\[ ξ(t) = [1 - Γ(t)]ξ_ε(t) + Γ(t)ξ_θ(t) \]

This problem is equivalent to two semi-infinite coaxial cables being connected together, one of characteristic impedance Z_0, the other with a time-dependent characteristic impedance Z_r(t). Eq. (2) immediately leads to a non-zero term \( ⟨ξ(ω)^2⟩ \) which reveals the existence of squeezing. The degree of squeezing depends on the shape of the time-dependence of Z_r(t). For a small sinusoidal variation, Z_r(t) = Z_0[1 + A\sin(4πft)] we find a degree of squeezing that is optimal for two values of Z_r that are on each side of Z_0, and equal to 0.3A, i.e. 3% for A = 0.1.

Experimentally, modulating the gate voltage while keeping V_{DS} is challenging because of capacitive coupling between the gate and the drain of the FET. To modulate significantly the resistance of the channel below 100Ω requires tens of mV of ac voltage. In contrast, we have observed shot noise for a drain-source voltage of some tens of microvolts. We have indeed tried to detect squeezing with gate modulation and could not observe it. Another, maybe more promising approach would be to have a macroscopic resistor and a fast shunt, possibly made with a Josephson junction. This is reminiscent of the setup of ref. [8] though conceptually quite different.

IV. CONCLUSION

We have reported measurements of: i) low frequency shot noise in the channel of a FET when gated to be high impedance, ii) high frequency quantum shot noise, photo-assisted noise as well as vacuum squeezing in the same transistor gated to be of 50Ω impedance. Our results show that existing commercial technology of micro-electronics could be used to generate quantum states of microwave radiation such as squeezed vacuum. We have also proposed another way to generate squeezed vacuum fluctuations by modulating the gate voltage of the FET in the absence of drain-source bias.

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Discussion of the flicker noise origin at very low temperature and polarisation operation

Dimitri Boudier and Bogdan Cretu
ENSICAEN, UNICAEN, CNRS, GREYC
University of Caen
Caen, France
bogdan.cretu@ensicaen.fr

Eddy Simoens and Anabela Veloso
Imec
Leuven, Belgium
eddy.simoens@imec.be

Cor Claeyts
ESAT-INSYS
KU Leuven
Leuven, Belgium
cor.claeyts@kuleuven.be

Abstract—Low frequency noise measurements are performed at very low temperature (i.e. 10 K) and for applied drain and gate voltages leading to step-like behaviors in the drain current transfer characteristics. It is shown that for the investigated low applied biases, the Coulomb scattering explains the 1/f behavior versus the applied voltages.

Keywords—Flicker noise, number fluctuations, mobility fluctuation, Coulomb scattering, very low polarization, cryogenic temperature

I. INTRODUCTION

Several studies at cryogenic temperatures concerning n-channel gate all around (GAA) nanowire (NW) FETs have been already reported (e.g. [1]). Moreover, in [2], a novel approach leading to a new theoretical formulation valid in moderate inversion for temperatures (T) lower than 40 K has been constructed for the mobility fluctuations (δμ) and for the carrier number with correlated mobility fluctuations (ων−δμ) models. These models take into account the mobility law that governs the carrier transport for T < 40 K, i.e., the Coulomb scattering at low vertical fields and surface roughness scattering at high vertical fields [3,4].

In this work, additional low frequency noise measurements are analyzed in order to have information on the flicker noise origin at very low applied biases and temperature operation.

An extension of the model of [2] is constructed, given a novel improved theoretical formulation valid also in weak inversion. This extended model may be applied when low frequency noise measurements are performed at fixed drain voltage without any additional considerations.

It is demonstrated that for the investigated applied biases, even if a step-like behavior is present in the drain current characteristics, the Coulomb scattering mechanism explains the 1/f behavior in the applied bias range. Moreover, it is suggested that in order to observe the inter/intra subband scattering impact on the flicker noise much lower applied biases are needed.

II. EXPERIMENTAL

Low frequency noise (LFN) measurements were performed at wafer-level in n-channel GAA NW FETs fabricated at imec on 300 mm wafers. The process description may be found in [5]. Details on the measurement set-up are given in [6].

In order to estimate the low frequency noise parameters, the same methodology as in [1,2,6,7] is applied, assuming that the white noise, flicker noise and generation-recombination noise are non-correlated noise sources. The LF noise is given by the general expression [6]

\[ S_n(f) = B_n + \frac{K_n}{f} + \sum_{i=1}^{3} \frac{A_i f_0}{f_0^2 + f^2} \]

where \( S_n \) is the input-referred voltage noise, \( B_n \) is related to the white noise level, \( K_n \) gives the flicker noise level, and the last term of the equation presents a sum of Lorentzian components, with \( A_i \) the plateau value and \( f_0 \) the characteristic frequency. The different parameters can be determined based on a comparison between the experimental data and the model of (1). Typical example of noise parameter estimation is given in Fig. 1.

\[ S_n(f) = B_n + \frac{K_n}{f} + \sum_{i=1}^{3} \frac{A_i f_0}{f_0^2 + f^2} \]

The LFN study was focused at very low temperature (i.e. 10 K) and was effectuated as a function of the applied gate voltage (VGS) for an applied drain voltage (VDS) of 500 μV and as a function of the applied drain voltage (VDS) for an applied gate voltage (VGS) of 0.5 V.

From Fig. 2, it may be observed that at these polarization conditions, the drain current normalized by the applied drain bias presents step-like effects, leading to oscillations in the transconductance behavior (Fig. 3). For VDS lower than the thermal energy, it may be expected that the drain current and transconductance are not independent on the applied drain voltage in the subthreshold operation. However, the drain current and the transconductance normalized by the applied drain bias not superpose at lower VDS Values even in strong inversion regime. This suggests that the step-like effects are related to the subband scattering effects. Using the same procedure as in [8], from the minima of the transconductance oscillations, the subband energy spacing ΔE between the successive energy subband fillings may be estimated. This technique was already successfully applied in UTBOX’s, FinFET’s and GAA NW FET’s [1,2,7]. In our

Fig. 1 Typical application of noise parameters estimation.
actual case, the estimated $\Delta E$ are between 2 - 7 meV, confirming that the conditions
where quantum transport may be observed are fulfilled, i.e., a $\Delta E$ higher than the energy
induced by the drain voltage ($qV_{GS}$) and higher than the thermal energy (about 862 $\mu$eV at 10 K).

![Graph](image_url)

Fig. 2. $I_{DS}/V_{GS}$ characteristics as a function of the applied $V_{GS}$.

![Graph](image_url)

Fig. 3. $g_{m}/V_{GS}$ characteristics versus $V_{GS}$.

III. RESULTS

In Fig. 4, the estimated input-referred gate voltage $1/f$ noise levels are plotted as a function of the applied gate voltage at fixed $V_{DS}$ of 500 $\mu$V.

Using the methodology presented in [2] the origin of the $1/f$ noise behavior is checked. As the $1/f$ noise increases with the applied gate voltage, one can use the $\delta n+\delta \mu$ model described by (13) of [2]. Except for the lowest $V_{GS}$ polarizations a good agreement may be observed between the experimental data and the $\delta n+\delta \mu$ model, considering a value of the flat-band noise level $S_{\text{fsh}}$ of about $10^{-7}$ $V^2$/Hz and a $c_{dn}$ factor of about 3.5 $V^{-2}$.

However, using a $c_{dm}$ factor of $7 \times 10^{-7}$V, a good agreement between the data and the $\delta \mu$ model described by (10) of [2] is observed, suggesting that the mobility fluctuations through the Coulomb scattering mechanism may be also responsible for the $1/f$ noise at this applied $V_{DS}$.

Even if the existence of both $\delta n$ and $\delta \mu$ fluctuations as independent contributions on the total noise was already evoked [9], it is surprising that both $\delta n+\delta \mu$ or $\delta \mu$ fluctuation models may explain the flicker noise behavior and this for polarization conditions in which a step-like behavior is observed in the transfer characteristics. One should note that the measured drain current and transconductance read all mobility mechanisms involved in the carrier transport. As the models of (10) or (13) of [2] consider only the Coulomb scattering mechanism (i.e. low vertical electric field), this suggests that even if the inter/intra subband scattering has an important impact on the DC characteristics (see Figs. 1 and 2), for the investigated polarization they do not affect the $1/f$ noise levels.

It may be noticed that the deviation between the data and the $\delta n+\delta \mu$ or $\delta \mu$ models of [2] at very low applied $V_{GS}$ may be related to the fact that the moderate inversion conditions are not fulfilled.

![Graph](image_url)

Fig. 4. Input referred $1/f$ noise levels versus $V_{GS}$ at fixed $V_{DS}$ of 500 $\mu$V.

![Graph](image_url)

Fig. 5. Input referred $1/f$ noise levels versus $V_{GS}$ at fixed $V_{DS}$ of 0.5 $\mu$V.

In Fig. 5, the estimated input-referred gate voltage $1/f$ noise levels are plotted as a function of the applied drain voltage at fixed $V_{GS}$ of 0.5 $\mu$V.

The behavior of the flicker noise levels shows a slight decrease with the increase of $V_{DS}$, followed by a quick decrease of more than one decade with the further increase of the applied $V_{DS}$ in the range of 1 mV – 100 mV. Assuming the carrier number fluctuation mechanism leads to consider a flat-band noise independent on the applied drain voltage at fixed $V_{GS}$. This implies that the behavior of the $1/f$ noise levels versus $V_{DS}$ cannot be explained in the framework of the carrier number fluctuation mechanisms. Considering the $\delta n+\delta \mu$ model, using the same parameters as derived from Fig. 4 (i.e. $S_{\text{fsh}}$ of $10^{-7}$ $V^2$/Hz and $c_{dn}$ of 3.5 $V^{-2}$), it is impossible to fit the experimental data. One should note that in moderate inversion operation, the ratio of the $I_{DS}/I_{DS}$ which is used in (13) of [2] to model the $\delta n+\delta \mu$...
mechanism, should increase with the applied V_{DD}, in
contradiction with the experimental shape. One can
conclude that considering moderate inversion operation, the
\( \delta n \) or \( \delta n+\delta m \) mechanisms are not responsible for the 1/f
noise behavior at this temperature and applied polarization.

Moreover, one can observe that using the \( \delta m \) model
described by (10) of [2] cannot explain the 1/f noise
behavior presented in Fig. 5. Taking into account the quasi-
constancy of the \( \log(V_{DS}) \) behavior with \( V_{DS} \) (see (10) of [2]) it
may be suggested that at a \( V_{DS} \) of 0.5 V we still operate in
weak inversion regime.

A methodology to describe the \( \delta n+\delta m \) or \( \delta m \) models in
weak inversion is therefore mandatory.

IV. THEORETICAL APPROACH

Taking into account the mobility law for temperatures
lower than 40 K (e.g. (3) of [2] derived from [3]), and the
expression of the transconductance related to the conductance
\( g_{so} \) (e.g. (5) of [2], derived from [9,10]), it may be easily demonstrated that:

- from weak to moderate inversion \( (Q_i < Q_m) \) the
conductance can be expressed as:

\[
G_{so} = 2\frac{W\mu_m}{L} Q_i^2
\]

where \( W \) and \( L \) are the effective channel length and width,
\( \mu_m \) is the maximum of the effective mobility and \( Q_m \) is the
corresponding inversion charge density; \( Q_i \) being the
inversion charge density at arbitrary \( V_{DS} \).
- while in weak inversion \( Q_i < Q_m \) and \( C_i \ll C_s, C_m, C_{on} \),
where \( C_s, C_m, C_{on} \) are the inversion, the depletion,
the interface state and the gate oxide capacitances per unit of area,
respectively), transconductance related to the conductance \( g_{so} \) will read:

\[
g_{so} = \left( 4\frac{W}{L} \frac{\mu_m}{Q_m} \right) Q_i = \left( 4\frac{W}{L} \frac{\mu_m}{k_BT Q_m} \right) Q_i^2
\]

Using the same methodology as in [2], the resulting expressions for the \( \delta n+\delta m \) [11] or \( \delta m \) [12] models may be
finally derived as in Table 1:

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Summary of ( \delta m ) and ( \delta n+\delta m ) models as a function of ( G_{so} ) and ( g_{so} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta m )</td>
<td>weak inversion</td>
</tr>
<tr>
<td>( S_{\delta m} )</td>
<td>( \frac{\delta m}{\delta \rho} \frac{d \rho}{\delta \rho} \frac{I_{DS}^2}{\text{w}} ) (a)</td>
</tr>
<tr>
<td>( S_{\delta m} )</td>
<td>( \frac{\delta m}{\delta \rho} \frac{d \rho}{\delta \rho} \frac{I_{DS}^2}{\text{w}} ) (c)</td>
</tr>
<tr>
<td>( \delta n+\delta m )</td>
<td>( S_{\delta n+\delta m} )</td>
</tr>
<tr>
<td>( S_{\delta n+\delta m} )</td>
<td>( 1 + c_{1n} \frac{I_{DS}^2}{\text{g}_{so}} ) (d)</td>
</tr>
</tbody>
</table>

where the different coefficients are summarized in Table 2:

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Summary of coefficients used in the equations presented in Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>( c_1 \frac{d \rho}{\delta \rho} \frac{I_{DS}^2}{\text{w}} ) (a)</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>( c_2 \frac{d \rho}{\delta \rho} \frac{I_{DS}^2}{\text{w}} ) (b)</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>( c_3 \frac{d \rho}{\delta \rho} \frac{I_{DS}^2}{\text{w}} ) (c)</td>
</tr>
<tr>
<td>( \delta n )</td>
<td>( \delta m )</td>
</tr>
<tr>
<td>( \delta n+\delta m )</td>
<td>( \delta n+\delta m )</td>
</tr>
</tbody>
</table>

where \( k_3 \) is the Boltzmann constant and \( q \) is the elementary charge.

Assuming as usual that the inversion charge from weak to
moderate inversion presents an exponential dependency with the applied drain bias, the equations presented in Table 1 becomes:

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Summary of ( \delta n ) and ( \delta m+\delta n ) models as a function of ( I_{DS} ) and ( g_{so} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta n )</td>
<td>( \delta n )</td>
</tr>
<tr>
<td>( \delta n+\delta m )</td>
<td>( \delta n+\delta m )</td>
</tr>
</tbody>
</table>

where \( c_4, c_5, c_6, \) and \( c_7 \) are related to \( c_1, c_2, c_3, d_1 \) and \( c_2, c_3, d_1 \) and \( c_4, d_1 \) are related to \( c_1, d_1 \) and
\( c_2, d_1 \) with the respect of the integration coefficient of the
inversion charge with the drain bias.

V. DISCUSSION

Assuming the weak inversion operation mode, the \( \delta n+\delta m \) model will be described by Eq. (d) or (e) presented in Table 3. Taking into account the experimental quasi-independence (in agreement with the expected theoretical independence) of the \( \log(V_{DS}) \) behavior with \( V_{DS} \) observed in Fig. 5, it is
obvious that the \( \delta n+\delta m \) model will provide 1/f noise levels which will increase with the increase of \( V_{DS} \), in
contradiction with the experimental behavior presented in
Fig. 5.

It should be noted that the \( c_d \) factor of about 3.5 V^{-2}
(used to model results presented in Fig 4 in the framework of the αm±δμ model) lead to a Coulomb scattering coefficient (αC) value of about 0.02×10⁻⁴ V/s/°C. Taking into account the generally reported value of αC in the range of 1×10⁻⁴ V/s/°C [11] this obtained value may be considered as an unphysical one.

All these arguments demonstrate that the hypothesis of the δμ or αm±δμ fluctuation mechanism in order to explain the flicker noise behavior can be completely dismissed.

Fig. 6. Input referred 1/f noise levels versus VGS at fixed VDS of 500 μV.

Eq. (c) of Table 3 permits to model correctly the experimental 1/f noise behavior and this from weak to moderate inversion operation as observed in Fig. 6. This confirms our hypothesis: the mobility fluctuations through the Coulomb scattering mechanism are responsible for the 1/f noise behavior versus VGS.

Applying Eq. (a) of Table 3 permits to obtain a good agreement between the model and data in weak inversion; the same result is obtained using Eq. (b) in moderate inversion. Employing Eq. (a) and (b) of Table 3 permits to identify the different operation modes at VDS = 500 μV: weak inversion till around an applied VGS of about 0.7 V and moderate inversion for VGS higher than about 0.75 V.

The first experimental point (at VGS = 0.6 V) cannot be modelled employing the equations of Table 3, suggesting that for much lower polarization than studied, the inter/intra subband scattering will impact the flicker noise levels.

Fig. 7. Input referred 1/f noise levels versus VDS at fixed VGS of 0.5 V.

Taking into account the same value of the coefficient c6_δμ as for Fig. 6, one can observe that the δμ model provides some trend for the flicker noise behavior as a function of VDS for VDS higher than 1 mV (Fig. 7). This corroborates with the hypothesis that Coulomb scattering mechanisms are responsible for the 1/f noise fluctuations.

However, a new physical model is necessary to clearly state the drain current evolution with VDS in order to explain the flicker noise behavior as a function of the applied VDS, in particular at the lowest applied VDS.

VI. CONCLUSION

It was clearly demonstrated that the carrier number or carrier number correlated to mobility fluctuations mechanism cannot explain conveniently the flicker noise behavior.

Even if the inter/intra subband scattering affect the DC measurements, there is a polarization interval in which they do not impact the 1/f noise. It was proved that in this low polarization range, the Coulomb scattering mechanism is responsible for the 1/f noise fluctuations.

REFERENCES


Generation-recombination noise of magnetic monopoles in spin ice

Alexey V. Klyuev
Stochastic Multistable Systems Laboratory
National Research Lobachevsky State University of Nizhni Novgorod
Nizhni Novgorod, Russia
klyuev@rf.unn.ru

Mikhail I. Ryzhkin
Laboratory of semiconductors surface spectroscopy
Institute of Solid State Physics (ISSP) Russian Academy of Sciences
Chernogolovka, Moscow District
ryzhkin@bk.ru

Arkady V. Yakimov
Stochastic Multistable Systems Laboratory
National Research Lobachevsky State University of Nizhni Novgorod
Nizhni Novgorod, Russia
yakimov@rf.unn.ru

Abstract—We theoretically investigate the fluctuations in the number of emergent magnetic monopoles in spin ice. The Langevin equation for these fluctuations is deduced. That allows us to calculate the spectrum of relative fluctuations, which can be measured experimentally.

Keywords—magnetic monopoles; spin ice; fluctuations; ice rules; generation-recombination.

I. INTRODUCTION

In the last time there was a lot of works, due to a great interest in the study of spin ice [1]. Such an interest was caused by the theoretical prediction of unusual magnetic defects in spin ice, which were called as emergent magnetic monopoles [2, 3].

Originally, the term "spin ice" was applied to the compounds such as HoTi2O7. Magnetic ions in them are located at the vertices of regular tetrahedra linked into a three dimensional pyrochlore lattice. Due to the strong anisotropy, the magnetic moments can be directed either toward the center of each tetrahedron, or away from this center. Note that dual lattice formed by the centers of the tetrahedra is a usual diamond lattice. The ground state of spin system is characterized by the ice rule: two spins of each tetrahedron are directed toward its center and two other ones away from its center (Fig.1 a). This corresponds to a distribution of electrical dipole moments hydrogen bonds in hexagonal ice, solid phase of water in natural conditions [1]. That justifies the term "spin ice". Subsequently, this term began to apply to any two-dimensional and three-dimensional systems in which the magnetic moments (spins) are distributed in accordance with the ice rules.

The flipping of an arbitrary spin in ground state configuration, which takes place at finite temperatures, breaks the ice rule in two neighboring tetrahedra. One tetrahedron has three spins directed “in”, and one – “out”, while its neighbor has one “in” and three – “out” (Fig. 1 b). These tetrahedra can be considered as positive and negative emergent magnetic monopoles, respectively. The positive magnetic monopole is a magnetic analogue of a positive ion in water ice. Accordingly, the negative magnetic monopole is a magnetic analogue of a negative ionic defect in water ice [2]. As one can see from Fig. 1, the flipping of any of three identical magnetic spins of the defect is equivalent to the defect movement to another site. This movement is not accompanied by the formation of new defect and by the increase of energy. By means of further spin flips the magnetic monopoles can move through the lattice and thereby change the spin configuration. Monopoles play the role of classical quasiparticles, which movement through the lattice change the spin configuration on all the distance traveled, resulting in the formation of a “spin spaghetti” [3]. Therefore, in comparison with defects in semiconductors, which may negatively influence on characteristics of devices [4–9], magnetic defects in spin ice [10–15] play the positive role of carriers of magnetic charge. But, in spin ice (in difference with ordinary ice [16–21]) a direct current of magnetic monopoles is impossible.

The purpose of our work is the investigation of the fluctuations in the number of magnetic monopoles. To implement this we derive the Langevin equation for these fluctuations by the Einstein–Fokker–Planck equation method. That allows us to calculate the spectrum of relative fluctuations in the number of magnetic monopoles. Then we discuss a possible relation of fluctuations in the number of pairs of magnetic monopoles with measurable characteristic of spin ice and an experimental display of obtained results.

II. GENERATION-RECOMBINATION NOISE OF MAGNETIC MONOPOLES

To analyze fluctuations in the number of magnetic monopoles, we consider the processes of generation and recombination of magnetic monopoles pairs [22]. It is the simplest stochastic analysis, which shows that these fluctuations can be described by the Langevin equation. Thus, these fluctuations can be considered as generation–recombination noise of quasiparticles (magnetic monopoles). The spectrum of this noise is determined.
(b) We assume that only one pair can be created or annihilated over a sufficiently small time $dt$.

(c) We discuss relatively small temperatures at which the concentration of monopoles is low. In this case, we can consider the monopoles as non-interacting quasiparticles.

At the first step we use standard treatment for generation–recombination noise, see, e.g., [23]. We consider probability $P(N,t)$ of the event that $N$ pairs of quasiparticles (magnetic monopoles) are present at time $t$. This probability is changed on value $dP$ during time $dt$ due to following transitions:

\[
\begin{align*}
N - \xrightarrow{\text{g}} & (N+1); \\
N - \xrightarrow{\text{r}} & (N-1); \\
(N-1) - \xrightarrow{\text{r}} & N; \\
(N+1) - \xrightarrow{\text{g}} & N.
\end{align*}
\]

Here transitions, which are specified in the first line, reduce the probability, i.e. $dP < 0$. On the contrary, remaining two transitions give $dP > 0$. Transitions, marked by symbol “$g$” above the arrow, are caused by the generation of a pair; respectively, the symbol “$r$” means recombination of the pair.

The rates of generation, $g = g(N)$, and recombination, $r = r(N)$, depend only on number $N$ of the pairs at time $t$. These rates determine the probability $g dt$ of generation and $r dt$ of recombination of one pair over an arbitrarily small time $dr$. As a result, the analysis of transitions (1) leads to the Einstein–Fokker–Planck equation [24] for $P(N,t)$:

\[
\frac{dP}{dt} = -\frac{\partial}{\partial N}[K_1(N) \cdot P] + \frac{1}{2}\frac{\partial^2}{\partial N^2}[K_2(N) \cdot P] .
\]

Kinetic coefficients in this equation are determined by the generation and recombination rates of monopole pairs:

\[
K_1(N) = g(N) - r(N), \quad K_2(N) = g(N) + r(N) .
\]

At this stage, we restrict the analysis by the stationary case. We consider the relative fluctuations in the number of pairs $\delta N = \delta N(t)$ near the stationary state $N_0$. Thus, the total number of pairs is $N = (1 + \delta N) N_0$. The variance $\sigma^{2}_\delta$ of these fluctuations is assumed to be sufficiently small, $\sigma^{2}_\delta << 1$.

Eq. (2) corresponds to the following equation for fluctuations in $N(t)$:

\[
\frac{d}{dt} \delta N = -\frac{1}{\tau_0} \delta N + 2 \sqrt{\frac{\sigma^{2}_\delta}{\tau_0}} \zeta(t) .
\]

Here $\zeta(t)$ is stationary white noise with normalized spectrum $S_{\zeta}(f) = 1$ [Hz/Hz]. Note that eq. (4) coincides in form with the Langevin equation, see, e.g. [23]. Here $\tau_0$ is the relaxation time, defined via the difference of derivatives from velocities of recombination and generation, taken in point $N_0$:

\[
\tau_0 = \left[ \frac{d(r-g)}{dN} \right]_{N_0}^{-1} .
\]

Eq. (4) allows determining of the spectrum $S_{\delta N}(f)$ of relative fluctuations in the number of pairs:

\[
S_{\delta N}(f) = \frac{4\sigma_{\delta N}}{1 + (2\pi f \tau_0)^2} \sigma^{2}_\delta .
\]

Now we can investigate the dependence of this spectrum on the magnitude $\tau_0$ (see Fig. 2), assuming the variance of the fluctuations constant, $\sigma^{2}_\delta = \text{const}$.

While relaxation time $\tau_0$ is increased the height of the spectrum is increased proportionally to $\tau_0$, and the width is decreased as $\tau_0^{-1}$. Thus, an increase of the lifetime of quasiparticles $\tau_0$ leads to increased low-frequency component of fluctuations of generation-recombination of quasiparticles and to attenuation of high frequency components.

III. RELATION OF FLUCTUATIONS IN THE NUMBER OF PAIRS OF MAGNETIC MONOPOLES WITH MAGNETIC MOMENT

Very often in literature pairs of magnetic monopoles, which are formed by inverting the shared spin (Fig.1 b), are called bound. On the other hand, the magnetic monopoles, which move through the lattice, are called unbound or free. But, the “unbound” or “free” monopoles are bounded by the “spin spaghetti”, which is formed by changing of the spin configuration along the way of monopole.

Following [10], we define number $n_b$ of bound, and unbound (free) $n_u$ pairs, $N_0 = n_b + n_u$ – the total number of pairs, and $\alpha = n_b/N_0$ – the degree of dissociation.

We now assume that the vast majority of pairs of quasiparticles remain in a bound state, i.e. $\alpha << 1$, and one pair of bound quasiparticles interacts weakly with another pair. Thus, $N_0 = n_b$ – the total number of pairs is approximately constant at a given temperature.

A small change, caused by small perturbation in concentration of unbound (free) pairs $\Delta n_u$ yields small change $\Delta M$ in the magnetic moment. In our case of a small change in concentration $n_u$ we have $N = n_b + n_u + \Delta n_u$, or $N = N_0 + \Delta n_u$. That is $N = N_0(1 + \Delta n_u/N_0) = N_0 + \Delta n_u$.

SciPy is a library for scientific computing with Python. It provides fast, user-friendly access to powerful libraries for numerical computing and scientific data analysis. It is built on a strong core of lower-level packages such as NumPy, which provides Python with its ability to perform array operations. NumPy also contains functions for linear algebra, Fourier transforms, and random number generation.

SciPy extends the capabilities of NumPy with a large number of user-defined functions that perform complex mathematical operations, such as integration, optimization, and signal processing.

One of the key features of SciPy is its ability to work with large arrays of data. This makes it an ideal tool for scientists and engineers who need to perform complex mathematical calculations on large datasets.

Fig. 2. Dependence of the spectrum on magnitude $\tau_0$. 

For a given $\alpha$ the spectrum $S_{\delta \mu}(f)$ is proportional to $\tau_0^{-1}$, and the width of the spectrum is approximately constant.
\[ \Delta \alpha = \delta N - \text{fluctuations in } N \text{ through the fluctuations in } \alpha \text{ (eq. (8) in [10]) lead to fluctuations in magnetic moment:} \]

\[ \Delta M \propto \frac{\mu_0 Q^3}{8\pi kT} \cdot \delta N . \]  

(7)

Here \( \mu_0 \) is the permeability of the vacuum, \( Q \) is the elementary magnetic charge, \( k \) is the Boltzmann's constant, \( T \) is the temperature.

For spectra we can write following equation:

\[ S_\Delta M(f) \propto \frac{\mu_0 Q^3}{8\pi kT} ^2 \cdot S_\delta N(f) . \]  

(8)

Here \( S_\Delta M(f) \) is the spectrum of absolute fluctuations of magnetic moment. It can be expected, that measurement of fluctuations in the magnetic moment may give access to information about fluctuations in the number of pairs of magnetic monopoles in spin ice.

Finally we note that in a spin ice a few relaxation times may exist. As a result, the total spectrum \( S_\Delta M(f) \) in eq. (8) of relative fluctuations in the number of pairs of monopoles may be a superposition of spectra (6) with different relaxation times.

Experimental proof of this result is given in [25]. The authors reported development of a high-sensitivity superconducting quantum interference device (SQUID) based flux-noise spectrometer, and consequent measurements of the frequency and temperature dependence of spectral density of magnetic-flux noise due to generation-recombination fluctuations of magnetic monopoles for \( \text{Dy}_{2/3}\text{Ti}_{1/3} \text{O}_3 \) samples. Virtually all the elements predicted in [22] for the fluctuations in the number of magnetic monopoles, including the existence of intense magnetization noise and its characteristic frequency, were detected experimentally.

IV. CONCLUSIONS

We investigated the fluctuations in the number of pairs of magnetic monopoles. It is shown that these fluctuations can be described by equation of Langevin type. The spectrum of the fluctuations is determined and analyzed. The way how to detect these fluctuations by the measurement of the magnetic moment fluctuations is suggested. Our theoretical results are in agreement with experimental results of other authors.

REFERENCES

Electronic noise due to temperature differences across nanoscale conductors: beyond standard thermal and shot noises

Oren Tal
Weizmann Institute of Science

Since the discovery of electronic thermal (Johnson–Nyquist) noise and shot noise almost a century ago, these two forms of fundamental electronic noise have had an enormous impact on science and technology. Here, we report on a new version of electronic noise that is generated by temperature differences across nanoscale conductors, which we term “delta-T noise” [1,2]. We experimentally demonstrate this noise in atomic and molecular junctions, and analyze it theoretically using the Landauer formalism. The delta-T noise reveals a peculiar combination of characteristics that makes it different from the known thermal noise and voltage-activated shot noise. This noise can be used to detect temperature differences across nanoscale conductors without the need for fabricating sophisticated local probes. Furthermore, delta-T noise should be considered when designing modern nanoscale electronics, since temperature gradients are often generated unintentionally across electronic components.

References:

Magneto-Electric Diffusion of Electrons in Gallium Nitride: a Monte Carlo Analysis

G. I. Syngayivska ∗, V. Korotyeyev ∗, V. A. Kochelap ∗, L. Varani†

∗V. Lashkaryov Institute of Semiconductor Physics, NAS of Ukraine, 41 prospect Nauky, 03680 Kyiv, Ukraine
†Institute of Electronics and Systems (CNRS UMR 5214), University of Montpellier, 860 rue St. Priest, 34095 Montpellier, France

Abstract—We present a Monte Carlo analysis of the diffusion coefficient tensor in compensated bulk Gallium Nitride for parallel and crossed configurations of the electric and magnetic fields. We found that at low lattice temperatures and low impurity concentrations, the electric-field dependences of the transverse-to-current components of the diffusion tensor are non-monotonic for both configurations, while the diffusion processes are mainly controlled by the magnetic field. With increasing the lattice temperature or impurity concentration, the behaviour of the diffusion tensor becomes more monotonic and less affected by the magnetic field. We showed that this behaviour is caused by the peculiar kinetics of hot electrons in polar semiconductors with strong electron–optical phonon coupling.

I. INTRODUCTION

The nitride compounds represent nowadays perspective materials for many practical applications like light-emitting diodes [1], optical switches [2], biosensors [3] and terahertz devices such as electrically pumped terahertz sources [4], detectors [5] and modulators [6]. For these reasons a lot of attention has been recently devoted to the investigation of their electron transport properties not only in the presence of electric but also magnetic fields in order to develop novel devices working as sensors and switches controlled by a magnetic field [7]–[9]. Furthermore, nitrides are also favorable materials for the realisation of a remarkable electron transport regime characterised by a quasi-periodic electron motion in the momentum space due to the strong threshold character of the electron–optical phonon emission [10]. This so-called streaming regime may produce plenty of interesting phenomena among which we can mention the saturation of the current-voltage characteristics, the appearance of a negative dynamic conductivity and a strong non-monotonic behaviour of the diffusion coefficient. Additional useful information about the streaming regime can be obtained investigating the galvano-magnetic characteristics at different configurations of the electric and magnetic fields. In particular, we have recently shown that, in compensated Gallium Nitride, the electron transport may take the form of a vortex-like motion in momentum space and the magnetic field may induce a collapse of the dissipative current due to the suppression of optical–phonon emission [11]. The present interest in the study of the diffusion processes is also inspired by the development of recent pure all-optical pump-probe techniques known as light-induced transient grating, allowing the experimental determination of the diffusion coefficient as well as the carrier recombination times [12].

In this dynamic scientific context it is noteworthy to present a study by Monte Carlo simulations of the diffusion properties of hot electrons in compensated bulk-like GaN samples for various lattice temperatures and impurity concentrations in the presence of parallel and crossed electric and magnetic fields.

II. THEORETICAL MODEL

We consider the electron transport in samples of cubic GaN under parallel and crossed configurations of electric, $E$, and magnetic, $H$, fields. At $E \parallel H$, we assume that $E$ and $H$ are directed along the $z$-axis (see Fig. 1 (a)). In the case $E \perp H$, we assume that $E$ and $H$ are applied along the $z$- and $y$-axis, respectively (see Fig. 1 (b)). Here, the GaN-samples are assumed to be with short-circuited Hall contacts. To calculate electron transport characteristics, we used the single-particle algorithm of the Monte Carlo procedure [11]. Three main scattering mechanisms were taken into account: electron scattering by ionized impurities, acoustic phonons and polar optical phonons. We considered a range of electric fields for which the intervalley transitions to the upper valleys are absent and the electron dispersion law can be considered as parabolic. The electron-electron scattering was not included into these calculations, because only small electron concentrations were considered. The components of the diffusion tensor $D_{ij}$ are calculated by means of the correlation functions according to the following equation:

$$D_{ij} = \frac{1}{2} \frac{d}{dt} \langle (r_i(t) - \langle r_i \rangle) (r_j(t) - \langle r_j \rangle) \rangle \quad (1)$$
where \( r_{i,j} \) are the simulated values of the three electron coordinates as functions of time which include random collisions (i.e. diffusion noise) associated with scattering processes and the angle brackets denote the time average.

### III. Transport properties without magnetic field

We present the transport properties for three particular examples, which differ by the lattice temperature \( T_0 \), the concentration of ionized impurities \( N_i \) and the electron concentration \( N_e \). For the case I we assume \( N_i = 10^{16} \) cm\(^{-3} \), \( N_e = 10^{15} \) cm\(^{-3} \) and \( T_0 = 30 \) K, for the case II we assume \( N_i = 10^{17} \) cm\(^{-3} \), \( N_e = 10^{16} \) cm\(^{-3} \) and \( T_0 = 30 \) K, and for the case III we assume \( N_i = 10^{16} \) cm\(^{-3} \), \( N_e = 10^{15} \) cm\(^{-3} \) and \( T_0 = 300 \) K. The results of calculation are shown in Fig. 2. As expected, the characteristic features of the streaming regime are observed only for the sample I. In the range of \( E = 3-10 \) kV/cm, the drift velocity and the average energy saturate (see Figs. 2(a) and (b)) reaching one half of the characteristic velocity \( V_0 = \sqrt{2\hbar\omega_0/m^*} \) and about one third of the characteristic energy \( \hbar\omega_0 \), respectively. In contrast, for the case II with larger impurity concentration, a well-developed streaming regime is not formed. At room temperature, in the range of \( E = 3-10 \) kV/cm, the electron gas remains almost at equilibrium, so \( V_d(E) \) shows a linear behaviour and \( \langle \varepsilon \rangle(E) \) is close to its equilibrium value of \( 3k_B T_0/2 \). The emergence of the streaming regime can be clearly identified by the strong non-monotonic field dependence of the transverse component of the average electron energy \( \langle \varepsilon_\perp \rangle \) for the case I (see Fig. 2(c)). The increase at \( E \) up to about 1 kV/cm is associated with heating of the electron gas, while with further increasing the field, \( \langle \varepsilon_\perp \rangle \) decreases due to the formation of a streaming-like distribution function elongated along the field direction [13]. This decrease tends to saturate in the field range \( 3-10 \) kV/cm. For the cases II and III, the streaming is not formed, and \( \langle \varepsilon_\perp \rangle \) exhibits only a slightly non-monotonic behaviour.

The field dependency of the transverse component of the diffusion coefficient \( D_{xx}(E) \) (shown in Fig. 2(d)) is qualitatively similar to that of \( \langle \varepsilon \rangle \) and is associated with the streaming regime that is realized for the case I. At \( E < 0.5 \) kV/cm, the magnitude of \( D_{xx}(E) \) rapidly increases from the equilibrium value of 13 cm\(^2\)/s to a maximum of about 250 cm\(^2\)/s due to the electrons isotropic spreading in the transverse direction. The maximum of \( D_{xx}(E) \) corresponds to the electric field at which the rapid spreading of the distribution function is terminated and a significant part of high-energy electrons loses its energy due to emission of optical phonons. At further increasing \( E \), the streaming-like distribution function begins to form and \( D_{xx} \) rapidly decreases approaching 20–25 cm\(^2\)/s at \( E > 5 \) kV/cm. For the cases II and III, \( D_{xx}(E) \) slowly decreases from 50 down to 30 cm\(^2\)/s with increasing \( E \) from 1 up to 10 kV/cm.

### IV. Diffusion coefficient for \( E \parallel H \)

It should be noted that for electrons with parabolic dispersion law, the application of a magnetic field along the direction of the electric field has no effect on \( V_d(E) \) and \( \langle \varepsilon_\perp \rangle(E) \) because the electron motions in the directions along and transverse to the fields are uncoupled [14]. However, the electron diffusion process shows a strong dependence on the magnetic field. Fig. 3 presents the electric field dependences of the transverse component of the diffusion coefficient \( D_{xx}(E) \), calculated for three values of \( H \) where panels (a), (b) and (c) correspond to the cases I, II and III, respectively. For comparison, \( D_{xx}(E) \) at \( H = 0 \) is shown by the dashed curve while the curves 1, 2 and 3 correspond to three magnitudes of the magnetic field: 1.1, 2.3 and 3.4 T, respectively. The behaviour of \( D_{xx}(E) \) in magnetic field has the following general peculiarities for all the cases:

(i) the dependences are non-monotonic with their maximum shifted to the higher electric fields with increasing \( H \); (ii) the magnetic field suppresses diffusion in transverse directions with respect to \( E \) and \( H \); (iii) the effect of magnetic field decreases at higher electric fields. The magnetic field effect on the electron diffusion is more important for the case I, for which the streaming regime is realized. Even at weak magnetic fields, we observe a strong suppression of the maximum diffusion coefficient, which decreases from \( \approx 250 \) cm\(^2\)/s at \( H = 0 \) down to \( \approx 60 \) cm\(^2\)/s at \( H = 1.1 \) T. With further increasing the magnetic field, the maximum \( D_{xx} \) progressively decreases down to 30 cm\(^2\)/s at \( H = 2.3 \) T and approximately...
15 \text{ cm}^2/\text{s} \text{ at } H = 3.4 \text{ T}. \text{ The position of the maximum diffusion coefficient corresponds to electric fields of 0.5, 1.5, 3 and 5.5 \text{ kV/cm for } H = 0, 1.1, 2.3 \text{ and } 3.4 \text{ T, respectively.}

For the cases II and III, for which the streaming regime does not occur, the magnetic field weakly modifies the diffusion coefficient. For example, the maximum $D_{xx}$ decreases only twice from $\simeq 60 \text{ cm}^2/\text{s}$ at $H = 0$ down to $\simeq 30 \text{ cm}^2/\text{s}$ at $H = 3.4 \text{ T}$.

Summarizing: the results of the modelling of diffusion processes in compensated GaN show that the largest variations of $D_{xx}$ with the magnetic field occur in the samples for which the streaming electron transport is formed.

V. DIFFUSION COEFFICIENT FOR $E \perp H$

In this section, we consider the field dependences of the components of the diffusion tensor corresponding to the transverse motion with respect to the electric field direction (see Fig. 1(b)): the $D_{xx}$ component describes the diffusion current perpendicular to $E$ and $H$ while the $D_{yy}$ component characterizes the diffusion in the direction parallel to $H$ and perpendicular to $E$. As previously discussed [11], [15], [16], the magnetic field for this configuration strongly affects the transport characteristics. In particular, the streaming transport regime can be destroyed by the magnetic field, forming a vortex-like distribution function in the momentum space. The result of the measurements of current-voltage characteristics depends on the form of the external circuits. Our transport model assumes the short-circuited Hall contacts. The behaviour of $D_{xx}$ has the same features (i)-(iii) listed in previous section. Indeed, as seen from Fig. 4(a), the impact of the magnetic field on the diffusion is largest for the case I. The dependences of $D_{xx}$ show non-monotonic behavior with a maximum shifted to the region of higher electric fields with increasing $H$. However there is also a region of electric fields where the diffusion is higher than at $H = 0$. This peculiarity was absent in the case of $E \parallel H$. Moreover, the shift of the maximum $D_{xx}$ is more evident for the configuration $E \perp H$. For the cases II and III (panels (b) and (c) in Fig. 4, respectively), the effect of the magnetic field is weak, the dependences of $D_{xx}$ are weakly non-monotonic with an extended region of electric fields $E = 1-6 \text{ kV/cm}$ (at $H = 2.3-3.4 \text{ T}$) where $D_{xx}$ is practically constant.

In contrast to the parallel configuration of $E$ and $H$, in the crossed configuration $D_{xx}$ and $D_{yy}$ are not equal. The electric

![Fig. 3. Dependences of $D_{xx}(E)$ at $E \parallel H$ for the case I (a); II (b); III (c) at $H = 0$ (dashed curves), $H = 1.1 \text{ T}$ (curves 1), $2.3 \text{ T}$ (curves 2), $3.4 \text{ T}$ (curves 3).](image1)

![Fig. 4. Dependences of $D_{xx}(E)$ at $E \perp H$ for the case I (a); II (b); III (c) at $H = 0$ (dashed curves), $H = 1.1 \text{ T}$ (curves 1), $2.3 \text{ T}$ (curves 2), $3.4 \text{ T}$ (curves 3).](image2)
field dependences of $D_{yy}$ at several values of $H$ are shown in Fig. 5. In general, the increase of $H$ leads to a growth of electron diffusion in the y-direction for a wide range of electric fields. In particular, for the case I (see panel (a) in Fig. 5) the amplitude and position of the maximum are weakly modified with increasing $H$ within the range 1.1–3.4 T. However, at weak magnetic fields, 0–1.1 T, the maximum $D_{yy}$ is twice increased. With increasing electric field, diffusion along the direction of magnetic field progressively decreases, and at $E > 6$ kV/cm it tends to the case corresponding to $H = 0$. For the case II, the impact of magnetic field on $D_{yy}$ is not so evident as for the case I: its maximum is increased by $\approx 30\%$ with increasing the magnetic field from 0 up to 3.4 T. For the case III, the magnetic field does not have any influence on $D_{yy}$.

Fig. 5. Dependences $D_{yy}(E)$ at $E \perp H$ for the case I (a); II (b); III (c) at $H = 0$ (dashed curves), $H = 1.1$ T (curves 1), 2.3 T (curves 2), 3.4 T (curves 3).

VI. Conclusions

We have presented a Monte Carlo analysis of the diffusion coefficient in Gallium Nitride as a function of electric and magnetic fields in parallel and perpendicular configurations. Three samples have been considered: sample I with low concentration and low temperature, sample II with high concentration and low temperature and sample III with low concentration and room temperature. We have found that the strongest impact of magnetic field on the diffusion properties of the electron gas takes place for the sample I. In the parallel configuration of $E$ and $H$, the electric field dependences of the transverse-to-current components of the diffusion tensor exhibit a non-monotonic behaviour with a maximum whose amplitude and position depend on the magnitudes of magnetic field. In particular, the maximum decreases and its position is shifted to higher electric fields at increasing values of the magnetic field. In the crossed configuration of $E$ and $H$, the transverse-to-fields component of the diffusion tensor has similar behaviour. However, the positions of the maximum are more sensitive to the variation of $H$. The suppression of electron diffusion in the transverse–to–$H$ direction with the increase of magnetic field is a general phenomenon that is observed in both configurations. However, in the crossed configuration, the magnetic field enhances electron diffusion along the $H$ direction. The electric field dependences of the longitudinal-to–$H$ component of the diffusion tensor also have the maximum but both position and amplitude of the maximum weakly depend on the values of $H$. The physical parameters of the samples II and III prevent the formation of the streaming transport regime and this is the main reason of the weak influence of electric and magnetic fields on diffusion properties in these samples. However, the main peculiarities observed in the field dependences of diffusion processes for the sample I take place for the samples II and III as well. We suggest that the streaming transport regime and related magneto-transport effects can be investigated by measurements of the diffusion effects of hot electrons in electric and magnetic fields using, for instance, electro-gradient and optical measurements of the diffusion coefficient. Finally we emphasise that the knowledge of the far–from–equilibrium dependences of the diffusion coefficient is important for the modelling of electronic devices operating in strong electric and magnetic fields.

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Probabilistic analysis of two models of ideal memristor with external noise

1st Anna A. Kharcheva
Radiophysics Department
Lobachevsky State University
Nizhny Novgorod, Russia
kharcheva@rf.unn.ru

2nd Alexander A. Dubkov
Radiophysics Department
Lobachevsky State University
Nizhny Novgorod, Russia
dubkov@rf.unn.ru

3rd Bernardo Spagnolo
Dipartimento di Fisica e Chimica “Emilio Segrè”
Group of Interdisciplinary Theoretical Physics
Università di Palermo and CNISM, Unità di Palermo
Palermo, Italy
bernardo.spagnolo@unipa.it

Abstract—This work is devoted to probabilistic analysis of two models of the ideal memristor with an external Gaussian noise. First we study the charge-controlled ideal memristor and analyze how the applied voltage in the form of a stationary Gaussian noise influences the probability density function of the charge and resistance. For comparison we pay attention to the case of applied current in the same form. Further the current-controlled ideal memristor with the external fluctuations in the form of a stationary Gaussian noise is under consideration. The exact results are based on well-known theorems of the probability theory.

Index Terms—ideal memristor, charge-controlled memristor, probability density function, external Gaussian noise

I. INTRODUCTION

The memristor is a new fundamental element of the electrical circuit that dissipates energy and has memory. It was theoretically predicted by Chua in 1971 [1], but found its hardware implementation only in 2008 [2]. According to the axiomatic definition of the ideal memristor [1], its properties are unambiguously determined by the memristance versus charge map. The original model of the “HP memristor” [2] introduces this map via a linear function that represents this memristor as a variable resistor whose resistance is linearly dependent on the amount of charge flowing through. However, a lot of subsequent experimental works in this area, the current-voltage characteristics and various physical considerations have indicated the need to use nonlinear models to correctly describe the properties of memristors, e.g. a nonlinear dependence of the resistance on an external controlling variable. Various nonlinear dependencies of the resistor on charge were proposed [3], [4], and attempts were done to construct the nonlinear characteristics by probing with special deterministic perturbations [5], [6].

Currently, much attention is focused on the study of the memristor behavior when applying to it external deterministic or random signals and its switching mechanism [7]–[10], whose resistance varies according to the voltage applied to them, or the current flowing through (see, for example, reviews [11]–[14]). Unfortunately, at the moment theoretical, and even more experimental papers on the study of the statistical properties of the memristor as a separate element of the electrical circuit is not enough. There are only a few studies of the steady-state probability distributions of the current flowing through the memristor in a circuit with a resistance or a capacitor, when a white Gaussian noise source is applied to it [8], [9].

In this paper we demonstrate how the Gaussian fluctuations of the applied voltage or current may change the memristor’s state. These changes are indicated on an example of the ideal memristor using the probability characteristics of its resistance.

II. GENERAL FORMULAS

The simplest model of a memory device is the ideal memristor proposed in [1]. For this nonlinear element of electrical circuit, the Ohm’s law and the associated equations of state are written as follows

\[
U(t) = R(q)I(t), \quad I(t) = \frac{dq}{dt} \tag{1}
\]

where \(U(t), I(t)\) and \(q(t)\) are the voltage, the current and the charge on the memristor respectively.

As follows from (1), the ideal memristor is an integrable model and, hence, can be defined by an equivalent algebraic function

\[
w(t) = \int_0^t U(t)dt = \int_0^{q(t)} R(q)dq = \Phi(q(t)). \tag{2}
\]

Let us apply to memristor the voltage \(U(t)\) in the form of a stationary Gaussian noise with non-zero mean \(U_0\) and the correlation function \(K(\tau)\). According to (2), the random process \(w(t)\) is again a Gaussian random process with the following probability distribution

\[
P_w(y, t) = \frac{1}{\sqrt{4\pi D(t)}} \exp \left\{ \frac{-(y - U_0t)^2}{4D(t)} \right\}, \tag{3}
\]

where

\[
D(t) = \int_0^t (t - \tau) K(\tau) d\tau. \tag{4}
\]

Then we can apply the theorem of the probability theory to calculate from (2)-(3) the probability density function (PDF) of the charge flowing through a memristor

\[
P_q(z, t) = \frac{\Phi'(z)}{\sqrt{4\pi D(t)}} \exp \left\{ -\frac{\left(\Phi(z) - U_0t\right)^2}{4D(t)} \right\} \tag{5}
\]
and as a consequence, the PDF of the resistance
\[ P_R(r, t) = \frac{r}{\sqrt{4\pi D(t)}} \sum_k \frac{1}{|\Phi'(q_k(r))|} \exp \left\{ -\frac{\Phi(q_k(r)) - U_0 t^2}{4D(t)} \right\}, \tag{6} \]
where \( q_k(R) \) is the k-th branch of uniqueness of the function \( R = \Phi(q) \). By measuring of this PDF in the experiment, we can find an unknown algebraic function \( \Phi(q) \) and determine all the statistical characteristics of the memristor which we are interested in.

III. CHARGE-CONTROLLED MEMRISTOR

In the case described in [3] the authors considered the following monotonic exponential dependence of the resistance on charge
\[ R(q) = R_{ON} + \frac{\Delta R}{e^{(-q+q_0)/\gamma_0} + 1}, \tag{7} \]
where \( \Delta R = R_{OFF} - R_{ON} \) and \( R_{ON} \ll R_{OFF} \). The parameter \( q_0 \) is a characteristic charge required to switch the memristor and specifies the steepness of the transition between low resistance state (LRS) \( R_{ON} \) and high resistance state (HRS) \( R_{OFF} \). The constant \( q_1 \) is a parameter determining the memristance at the initial moment of time \( t = 0 \).

The inverse to (7) monotonic function reads
\[ q(R) = -q_1 - q_0 \ln \left( \frac{R_{OFF} - R}{R - R_{ON}} \right). \tag{8} \]

According to the definition (2) of the algebraic function, from (7) we find
\[ \Phi(q) = qR_{ON} + q_0 \Delta R \ln \left( e^{(q+q_1)/\gamma_0} + 1 \right), \tag{9} \]
\[ \Phi'(q) = R'(q) = \frac{\Delta R}{q_0} e^{-(q+q_1)/\gamma_0} \left( e^{(q+q_1)/\gamma_0} + 1 \right). \tag{10} \]
Substituting (8)-(10) in (6), we obtain finally the exact expression for the PDF of the resistance \( R(t) \) in the following form
\[ P_R(r, t) = \frac{q_0 \Delta R}{\sqrt{4\pi D(t)}} \frac{r}{(R_{OFF} - r)(r - R_{ON})} \exp \left\{ -\frac{\Phi(q(r)) - U_0 t^2}{4D(t)} \right\}, \tag{11} \]
where
\[ \Phi(q(r)) = -q_1 R_{ON} - q_0 \Delta R \ln \left( e^{q_1/\gamma_0} + 1 \right) + q_0 R_{OFF} \ln \left( \frac{\Delta R}{R_{OFF} - r} - q_0 R_{ON} \ln \left( \frac{\Delta R}{r - R_{ON}} \right) \right). \]

Let’s consider two kinds of the applied random voltage. For the white Gaussian noise \( U(t) \) with the correlation function
\[ K(\tau) = 2\delta(\tau), \tag{12} \]
where \( 2D \) is the noise intensity, we have to substitute \( D(t) = Dt \) in (11) in accordance with (4). The plots of corresponding PDF of the resistance are depicted in Fig. 1. As seen from Fig. 1, the noise causes the memristor to switch to high resistance state. In the case of the noise with zero mean one can observe both states.

For the case of colored Gaussian noise \( U(t) \) with the exponential correlation function
\[ K(\tau) = \sigma^2 \exp(-\tau/\tau_c), \tag{13} \]
where \( \sigma^2 \) and \( \tau_c \) are the variance and the correlation time respectively, equation (4) gives
\[ D(t) = \sigma^2 \tau_c \left( t - \tau_c \left( 1 - e^{-t/\tau_c} \right) \right). \tag{14} \]
The plots of corresponding PDF (11) of the memristance are shown in Fig. 2. In Fig. 2 we observe the same tendencies as in Fig. 1.

For comparison we apply to memristor the current \( I(t) \) in the form of a stationary Gaussian noise with zero mean and the correlation function \( K(\tau) \). In accordance with (1), the charge \( q(t) \) is a Gaussian process with the following probability distribution
\[ P_q(z, t) = \frac{1}{\sqrt{4\pi D(t)}} \exp \left\{ -\frac{z^2}{4D(t)} \right\}. \tag{15} \]
Corresponding to (15) the PDF of the memristance in this case reads

$$P_R(r,t) = \frac{1}{\sqrt{4\pi D(t)}} \sum_k \frac{1}{|R'(q_k(r))|} \exp \left\{ -\frac{q_k^2(r)}{4D(t)} \right\}. \quad (16)$$

Here we used the same notations as in (6).

For considering case of an exponential dependence of the memristance on charge from (8) and (16) we obtain

$$P_R(r,t) = \frac{1}{\sqrt{4\pi D(t)}} \frac{q_0 \Delta R}{(R_{OFF} - r)(r - R_{ON})} \times \exp \left\{ -\frac{1}{4D(t)} \left[ q_1 + q_0 \ln \frac{R_{OFF} - r}{r - R_{ON}} \right]^2 \right\}. \quad (17)$$

The 2D plot of PDF of the memristance for the case of white Gaussian noise with non-zero mean $\delta_0$ and the intensity $2D_1$. The charge $q(t)$ is again Gaussian process, but according to (19), the probability distribution of the bounded random process $l(t)$ is non-Gaussian and contains two delta functions

$$P_1(y,t) = p_1(t)\delta(y) + p_2(t)\delta(y-1) + \frac{1}{\sqrt{4\pi D_1 c_0^2 t}} \exp \left\{ -\frac{(y - c_0 I_0 t)^2}{4D_1 c_0^2 t} \right\} 1_{[0,1]}(y), \quad (20)$$

where: $c_0 = \mu_V R_{ON}/L^2$, $1_A(y)$ is the indicator of set $A$ and

$$p_1(t) = \int_{-\infty}^{0} \frac{1}{\sqrt{4\pi D_1 c_0^2 t}} \exp \left\{ -\frac{(y - c_0 I_0 t)^2}{4D_1 c_0^2 t} \right\} dy,$$

$$p_2(t) = \int_{1}^{\infty} \frac{1}{\sqrt{4\pi D_1 c_0^2 t}} \exp \left\{ -\frac{(y - c_0 I_0 t)^2}{4D_1 c_0^2 t} \right\} dy.$$
V. CONCLUSIONS

Two models of an ideal memristor with the external Gaussian noise have been under consideration. For both cases we have obtained exact analytical expressions for the PDF of the memristance. We have shown that for charge-controlled memristor the shape of the probability density function of resistance depends on what is applied in the form of Gaussian noise, the voltage or the current. Also, different noise excitations in the form of white and colored Gaussian noise have been analyzed. In the specific example of an exponential dependence of the resistance on the charge the influence of the noise mean value and the type of driven Gaussian noise on the memristor’s switchings between two states has been found.

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Noise-delayed decay for Lévy flights in unstable parabolic potential

1st Alexander A. Dubkov
Radiophysics Department
National Research Lobachevsky State University
Nizhny Novgorod, Russia
dubkov@rf.unn.ru

2nd Bartłomiej Dybiec
Marian Smoluchowski Institute of Physics
Jagiellonian University
Kraków, Poland
bartek@th.if.uj.edu.pl

3rd Bernardo Spagnolo
Dipartimento di Fisica e Chimica “Emilio Segré”
Group of Interdisciplinary Theoretical Physics
Università di Palermo and CNISM, Unità di Palermo
Palermo, Italy
bernardo.spagnolo@unipa.it

4th Davide Valenti
Dipartimento di Fisica e Chimica “Emilio Segré”
Group of Interdisciplinary Theoretical Physics
Università di Palermo and CNISM, Unità di Palermo
Palermo, Italy
davide.valenti@unipa.it

Abstract—We obtain the exact formula for the nonlinear relaxation time (NLRT) of anomalous diffusion of a particle in the form of Lévy flights in an unstable parabolic potential. Some peculiarities of the noise-delayed decay phenomenon in such a system compared with Brownian diffusion are discussed. In particular, we show how the NLRT depends on the stability index of the noise and initial position of a particle.

Index Terms—Lévy flights, fully unstable potential, nonlinear relaxation time, noise-delayed decay

I. INTRODUCTION

Recently, interest in the theoretical and experimental investigations of unstable systems has been renewed [1]–[3]. For example, the sensitivity of optomechanical sensors is commonly limited by parametric instability [1]. To suppress this instability in a micron-scale cavity optomechanical system the feedback via optomechanical transduction and electrical gradient force actuation is applied.

Unstable systems have a strong tendency to diverge during a short period of time. This is manifested, for example, in the divergence of all statistical moments of their phase variables. As a result, these characteristics cannot provide exhaustive information about stochastic dynamics of unstable systems. Additional information can be given by studying the evolution of probability distributions [2], [3], as well as by analysis of various temporal characteristics. Moreover, in such systems the constructive role of noise can be manifested. In particular, this role consists in delaying the decay from unstable nonequilibrium states.

The phenomenon of noise-delayed decay of unstable states was well-investigated for the Brownian diffusion [4], [5] (see also [6]) using the theory of first-passage times (FPT) and apparatus of nonlinear relaxation time (NLRT). It was shown that by choosing the optimal value of the noise intensity it is possible to slow down the decay of the initially unstable states of the system. This phenomenon for systems with metastable states is known as noise-enhanced stability (NES) [7]–[13]. However, there are still no studies of the peculiarities of noise-delayed decay phenomenon for anomalous diffusion, in particular for Lévy flights.

In this work we obtain exact results for the NLRT of anomalous diffusion in the form of Lévy flights with an arbitrary Lévy index \( \alpha \) in an unstable parabolic potential. A behavior of the NLRT versus the noise intensity parameter \( \sigma \) for different values of Lévy index \( \alpha \) and initial positions of a particle are discussed in detail. The effect is compared with the case of overdamped Brownian motion.

II. GENERAL RELATIONS

The anomalous diffusion in the form of Lévy flights for a particle moving in a potential profile \( U(x) \) is described by the following fractional Fokker-Planck equation for the probability density of transitions \( P(x,t|x_0,0) \) [14]–[16]

\[
\frac{\partial P}{\partial t} = \frac{\partial}{\partial x} \left[ U'(x) P \right] + \sigma^\alpha \frac{\partial^\alpha P}{\partial |x|^{\alpha}},
\]

where \( \alpha \) is the Lévy index \((0 < \alpha < 2)\) and \( \sigma \) is the parameter characterizing the intensity of Lévy noise. Equation (1) is associated with the following Langevin equation

\[
\frac{dx}{dt} = -U'(x) + \xi_\alpha(t),
\]

where \( x(t) \) is the particle coordinate and \( \xi_\alpha(t) \) is a symmetric white \( \alpha \)-stable noise.

The nonlinear relaxation time for a diffusion in a potential profile with a sink is defined as [6]

\[
T(x_0) = \int_0^\infty \Pr(t|x_0) dt,
\]
where
\[ \Pr(t, x_0) = \int_{L_1}^{L_2} P(x, t | x_0, 0) \, dx \quad (4) \]
represents the probability to find a particle in the interval \((L_1, L_2)\) at the time \(t\), if it starts from the point \(x_0 \in (L_1, L_2)\). Substituting (4) in (3) and changing the order of integration we arrive at
\[ T(x_0) = \int_{L_1}^{L_2} Y(x, x_0) \, dx, \quad (5) \]
where
\[ Y(x, x_0) = \int_0^\infty P(x, t | x_0, 0) \, dt. \quad (6) \]
Integrating both parts of (1) with respect to \(t\) from 0 to \(\infty\) and taking into account the obvious initial condition \(P(x, 0 | x_0, 0) = \delta(x - x_0)\) and asymptotics (for a potential with a sink) \(P(x, \infty | x_0, 0) = 0\), we obtain the following equation for the function \(Y(x, x_0)\)
\[ \frac{d}{dx} [U^\prime(x)] Y + \sigma^2 \alpha |k|^\alpha Y = -\delta(x - x_0). \quad (7) \]
To solve equation (7) it is better to introduce the Fourier transform of the function \(Y(x, x_0)\), i.e.
\[ \widetilde{Y}(k, x_0) = \int_0^\infty Y(x, x_0) e^{ikx} \, dx. \quad (8) \]
For smooth potential profiles \(U(x)\), after the Fourier transform (7) can be written in the following form
\[ \frac{d}{dx} [U^\prime(x)] \tilde{Y} + \sigma^2 |k|^\alpha \tilde{Y} = e^{ikx_0}. \quad (9) \]
Substituting \(Y(x, x_0)\) from the backward Fourier transform of (8) in (5) and changing the order of integration we have for the NLRT
\[ T(x_0) = \frac{1}{2\pi} \text{Re} \left\{ \int_0^\infty \tilde{Y}(k, x_0) \frac{e^{-ikL_1} - e^{-ikL_2}}{ik} \, dk \right\}. \quad (10) \]
One can easily check that after replacing \(k\) with \(-k\) (9) coincides with the equation for the complex conjugate function \(\tilde{Y}^\ast(k, x_0)\), i.e. \(\tilde{Y}(-k, x_0) = \tilde{Y}^\ast(k, x_0)\). As a result, (10) can be rearranged in a simpler form
\[ T(x_0) = \frac{1}{\pi b} \text{Re} \left\{ \int_0^\infty \tilde{Y}(k, x_0) \frac{e^{-ikL_1} - e^{-ikL_2}}{ik} \, dk \right\}, \quad (11) \]
where \text{Re} denotes the real part of expression. Thus, according to (11), it is sufficient to solve equation (9) only in the area of \(k > 0\), i.e. the following one
\[ U^\prime \left( -i \frac{d}{dk} \right) \tilde{Y} - i\sigma^2 k^{\alpha - 1} \tilde{Y} = e^{ikx_0} \frac{\pi b}{ik}. \quad (12) \]

### III. NLRT for a Particle in the Unstable Parabolic Potential

Let us solve the problem for the inverse parabolic potential \(U(x) = -bx^2/2\), see Fig. 1. Substituting this potential in (12) we arrive at the following first-order differential equation
\[ \frac{d}{dk} \tilde{Y} - \frac{\sigma^2 k^{\alpha - 1}}{b} \tilde{Y} = \frac{e^{ikx_0}}{bk}. \quad (13) \]
The general solution of (13) has the form
\[ \tilde{Y}(k, x_0) = e^{-\frac{\sigma^2 k^{\alpha}}{b}} \left( C + 1 \int_k^\infty e^{i\sigma k^{\alpha} - \frac{\sigma^2 k^{\alpha}}{b}} \frac{dq}{q} \right), \quad (14) \]
where the integral converges. In accordance with the properties of Fourier transform (8): \(\tilde{Y}(k, x_0) \to 0\) at \(k \to \infty\). As a result, we find from (14) the unknown constant: \(C = 0\) and, as a consequence,
\[ \tilde{Y}(k, x_0) = \frac{1}{b} e^{-\frac{\sigma^2 k^{\alpha}}{b}} \int_k^\infty e^{i\sigma k^{\alpha} - \frac{\sigma^2 k^{\alpha}}{b}} \frac{dq}{q}. \quad (15) \]
Substituting (15) in (11) and changing the order of integration we get
\[ T(x_0) = \frac{1}{\pi b} \text{Re} \left\{ \int_0^\infty e^{i\sigma k^{\alpha} - \frac{\sigma^2 k^{\alpha}}{b}} \frac{dq}{q} \int_k^\infty e^{i\sigma k^{\alpha} - \frac{\sigma^2 k^{\alpha}}{b}} \frac{dq}{q} \right\}. \quad (16) \]
If we put in (16) to simplify \(L_1 = -L, L_2 = L\), we arrive at
\[ T(x_0) = \frac{2}{\pi b} \int_0^\infty \cos(qx_0) e^{-i\frac{\sigma^2 q^{\alpha}}{b}} \frac{dq}{q} \int_0^q \sin kL \frac{ek^{\alpha}}{k} \, dk. \quad (17) \]
Equation (17) is the exact result obtained in quadratures for the NLRT of Lévy flights with an arbitrary Lévy index \(\alpha\) in the unstable parabolic potential.

### IV. General Formula for NLRT in Special Cases

Let us verify the result given by (17) in the absence of noise \(\xi_0(t)\). Putting \(\sigma = 0\) in (17) we arrive at
\[ T_{dyn} = \frac{2}{\pi b} \int_0^\infty \cos(qx_0) \frac{dq}{q} \int_0^q \sin kL \frac{ek^{\alpha}}{k} \, dk = \frac{2}{\pi b} \int_0^\infty \cos(qx_0) \frac{dq}{q} \int_0^q \sin kL \frac{ek^{\alpha}}{k} \, dq. \quad (18) \]
where \( \text{si}(x) \) in the sine integral function. Using the following auxiliary integral \((\alpha > 0, \beta > 0)\)
\[
\int_0^\infty \frac{\cos (\beta x)}{x} \text{si}(\alpha x) \, dx = \left\{ \begin{array}{ll} \frac{\pi}{2} \ln (\alpha/\beta), & \alpha > \beta, \\ 0, & \alpha < \beta \end{array} \right.
\]
and taking into account that \(|x_0| < L\), we find the dynamical time in which the particle from its initial position \(x_0\) reaches one of the boundaries \(\pm L\), rolling down the potential profile (see Fig. 1)
\[
T_{\text{dyn}} = \frac{1}{b} \ln \frac{L}{|x_0|}.
\] (19)

On the other hand, the direct integration of (2) with \(U(x) = bx^2/2\) and \(\xi_0(t) = 0\),
\[
\int_{x_0}^L \frac{dx}{x} = \int_0^{T_{x_0}} b \, dt
\]
gives the same result.

Let us show now that in the case of Lévy index \(\alpha = 1\) the NLRT (17) can be written in the form of a single integral. Substituting \(\alpha = 1\) in (17) and changing the order of integration we arrive at
\[
T(x_0) = \frac{2}{\pi b} \int_0^\infty \frac{\sin kL}{k} e^{\sigma k/b} dk \int_0^\infty \frac{\cos (q x_0)}{q} e^{\sigma q/b} dq.
\] (20)

Differentiating both sides of (20) with respect to the parameter \(x_0\) and calculating the internal integral, we have
\[
T'(x_0) = \frac{2}{\pi} \int_0^\infty \frac{bx_0 \cos k x_0 + \sigma \sin k x_0}{k (\sigma^2 + b^2 x_0^2)} \sin kL \, dk.
\]

Applying Dirichlet
\[
\int_0^\infty \frac{\sin \alpha x}{x} \, dx = \frac{\pi}{2} \text{sgn}(\alpha)
\]
and Frullani formulas
\[
\int_0^\infty \frac{\cos \alpha x - \cos bx}{x} \, dx = \ln \frac{|b|}{|\alpha|},
\]
where sgn\((x)\) is the sign function, we obtain from (21)
\[
T'(x_0) = \frac{\sigma}{\pi} \ln \frac{L-x_0}{L+x_0} - \frac{bx_0}{\sigma^2 + b^2 x_0^2} (L - x_0),
\] (22)

where \(\mathbb{I}(x)\) is the step function. According to (17),
\[
\lim_{x_0 \to \infty} T(x_0) = 0.
\]

As a consequence, we find from (22) the NLRT for the case \(\alpha = 1\)
\[
T(x_0) = \frac{1}{2b} \ln \frac{\sigma^2 + b^2 L^2}{\sigma^2 + b^2 x_0^2} + \frac{\sigma}{\pi} \ln \int_{x_0}^\infty \frac{L + z}{L - z} \, dz
\]
\[
\int_{L-x_0}^{L+x_0} \frac{L - z}{L + z} \, dz.
\] (23)

where \(|x_0| < L\). Obviously, in the case of \(\alpha = 0\) (23) coincides with (19).

![Fig. 2. Normalized nonlinear relaxation time as a function of the noise intensity parameter \(\sigma\) for \(L = 1\), different values of Lévy index \(\alpha\) and various initial positions of a particle \(x_0 = 0.01\) (top panel), \(x_0 = 0.1\) (middle panel) and \(x_0 = 0.5\) (bottom panel). The case \(\alpha = 2\) corresponds to usual Brownian motion. Error bars, which are standard deviations of the mean, are within the symbol size.](image)

### V. Discussion

The plot in semi-log scale of the normalized nonlinear relaxation time \(T(x_0)/T_{\text{dyn}}\) as a function of the scale parameter \(\sigma\) is depicted in Fig. 2, where points correspond to the Monte Carlo simulation of the Langevin equation (2) while solid lines present numerical integration of the exact formula (17). As seen from Fig. 2, Monte Carlo simulations nicely corroborate exact results. Subsequent rows correspond to various values of \(x_0\): \(x_0 = 0.01\) (top panel), \(x_0 = 0.1\) (middle panel) and \(x_0 = 0.5\) (bottom panel).

As it can be seen in Fig. 2, for the fixed interval half-width \(L\) the NLRT \(T(x_0)\) depends both on the stability index
\(\alpha\) and the initial condition \(x_0\). Moreover, it can be a non-monotonous function of the scale parameter \(\sigma\). Middle and bottom panel of Fig. 2 demonstrate that analogously like in the noise enhanced stability [8] the increasing noise can extend the lifetime of unstable states, i.e. it results in the noise delayed decay (NDD) phenomenon. In such a case, on the one hand, the scale parameter is large enough that particle can surmount the potential barrier. On the other hand, the noise is weak enough thus majority of escapes take place into the direction of the deterministic force. Therefore, the delay in the decay is caused by excursions into the direction of the origin, which slow down the escape rate.

As follows from Fig. 2, the region of NDD (area of increasing NLRT compared with the dynamical decay time \(T_{dyn}\)) is most clearly visible at initial position of a particle \(x_0\) close to the boundary \(L = 1\) (bottom panel). At the same time, the NDD phenomenon for the noise with Lévy index \(\alpha < 1\) can be observed only for very small values of the noise intensity parameter \(\sigma\). It should be noted also that the effect is great for Brownian diffusion \((\alpha = 2)\) than for anomalous one.

VI. Conclusions

Using analytical considerations we have derived the closed formula for the nonlinear relaxation time of Lévy flights in fully unstable potential. In the case of Cauchy noise excitation, i.e. Lévy noise with the stability index \(\alpha = 1\), the general formula for the NLRT can be significantly simplified.

In case of unstable potentials, the NLRT is sensitive both to the initial position and noise parameters. In particular, it can be a non-monotonous function of the scale parameter \(\sigma\). Therefore, for a fixed initial position and the value of the Lévy index it is possible to find such a value of the scale parameter which maximally slows down the escape process, i.e. it extends the lifetime of the fully unstable state.

Acknowledgment

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References

RF Noise Benchmark Tests for MOSFETs

NXP Semiconductors, High Tech Campus 46, 5656 AE Eindhoven, The Netherlands

Introduction. The understanding and modeling of thermal noise is essential for the usage of highly scaled CMOS in RF circuit applications. In this paper, we present a set of RF noise benchmark tests, which can be used in (i) assessing the correctness of RF noise measurements, (ii) developing RF noise model equations, and (iii) assessing the validity of model libraries for a given CMOS process.

Benchmark tests. An overview of the benchmark tests is given in Table 1. Apart from the noise power spectral density of the drain current, $S_{I_D}$, we make use of two auxiliary quantities, which are (i) the Fano factor $F = S_{I_D}/(2qI_D)$, and (ii) the white noise gamma factor $\gamma = S_{I_D}/(4k_BTg_{DS})$, where $g_{DS}$ is the zero-Vth output conductance [1]. Here, we illustrate some of them with the help of measurements and simulations on a high-k metal gate 28-nm CMOS technology from a foundry. This technology provides various threshold voltage ($V_{th}$) flavors. In Figure 1(a), we compare the three flavors for NMOS and PMOS, by plotting $S_{I_D}$ versus $I_D$. Clearly, the measurements on three flavors overlap nicely for both NMOS and PMOS, as demanded by test #7 in Table 1. In Figure 1(b), we plot $\gamma$ as a function of $V_{DS}$ for the same devices. First, we observe that for $V_{DS} = 0 \, \text{V}$, $\gamma \approx 1$ in all the measurements, as demanded by test #1 in Table 1 (which is simply the Nyquist law). This provides a lot of confidence in the measurement system used. At higher $V_{DS}$, both for NMOS and PMOS, we observe a strong noise enhancement in line with Refs. [2,3], as demanded by test #8 in Table 1. Again, the measurements nearly overlap for the different $V_{th}$ flavors.

Simulations were carried out with the model library provided by the foundry (colored lines in Figs. 1(a) and (b)). As can be seen, the model deviates from the measurements and shows some differences between the $V_{th}$-flavors that are not observed in the measurements. It is important to note, however, that even when measurements would not have been available, our benchmark tests reveal the shortcomings in the simulation model, and can be used to provide feedback to the foundry in question. This demonstrates the usefulness of this set of simple benchmark tests.

Table 1: Overview of benchmark tests for thermal noise.

<table>
<thead>
<tr>
<th>bias $V_{DS}$</th>
<th>channel length</th>
<th>quantity</th>
<th>test</th>
<th>remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 $V_{DS} = 0 , \text{V}$</td>
<td>all</td>
<td>$S_{I_D}$</td>
<td>$\gamma = 1$</td>
<td>Ref. [4]</td>
</tr>
<tr>
<td>#2 $V_{DS} = 0 , \text{V}$</td>
<td>all</td>
<td>$S_{I_D}$</td>
<td>$\gamma = 1$</td>
<td>Ref. [4]</td>
</tr>
<tr>
<td>#3 $V_{DS} = 0 , \text{V}$</td>
<td>all</td>
<td>$c$</td>
<td>$c = 0$</td>
<td>in the limit $f \rightarrow 0 , \text{Hz}$; Ref. [5]</td>
</tr>
<tr>
<td>#4 weak inversion</td>
<td>all</td>
<td>$S_{I_D}$</td>
<td>$F = 1$</td>
<td>disregard $S_{I_D}$ contributions from gate to drain; Ref. [6]</td>
</tr>
<tr>
<td>#5 saturation</td>
<td>long</td>
<td>$S_{I_D}$</td>
<td>$\gamma = 2/3$</td>
<td>Ref. [1]</td>
</tr>
<tr>
<td>#6 saturation</td>
<td>long</td>
<td>$c$</td>
<td>$c = 0.4j$</td>
<td>Ref. [1]</td>
</tr>
<tr>
<td>#7 all</td>
<td>all</td>
<td>$S_{I_D}$</td>
<td>different $V_{th}$ flavors should nearly coincide when plotted against $I_D$</td>
<td></td>
</tr>
<tr>
<td>#8 saturation</td>
<td>short</td>
<td>$S_{I_D}$</td>
<td>$\gamma$ enhancement in line with Figs. 1 in Refs. [2,3] switch off gate resistance</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: RF noise, measured at $f = 30 \, \text{GHz}$, for NMOS and PMOS devices in high-k metal gate 28-nm CMOS technology. The devices consist of 20 fingers, each having gate width $W = 0.8 \, \mu\text{m}$ and gate length 30 nm. Different colors represent different $V_{th}$-flavors, which are super-low VT, low VT, and regular VT. Markers represent measurements, colored lines represent corresponding simulations done with the foundry model library. Figure (a) shows $S_{I_D}$ as a function of the drain current $I_D$, at a fixed drain-source voltage $V_{DS} = 0.9 \, \text{V}$. Figure (b) shows the white-noise gamma factor $\gamma$ as a function of the drain-source voltage $V_{DS}$, at a fixed gate-source voltage $V_{GS} = 0.5 \, \text{V}$.

1/f Noise in Fully Integrated Electrolytically Gated FinFETs with Fin Width Down to 20nm

Koen Martens, Bert Du Bois, Yong Kong Siew, Anshul Gupta, Anabela Veloso, Emmanuel Dupuy, Dunja Radisic, Efrain A. Sanchez, Willem Van Roy, Simone Severi, Eddy Simoen
Imec, Leuven, Belgium, koen.martens@imec.be

Abstract—The low frequency noise of nanoscale electrolytically gated FETs will determine the ISFET or BioFET sensor signal-to-noise ratio and is hence of key importance for FET-based biosensing applications. We report on the 1/f noise of fully integrated electrolytically gated FETs fabricated in a 300nm fabrication facility with fin widths down to 20nm. For a 20nm wide, 10μm long device we observe a gate referred noise Power Spectral Density (PSD) down to ~1x10^-10V^2/Hz at 10Hz, which is significantly lower than previously reported for electrolytically gated top-down fabricated FETs. We find the gate referred noise PSD to increase with increasing overdrive, indicating that lower overdrive is better for sensor signal-to-noise ratio.

Keywords—ISFET, BioFET, FinFET, 1/f noise, pink noise

In this work the low frequency noise of electrolytically gated pMOS finFETs was characterized. The inversion mode pMOS finFET devices were fully fabricated in a 300nm processing facility in a process derived from a solid gate finFET process [1]. Fin widths down to 20nm were reached. Unlike MOSFETs, ISFETs (or bioFETs) use an electrolyte instead of metal or polysilicon as gate electrode. The devices had a 4nm SiON gate dielectric exposed to a 15mM phosphate buffered saline (PBS) electrolyte solution contacted by a Ag/AgCl electrode.

To accommodate the electrolyte a circular silicone flow cell with diameter 8 mm and depth 1 mm (by Grace Bio-Labs) was mounted on the die (DUT). The flow cell was connected to a flow-through true Ag/AgCl reference electrode (Microelectrodes Inc). The reference electrode contains a Ag/AgCl wire in contact with a 3 M KCl solution, separated from the buffer solution flowing in the flow-through part by a ceramic frit. The connections were made using polyvinyl chloride (PVC) and polytetrafluoroethylene (PTFE) tubes with inner and outer diameter of 1/16 inch x 1/8 inch (for PVC) and 0.023 inch x 1/16 inch (for PTFE), connected to a syringe containing the diluted PBS buffer solution. A Keysight E4727A low frequency noise analyzer was used which can measure noise down to 0.67 nV/√Hz at 10kHz. The drain to source bias Vds was -0.1V and the bulk to source bias Vbs was 0V.

The typical drain current vs. gate voltage characteristic (I_d-V_g) is shown in the inset of fig. 1. The threshold voltage of the device is -0.25V. The gate voltage referred noise spectrum exhibits 1/f behavior across the measured gate bias range (see fig. 1).

Fig. 1. Gate voltage referred noise power spectrum showing 1/f noise. A pMOS with 20nm fin width and 1μm channel length is shown. I_d-V_g of the device is shown in the inset (V_T=-0.25V).

We have investigated the geometry dependence of the gate voltage referred noise power and find the noise power to scale inversely proportional to channel length (L), as expected. The device-to-device variability in noise power was too large to assess any trends due to fin width (W) which had a limited range of 20-40nm. The noise power is expected to scale inversely with effective fin width. In terms of effective fin width (W+2H) the range was 80-100nm. See figures 2 and 3 for the geometry dependence of noise power for overdrives (V_G-V_T) of 0 and -0.5V resp.

Figure 2. The channel length dependence of the gate voltage referred noise power at 0V overdrive (V_G-V_T) shows the expected 1/L scaling.
The channel length dependence of the gate voltage referred noise power at -0.5V overdrive (\(V_G-V_T\)) shows the expected \(1/L\) scaling.

The gate bias dependence of the gate voltage referred noise power at 10Hz is shown in fig. 4 for 20nm finFET devices with length 1\(\mu\)m and 10\(\mu\)m. We observed that the noise power decreases with decreasing overdrive. This is in agreement with the findings of Kim et al. [2] who reported the noise characteristics of 50nm wide top-down fabricated electrolytically gated nFETs, and who also observed increasing gate bias referred noise power at higher overdrive. This is also in agreement with the carbon nanotube work [4] and the theory in [5]. Rajan et al. [3], however, reported a different gate bias dependence of noise power for 100nm wide top-down nMOSs.

We found the gate bias dependence of the noise power of the fabricated electrolytically gated FETs to be consistent with the description of number fluctuation noise with correlated mobility according to Hung et al. [5]. The gate voltage referred noise power \(S_{\nu G}\) increased quadratically below the threshold voltage with gate bias. In figure 5 the square root of \(S_{\nu G}\) is plotted vs gate bias and shows a linear dependence on gate bias below threshold. This behavior has also been observed for electrolytically gated FETs by Kim et al. [2]. The gate referred noise power below pMOS \(V_T\) is described by the following formula according to the number fluctuation model with correlated mobility fluctuations [5]:

\[
S_{\nu G} = \frac{kTq^2}{\gamma J W L C_{ox}} (1 + \alpha \mu N)^2 N_t
\]

\(k\) is Boltzmann’s constant, \(T\) is temperature, \(q\) is elementary charge, \(\gamma\) is the attenuation coefficient of the carrier wave function in the oxide, \(C_{ox}\) is oxide capacitance, \(\alpha\) is the scattering coefficient, \(\mu\) is the mobility, \(N\) is mobile carrier charge density, and \(N_t\) is trap density.

Here we report gate bias referred noise powers down to \(1\times10^{-10}\text{V}^2/\text{Hz}\) at 10Hz for a 20nm wide 10\(\mu\)m long single fin device. The gate bias referred noise PSD reported by Kim et al. [2] is significantly higher \((1\times10^{-8}\text{V}^2/\text{Hz})\) at 10Hz for a much larger device with an effective width of \((50nm+2\times40nm)\times10\) fins and a 10\(\mu\)m length. Converting Kim et al.’s figure to the 10\(\mu\)m length and 20nm effective width geometry reported in this work would amount to \(~1.6\times10^{-7}\text{V}^2/\text{Hz}\). Rajan et al.[3] report a minimum noise power of \(~1\times10^{-9}\text{V}^2/\text{Hz}\) (already converted to the frequency and 10\(\mu\)m geometry reported in this work).

The bias dependence of the noise power determines the bias at which the signal-to-noise ratio (SNR) becomes optimal. The signal-to-noise ratio was investigated in this work for threshold shift signals (as in e.g. ISFETs) by making use of the square rooted inverse gate referred noise power as figure-of-merit for SNR:

\[
\text{SNR'} = \frac{R_m[\Delta V]}{\sqrt{\int_{10Hz}^{100Hz} S_{id}[\Delta V/hz]d\nu}} = \frac{1}{\int_{10Hz}^{100Hz} S_{\nu G}[\Delta V/hz]d\nu}
\]

in which \(S_{id}\) is the drain current referred noise power spectral density and \(g_m\) is the transconductance.
We found this SNR’ figure-of-merit (see figure 6) to increase for lower overdrive in contrast to the findings of Rajan et al. [3] who reported an optimum at maximum transconductance and we found the SNR’ to be in agreement with the findings of Heller et al. [4] for carbon nanotube electrolytically gated FETs.

In this work we report on the noise characteristics of nanoscale top down fabricated electrolytically gated FETs with fin widths down to 20nm. We found the expected 1/f low frequency noise spectrum and noise power length scaling behavior. For a 20nm wide, 10µm long device gate referred noise power down to ~1x10^{-10}V^2/Hz at 10Hz was observed, which is significantly lower than previously reported for electrolytically gated top-down fabricated FETs. We find the gate referred noise power to increase with increasing overdrive, indicating the SNR to be optimal for threshold shift signals for the lowest overdrives. We have also found the dependence of gate referred noise power to be in agreement with a number fluctuation model with correlated mobility fluctuations.

Processing Impact on the Low-Frequency Noise of 1.8 V Input-Output Bulk FinFETs

Cor Claeys
ESAT Dept.
KU Leuven
Leuven, Belgium
cor.claeys@kuleuven.be

Geert Hellings, Hiroaki Arimura,
Bertrand Parvais, Lars-Åke
Ragnarsson, Harold Dekkers, Tom
Schram, Dimitri Linten, Naoto
Horiguchi and Eddy Simoen
imec
Leuven, Belgium

Dimitri Boudier and Bogdan Cretu
ENSICAEN, UNICAEN, CNRS, GREYC
University of Caen
Caen, France
Bogdan.Cretu@ensicaen.fr

Abstract—The low-frequency noise of input-output (I/O) FinFETs with 3.5 nm SiO2 gate dielectric is studied for different processing conditions. It is shown that a high-pressure (HP) deuterium anneal can improve the noise Power Spectral Density (PSD). There is no significant impact on n-channel devices, while a pronounced effect is observed for p-channel devices, especially for a HP anneal at 400 °C and 20 atm. Results are also presented on the use of a Si/SiGe superlattice architecture and it is shown that the same gate stack quality as for standard devices can be maintained.

Keywords—Flicker noise, number fluctuations, oxide trap density, deuterium annealing, input/output FinFETs, superlattice

I. INTRODUCTION

System-on-a-chip (SoC) applications of CMOS technology require the presence of different flavors of transistors. Besides devices for high speed and low power logic, high-voltage Input-Output (I/O) transistors, which are able to operate at higher supply voltage, have to be implemented [1]. The corresponding gate oxide thickness (t oxide) is currently in the range of 3 nm, so that standard SiO2 can serve as a high-reliability gate dielectric for I/O MOSFETs. In order to be compatible with a bulk FinFET process flow, alternatives to standard high-temperature thermal oxidation are being explored. An attractive option in this context could be the Atomic Layer Deposition (ALD) of SiO2 yielding high-quality and highly reliable I/O transistors [2]. It has also been demonstrated that a high-pressure post-deposition annealing (PDA) in deuterium (D2) can improve the interface properties, i.e., the subthreshold slope SS and the density of interface states (Dit) of Si-cap-free SiGe p-channel FinFETs and the overall device performance [3].

Further along the roadmap, vertically stacked horizontal gate-all-around nanowire FETs are promising candidates because they allow a more aggressive gate length scaling [4]. However, the spacing between stacked nanowire devices does not accommodate the thick oxide for I/O FETs. Alternatively, one can consider a Si/SiGe/Si/SiGe/Si superlattice FinFET for I/O applications.

In this work, the impact of high-pressure (HP) PDA on the low-frequency (LF) noise performance of 1.8-V compatible I/O bulk FinFETs, with 3.5 nm ALD SiO2 and metal gate is reported. It is well-known that LF noise is a good quality and reliability indicator for the gate stack [5],[6], which has recently also been applied to planar I/O transistors for DRAM peripheral applications [7],[8]. It is shown here that while the HP annealing has little impact on the 1/f noise Power Spectral Density (PSD) of the nMOSFETs, a significant reduction can be observed for the p-channel counterparts, especially, for an anneal at 400 °C. The responsible mechanism for the 1/f noise reduction is investigated, showing that the main contribution comes from the lowering of the access resistance noise. A secondary effect may be the result of a lowering of the density of traps in the SiO2, with energy close to the silicon valence band edge.

Furthermore, the noise spectra of Si/SiGe superlattice I/O n-channel FinFETs are investigated, pointing out that the spectra contain 1/f and generation-recombination (GR) noise components. The flicker noise is related to remote Coulomb scattering impacting the carrier mobility, while traps in the depletion later cause the second component [9].

II. EXPERIMENTAL DETAILS

I/O FinFETs with a supply voltage of 1.8 V have been processed in a 16 nm technology with an Equivalent Oxide Thickness (EOT) of 3.5 nm. The gate stack consists of a SiO2 gate dielectric processed by Atomic Layer Deposition and a TiN metal gate. One quarter of a 300 mm wafer received a 20 atm H2 anneal at 450 °C for 20 min (devices are labeled HP1), while another quarter underwent a 20 atm H2 anneal at 400 °C for 20 min (HP2).

Noise measurements have been executed on 1 μm long FinFETs with 4 fins, with a height of 35 nm and a width of 10 nm. The noise was measured on at least four devices per wafer in linear operation (|VDS|=50 mV) by stepping the front gate voltage VGS from weak to strong inversion. The input-referred voltage noise PSD was calculated from $S_{V_{TH}} = S_{I} / g_{m}$ with g m the measured transconductance in each operation point and $S_{I}$ the measured drain current noise PSD.

The Si/Si0.75Ge0.25 superlattice I/O FinFETs operating at 1.8 V have a gate stack of 2 nm ALD high-k dielectric (HISOI) on top of 3 nm interfacial SiO2. In the case of 2.5 V operation the thickness on the interfacial SiO2 is increased to 5 nm, resulting in an equivalent oxide thickness (EOT) of 5.6 nm. Full process details are described in [4].

III. RESULTS AND DISCUSSION

A. I/O Bulk FinFETs

The input I D-V GS characteristics in linear operation corresponding with the noise measurements are represented in Fig. 1, together with the transconductance. For the n-channel FinFETs in Fig. 1a, rather similar g m and I D are found, the main impact being an increase of the threshold voltage V TH upon application of a HP anneal. No systematic trend is found for the pFinFETs in Fig. 1b, whereby it should be remarked that the observed changes fall within the variability range of the device performance.
Typical LF noise spectra are shown in Fig. 2, in linear operation and at a current of about 1.1 μA for n- (Fig. 2a) and 1 μA for pFinFETs (Fig. 2b) for a reference (no HP anneal), a HP1 and a HP2 device, respectively. As can be seen, the spectra are predominantly 1/f-like with the frequency exponent γ close to 1. It is clear from Fig. 2a that there is little difference in the current noise PSD (and also in the normalized PSD given by $S_i/I_D^2$) for the n-channel transistors. For the pFinFETs, on the other hand, there is a clear reduction of $S_i$ for HP2, while the spectrum for HP1 coincides with the reference non-annealed device.

The input-referred voltage noise PSD represented in Fig. 4a for the nFinFETs is constant for most of the gate voltage overdrive $V_{GTh}=V_{GS}-V_T$ range studied and corresponds with the value at flat-band voltage, i.e., $S_{VFB}$, with negligible correlated mobility fluctuations. It is rather similar for the three nFinFETs studied. According to the number fluctuations theory, the oxide trap density $N_{ox}$ can be extracted from [5],[10]:

$$S_{VFB} = \frac{q^2 kT \lambda e_{ox}^2}{C_{ox} n_{ox}}$$

with $C_{ox}$ the oxide capacitance per unit area, $q$ the electron charge; $k_B$ Boltzmann’s constant, $T$ the temperature, $W/L$ the effective transistor area and $\lambda$ the attenuation length of the electron (hole) wave function in SiO$_2$.

The pFinFETs exhibit a much stronger dependence of $S_{VG}$ on the gate voltage overdrive in Fig. 4b. This indicates that besides the number fluctuations and associated correlated mobility fluctuations (CMF) another mechanism becomes important in strong inversion. In fact, it has been shown in the past that at larger currents, there could be a strong impact of the series or access resistance ($r_{tot}$) on the noise PSD [11], which can be represented by [12]:

$$S_{Vg} = \frac{kT \lambda}{r_{tot} C_{ox} n_{ox}} S_{FB} (1 + \alpha C_{OV} \sigma T K r_{tot} C_{ox} n_{ox}^2 + \frac{r_{tot} R}{f^2} \frac{r_{tot} R}{f^2} + \frac{r_{tot} R}{f^2} \frac{r_{tot} R}{f^2})$$

with $r_{tot}$ is the total channel resistance ($=V_{pd}/I_D$ in linear operation), $r_{acc}$ the access resistance, $K$ the strength of the access resistance noise source, $\mu_{eff}$ the effective mobility and $e_{ox}$ the Coulomb scattering factor. The + or – sign in Eq. (2) depends on the donor or acceptor character of the oxide traps. Narrow-fin architectures are particularly prone to a high access resistance and associated increase in input-referred voltage noise PSD [12],[13].

According to the second term in Eq. (2), the access resistance noise $S_i$ increases proportionally with $I_D^2$, which is represented by the dashed curves in Fig. 4b and well fits the experimental data at more negative gate voltage overdrives. It is also evident that the main effect of the HP anneal is to reduce this $S_i$ contribution to the total noise PSD by approximately 20 % going from the reference to HP2. Note that the increase with $V_{GTh}$ in Fig. 4a for the nFinFETs of $S_{Vg}$ could also be related to the series resistance noise, although in this case there is no systematic trend between annealed and reference nFinFETs.
Besides the clear reduction of $S_v$ at larger $V_{GT}$ for the pFinFETs, there is also a tendency for lower noise PSD around $V_f$ (the flat-band value) in Fig. 4b, suggesting a reduction in $N_{tr}$ around the valence band energy of silicon. However, considering that three different devices are being measured - and not the same device after different HP PDA – some caution in this interpretation is required, in view of the significant noise dispersion or variability which exists for scaled technologies [6],[13]. Therefore, in Fig. 5 the average flat-band noise PSD at 10 Hz is represented for all n- and p-channel FinFETs studied, showing little change for the n-channel devices and a marked reduction for the pFinFETs after a HP2 anneal at 400 °C.

Fig. 4. Input-referred voltage PSD at $f=10$ Hz and in linear operation at $|V_{GS}|=0.05$ V of 1 μm long n- (a) and p-channel (b) FinFETs, without HP annealing (ref), HP1 and HP2. The dashed lines in (b) represent the series resistance related noise PSD.

Fig. 5. Flat-band voltage noise PSD at 10 Hz and linear operation for 1 μm I/O n- and pFinFETs without annealing, after HP1 at 450 °C and HP2 at 400 °C.

B. I/O Si/SiGe/Si superlattice FinFETs

The input $I_{D}$-$V_{GS}$ characteristics of 1.8 V superlattice I/O FinFETs are shown in Fig. 6a for n- and p-channel devices [4]. There is a good electrostatic control with $I_{on}$ >1200 μA/μm and for the pFETs the SS = 69 mV/dec and DIBL = 30 mV/V [4]. The gate voltage noise PSD (experimental and model) is shown in Fig. 6b. for two devices [9]. The developed noise model is based on [14]

$$S_v(f) = B_v + K_v \frac{f^n}{f^2} + \sum_{i=0}^{N} A_i \left[ 1 + \left( \frac{f}{f_i} \right)^{2n} \right]$$

with as noise components the $1/f$ noise (noted $K_v$), the generation-recombination Lorentzians with corner frequency ($f_i$) and plateau value ($A_i$) and the white noise $B_v$.

Using noise spectroscopy whereby the low frequency noise at a fixed polarization is studied as a function of the temperature enables to obtain more information on the nature of the traps by determining for each trap the energy level, the concentration and the capture section. The analysis includes the evolution of both the corner frequencies and the plateau values. Some typical results are illustrated in Fig. 8 resulting in four trapping levels, i.e., $E_{C,T1}=0.45$ eV (T1), $E_{C,T2}=0.44$ eV (T3), $E_{C,T3}=0.42$ eV (T2) and $E_{C,T4}=0.17$ eV (T4) [9]. These levels can be associated with $V_{2H}$, $V_{P}$, $V_{2H}^{(0)}$ and $CC_{S}$ complexes, respectively. Further investigations are ongoing.

Similar as for the standard I/O bulk FinFETs discussed in the previous section it is interesting to analyze the flat-band noise PSD at 10 Hz as shown in Fig. 9. It can clearly be seen that the median value of the noise is only slightly increased for the superlattice FinFETs compared to the reference device architecture, although there is a higher spread in the data.

Fig. 6. Input $I_{D}$-$V_{GS}$ characteristics for 1.8 V superlattice I/O n- and p-channel FinFETs (5 nm SiO$_2$ and 70 nm gate length) (a) [4], and gate voltage noise PSD for two devices (b). For device 1 the noise modeling only requires white noise and flicker noise, while device 2 also requires a Lorentzian generation-recombination noise component [9].

The trap density can be calculated using Eq. (1). It is important to remark that for the studied devices there is a correlation between the trap density and the inverse mobility of the devices as illustrated in Fig. 7 [9]. A similar trend has been reported before for e.g. HfO$_2$ nMOSFETs [15] and TiN/TaN/HfO$_2$ Ge pMOSFETs [16] and was associated with the impact of the remote Coulomb scattering. Shorter devices lead to a more pronounced reduction of the mobility.

Fig. 7. Correlation between the trap density and the inverse of the carrier mobility for 70 nm superlattice I/O devices.
It has been shown that the 1/f noise PSD of I/O pFinFETs with 3.5 nm ALD SiO₂ is significantly reduced after a post-deposition high-pressure deuterium anneal at 400 °C. This is mainly related to the reduction of the series resistance noise. The n-channel counterparts show little impact of the HP annealing on the flat-band voltage noise PSD, while in strong inversion suppression of the access resistance noise could be found for some devices. For Si/SiGe superlattice devices the presence of traps in the depletion layer impacts the low frequency noise of Si/SiGe superlattice devices. Although there is a higher spread in the noise from device to device compared to standard I/O devices, there is only a limited impact on the median low-frequency noise values.

IV. CONCLUSIONS

This work has been performed in the frame of the imec Core Partner program on Logic FinFET devices.

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A 4-Terminal Method for Oxide and Semiconductor Trap Characterization in FDSOI MOSFETs

Hung Chi Han
IMEP-LAHC
Univ. Grenoble Alpes, Univ. Savoie
Mont Blanc, CNRS, Grenoble INP
Grenoble, France
hung.han@epfl.ch

Christoforos Theodorou
IMEP-LAHC
Univ. Grenoble Alpes, Univ. Savoie
Mont Blanc, CNRS, Grenoble INP
Grenoble, France
christoforos.theodorou@grenoble-inp.fr

Gérard Ghibaudo
IMEP-LAHC
Univ. Grenoble Alpes, Univ. Savoie
Mont Blanc, CNRS, Grenoble INP
Grenoble, France
gerard.ghibaudo@grenoble-inp.fr

Abstract—In this work, we present an experimental method that allows for a proper distinction between oxide (front or back) and semiconductor traps in FDSOI MOSFETs, using all four bias terminals. To this end, two cases of Random Telegraph Noise (RTN) measured signals are studied. It is shown that this method can also be used to localize the trap across the channel. Furthermore, the trap’s electrostatic impact is proven to be misleading when it comes to trap localization in nanoscale devices.

Keywords—Random Telegraph Noise, FDSOI, MOSFET, Characterization, Traps;

I. INTRODUCTION

Random Telegraph Noise (RTN) fluctuations [1], related to discrete traps that cause strong conductance modulation, have become very important in nano-scale devices, because their amplitude increases with surface reduction [1]. In Fully Depleted Silicon-On-Insulator (FDSOI) MOSFETs [2] in particular, apart from the fact that the traps may lay inside the bottom interface oxide, it has been suggested that RTN signals may also be related to traps in the silicon film [3],[4]. Recently, Marquez et al. [5] presented a RTN characterization method to help localize an oxide trap both vertically (in depth) and laterally (across the channel). In this paper, we extend this method by combining front (FG) and back gate (BG) bias modes, to achieve the detection of both oxide (front/bottom) and semiconductor traps in FDSOI MOSFETs, using all four bias terminals (front gate, bottom gate, drain and source). In this paper, we extend this method by combining front (FG) and back gate (BG) bias modes, to achieve the detection of both oxide (front/bottom) and silicon traps that can induce RTN, as well as their localization vertically and laterally.

II. RTN AMPLITUDE AND KINETICS

In order to study the behavior of RTN, there are three main parameters that should be first extracted from the time domain series and histogram, i.e. the higher current level duration, lower current level duration, and switching amplitude. Fig. 1(a) shows a typical RTN signal with two current levels in its time domain. The duration of the high current level is called capture time, $\tau_c$, meaning the time required for the carrier to be captured into the trap. On the other side, the period of low current level is called emission time, $\tau_e$, time constants.

The carrier capture and emission process by a trap can be defined by the Shockley-Read-Hall theory [6], [7]. First, the average capture time $\bar{\tau}_c$ for an electron in the inversion layer is given by:

$$\bar{\tau}_c = \frac{1}{\sigma_n v_p} (a)$$

and the average emission time $\bar{\tau}_e$ by:

$$\bar{\tau}_e = \frac{1}{\sigma_n v_h} (b)$$

where $\sigma_n$ is the carrier concentration near the trap in cm$^{-3}$, $v_p$ is the surface carrier concentration when the Fermi level $E_F$ crosses the trap energy $E_T$, $v_T$ is the thermal velocity for electrons, $\sigma$ is the trap cross-section [6], [8]. For oxide traps near the interface, and considering constant mobility and drain-source voltage, $n_e$ is proportional to the drain current, $I_D$, which increases with the front gate bias $V_F$ or back bias $V_B$, thus $\tau_e$ is inversely proportional to $I_D$. On the other hand, the trap energy level is modulated by gate voltage bias, affecting by turn $\tau_c$ through $n_e$ [1].

In the histogram of Fig. 1, there are two main peaks indicating $I_{high}$ and $I_{low}$. The difference between the two current levels is defined as $\Delta I$, describing the average amplitude of the RTN fluctuation. When a single electron with charge $q$ is trapped into the gate oxide, it produces a local change of the flat band voltage [9]. The corresponding model predicts the RTN amplitude through:

$$\frac{\Delta I}{I_0} = \frac{g_m}{l_0 W L C_{ox}} \left(1 - \frac{x_t}{L} \right)$$

where $g_m$ is the transconductance, $W$ and $L$ are the width and length of channel of MOSFET, $C_{ox}$ is the capacitance of gate oxide per unit area, $x_t$ is the depth of trap from the Si/SiO$_2$ interface, and $t_{ox}$ is the thickness of oxide layer. In principle, (2) can be applied for both the ohmic and non-ohmic regions, demonstrating that the RTN amplitude will vary with the transistor gain, $g_m/I_d$. As we demonstrate in the next section, however, this approach can be misleading.

III. RESULTS AND DISCUSSION

The devices were issued from a 14 nm FDSOI technology [2], provided by STMicroelectronics, with channel width $W = 60$ nm and length $L = 20$ nm. In total, we measured 88 dies, from which the majority had multiple RTN signals, which makes the time domain analysis complex to be processed for discrete trap parameter extraction, as needed for our study. Therefore, we chose to address two single RTN cases: a typical RTN (shown in Fig. 2(a)), in the sense that $\tau_c$ and $\tau_e$ change rapidly with gate bias (device #1) and a non-typical RTN (shown in Fig. 2(b)) that seemed to have a gate bias independent occupancy (device #2).
A. Typical RTN case (device #1)

In Fig. 3, the relative RTN-induced drain current shift is plotted along with the $g_m/I_d$ transistor gain, multiplied by a constant. In principle, if the two trends are similar, it means that the carrier number fluctuations (CNF) model can explain the RTN signal, through (2) [1]. However, this is in the case of classic bulk MOSFET structures, where only one silicon/oxide interface is present. In FDSOI MOSFETs, since there is a strong coupling between the two interfaces (front and back), it is not so simple to localize the trap in one of the two.

Indeed, as can be seen in Fig. 3, while we were expecting that in front gate (FG) mode, $\Delta I_d/I_d$ would be proportional to $g_m/I_d$, this is only true when the device is biased in back gate (BG) mode. This finding can be misleading, as one might conclude that the trap is located in the BOX (buried oxide). To further clarify the situation, we examined the drain current dependencies of $\bar{\tau}_c$ and $\bar{\tau}_e$ for both FG and BG modes.

As shown in Fig. 4, the current dependence of $\bar{\tau}_c$ is much stronger in FG mode, compared to the BG mode. In fact, the trend is very close to $-1/I_d$. This can be explained through (1), only if the carrier density near the trap, $n_s$, is proportional to the drain current. The only requirement for the latter to be true is that the trap lies very close to the main channel interface. Therefore, in our case, this is a very strong indicator that the trap is inside the front oxide, not the back. The small deviation from the $1/I_d$ trend can be attributed to the mobility degradation, present in higher currents. On the other side, the $\bar{\tau}_e - I_d$ dependence in BG mode is far from following the $1/I_d$ trend. This happens because the main channel is located close to the back interface, thus the carrier density at the front interface does not follow the $I_d$ changes linearly.

Combining all the above, we can safely conclude that the RTN signal observed in device #1 is related to a trap located inside the front gate oxide. However, as noted before regarding Fig. 3, this is a surprising finding. First, it reveals that (2) in its present form cannot be used to extract the oxide trap depth in FDSOI MOSFETs. Had we followed this approach, we would have falsely located the trap in the BOX and confidently extracted $x_t$. Second, the FG mode trend in Fig. 3 suggests that the trapped charge gives rise to a very strong correlated mobility fluctuation, due to the very aggressively scaled gate area [10].

Fig. 2. The two RTN case studies: (a) typical RTN where the trap occupancy is directly affected by the gate voltage, and very rapidly and (b) non-typical RTN with a gate voltage independent occupancy.

Fig. 3. Relative RTN amplitude versus drain current for front (top) and back (bottom) gate mode (case #1) under $V_d = 30$ mV.

Fig. 4. Extracted capture (circles) and emission (triangles) time versus drain current under front (top) and back (bottom) gate mode for device #1 biased under $V_d = 30$ mV.
case -with a RTN trap in the BOX- although $\Delta I_d/I_d$ would follow the $g_m/I_d$ trend in FG mode, this should not lead to the conclusion that the trap is in the front oxide. Unfortunately because of this misunderstanding, there are many publications with invalid conclusions.

Concerning the trap’s lateral position across the channel, one has to follow a similar procedure, by checking the amplitude and time constant dependencies on the drain-source voltage [1], [5], [11]. We biased the device varying first $V_d$ (common source - CS) and then $V_s$ (common drain - CD) from linear to saturation region. The corresponding extracted $\Delta I_d/I_d$ is plotted in Fig. 5, revealing a complete lack of symmetry between the two bias methods. In fact, in CD mode, the amplitude reaches a maximum of 35% (!), while in common source mode it drops very quickly from 10 to 3%, before making the RTN pulse indistinguishable.

As suggested in [1] and [5], if $\Delta I_d/I_d$ increases with drain-source voltage, $V_{ds}$, the trap is located closer to the source. In our case, $\Delta I_d/I_d$ increases with $V_{ds}$, meaning that the trap lies on the side of the drain. Moreover, the sudden decrease with $V_{ds}$, combined with the fact that the RTN disappears after $V_{ds}$ reaches 0.1 V (end of linear region) leads us to assume that the trap is very close to the drain/channel junction.

Once again, an assumption like this can be only verified by exploring the dependence of $\tau_e$ and $\tau_c$ on $V_{ds}$. Indeed, as shown in Fig. 6(top), $\tau_e$ is increased suddenly in positive $V_{ds}$ whereas $\tau_c$ follows the opposite trend. As already pointed out, $\tau_e$ can change only when the carrier density, $n_s$, near the trap is modified and $\tau_c$ only when the trap energy level is modulated. Therefore, the sudden increase of $\tau_e$ can be related to the carrier depopulation near the drain/channel junction, responsible for a decrease of $n_s$, while the decrease of $\tau_c$ can only happen if $V_d$ can modulate the trap energy level, which is also true for traps very close to the drain. This becomes even clearer if we plot the trap occupancy factor $f_e = \tau_e/\tau_e^c$. As one can notice in Fig. 6(bottom), the trap maximum activity (around 55%) is only when the region near the drain is not in depletion. The decrease of $f_e$ in high $V_d$ values under CD mode can be regarded as a short channel effect: a source-induced barrier lowering (SIBL) near the drain. In conclusion, the initial assumption is confirmed by all the experimental results: the device #1 discrete oxide trap is next to the drain/channel junction.

B. Non-typical RTN case (device #2)

Concerning device #2, where the RTN signal behavior in terms of duty cycle seemed independent of the gate bias, as demonstrated in Fig. 2(b), we followed the exact same biasing methodology as for device #1, in order to extract all the dependencies (vs $V_g$, $V_b$, $V_{ds}$) of $\Delta I_d/I_d$, $\tau_e$, and $\tau_c$. The corresponding results are plotted in Figs. 7-10.

First of all, $\Delta I_d/I_d$ follows the $g_m/I_d$ trend for both FG and BG modes (Fig. 7), which initially led us to assume that it is an oxide trap. Nonetheless, in Fig. 8 we notice that $\tau_e$ and $\tau_c$ do not follow the $1/I_d$ trend, neither in FG nor in BG mode. On the contrary, both seem to be almost completely gate bias independent, which is an indicator for semiconductor traps [4], [10], [12]. Regarding the capture time, this means that $n_s$ is constant near the trap and from the emission time stability it figures that the trap energy is also constant with $V_g$ and $V_b$. The above can only be valid when a trap is in a depletion or accumulation region that is not affected by gate bias, which however is impossible in the channel of FDSOI MOSFETs, because of the strong front/back gate coupling. Therefore, we have to assume that the trap is located in one of the n+d doped junction regions, either source or drain.

In order to localize the semiconductor trap in one of the two regions, one has to investigate the drain-source voltage dependencies. Indeed, Fig. 9 illustrates how $\Delta I_d/I_d$ is maximized under positive source bias (CD) and how the RTN disappears for $V_{gs}$>0.1V (CS), as for the trap of device #1. Nonetheless here, $\tau_e$ and $\tau_c$ (Fig. 10) remain constant with $V_{ds}$, for both CD and CS modes, meaning that neither $n_s$ nor $E_t$ are affected. The $n_s$ invariance confirms that the trap is located in the junction regions, where the doping prevails...
over any carrier density modulations occurring in the channel.

Therefore, the device #2 semiconductor trap is with certainty localized inside the drain/channel junction. The only reason the RTN disappears above $V_d = 0.1V$ (in CS) is the pinch-off effect which results in a longer distance between the trap and the main channel, leading to a negligible electrostatic impact.

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Low Frequency Noise in Electrochemical Sensors for Water Quality Monitoring

Abstract—In the past several decades, water pollution has increased drastically due to rapid industrialization and population growth. Water contamination with pharmaceuticals is becoming an emerging problem as even a very low concentration may pose risks to human health and aquatic lifeforms. Since the safe limit of some chemicals such as painkillers and hormones in drinking water are in the range of ppm, the requirements for low-level detection of pharmaceuticals in water are demanding. Therefore, the development of new water quality monitoring sensors with improved limit-of-detection and sensitivity are critical. State-of-the-art water quality monitoring systems include sensors for pH, free chlorine and dissolved oxygen. As opposed to conventional techniques, many novel sensors are based on electrochemical redox reactions, which are described by the Nernst equation. However, one important practical problem in realizing a highly sensitive and wide dynamic range sensor is that few ppm changes of activity or concentration results in a signal of only several microvolts. This results in a poor signal-to-noise (SNR) ratio, making the signal indistinguishable from the low frequency noise (LFN). Therefore, characterization of the sensor performance is indispensable. In this paper, we present our approach to the fabrication and results on the noise and sensitivity of several sensors (acetaminophen and estrogen, pH, free chlorine and temperature) for water quality monitoring.

Keywords—Low frequency noise, electrochemical sensors, water monitoring, pharmaceuticals sensor, pH, free chlorine, temperature

I. INTRODUCTION

The study of noise mechanisms in electrochemical sensors is critical for accurate in-vivo detection of analytes [1]. The detection efficiency of an electrochemical sensor depends on its ability to recognize small changes in the transduced signals that also brings uncertainty [2]. Therefore, an improved understanding of the sources of noise in an electrochemical sensor is crucial to accomplish high sensitivity and low limit-of-detection. Understanding of noise sources can help in developing new models and designs for the electrochemical sensors towards an improved system integration. For example, the noise behavior of an electrochemical sensor can provide insights into optimization of the sensor performance. The sensor performance is generally modified with different nanomaterials and functionalization methods. While noise phenomena are widely studied in solid-state electronic and photonic devices, noise in electrochemical sensing systems with solid-liquid interfaces are rarely investigated.

Noise fluctuations have been well-understood in common electronic components [3]–[5]. Based on their spectral densities, these fluctuations are generally classified in different types, such as thermal noise, shot noise and flicker noise [6]. The noise power spectral density (PSD) of the Johnson-Nyquist thermal voltage noise of a resistor is expressed as:

$$S_n = 4k_BT$$

where $k_B$ is the Boltzmann’s constant, $T$ is the absolute temperature and $R$ is the resistance of the resistor. Shot noise is an electronic noise that comes from the discrete nature of the electric charges while overcoming a potential barrier. Equation (1) shows that thermal noise is proportional to the temperature. On the other hand, shot noise does not depend on temperature, but on the current. The frequency independent behavior of the thermal and shot noise up to relatively high frequencies gave rise to their name white noise, in analogy with the spectrum of the white light. In contrast, the flicker noise PSD depends reciprocally on the frequency, so it is often called 1/f noise. Generally, 1/f noise is dominant at low frequencies.

Many experimental and theoretical studies on 1/f noise suggested that the 1/f noise is inversely proportional to the active area in electronic devices [7], and the electrode area of electrochemical devices [8]. Another study made some conclusions about the type of graphite electrode material that can minimize flow-associated noise [9]. The major contribution to the understanding of noise in electrochemical systems originates from Faradaic interfacial processes, which were divided into two fundamental areas such as: (1) the method of creating perturbation of an equilibrium followed by measuring the induced current and the associated transfer function of the interface phenomena [10], [11], and (2) the noise that naturally occurs in the interfaces as homogeneous processes [12]–[19]. In both cases, a few fundamental insights into noise processes were found. Specifically, fundamental noise sources such as Johnson-Nyquist thermal noise and shot noise were neglected due to their very low magnitudes compared to 1/f noise.

In recent years, nanomaterials have attracted great attention in electrochemical sensing and biosensing applications due to their unique electronic, physical and chemical properties leading to highly sensitive and robust sensing/biosensing platforms [20]–[22]. In particular, carbon based nanomaterials such as carbon nanotubes were widely investigated as an efficient electrode modifier that enhanced the efficiency of electrochemical sensors/biosensors [23]. For example, modification of the sensing electrodes with multiwalled carbon
nanotubes (MWCNTs) with/without β-cyclodextrin (βCD) resulted in improved charge transfer efficiency and better sensing performance [24], [25]. However, the behavior of noise in electrochemical sensors that operate in the voltammetric modes with or without nanomaterial modifications is still not well studied and well understood.

In this work, we realize an experimental setup that mimics the voltammetric sensing operation, and then study the low frequency noise of the unmodified/modified Screen Printed Electrodes (SPE) under different conditions. The effect of the bias voltage and two types of electrolyte solutions (Phosphate Buffer Saline (PBS) and Potassium Ferricyanide (K₃[Fe(CN)₆])) on the noise level is investigated. The effect of using sensing electrodes functionalized with MWCNTs and βCD on the electrochemical potential and noise level is investigated. We also study the noise of palladium/ palladium oxide (Pd/PdO) based pH sensor (in potentiometric configuration), amine modified carbon electrode based free chlorine sensor (in voltammetric configuration), and PEDOT:PSS/silicon based thermistors. The experimental setup is described in section II, followed by results and discussions in section III.

II. EXPERIMENTAL SETUP

A. Materials and reagents

Carbon based SPEs (Zensor), a glass based Ag/AgCl reference electrode and platinum counter electrode were purchased from CH Instruments. Potassium Ferricyanide (K₃[Fe(CN)₆]) in the form of powder and PBS tablets were purchased from Sigma. The K₃[Fe(CN)₆] powder was dissolved into deionized (DI) water to get 5 mM concentration. Each PBS tablet was dissolved into 200 ml of DI water to get 0.01 M PBS buffer with pH 7.4.

B. Preparation of electrodes

Modifications of the SPEs were done with MWCNTs and β-cyclodextrin to get three types of modified SPEs such as: MWCNT/SPE, MWCNT-βCD/Phys/SPE and MWCNT-βCD(SE)/SPE. The MWCNT/SPE electrode was modified with only pure MWCNT. The MWCNT-βCD/Phys/SPE electrode was modified by MWCNT which was non-covalently (i.e., physically) functionalized with βCD. In MWCNT-βCD(SE)/SPE electrode, MWCNT modified by which was covalently (i.e., Steglich esterification) functionalized with βCD. The detailed MWCNT modification processes were described in previous paper [25].

C. Apparatus

The noise measurements of the electrochemical sensors were done in a three-electrode configuration similar to a potentiostat configuration that performs cyclic voltammetry or differential pulse voltammetry, as shown in Figure 1 and details in Table 1. A dynamic signal analyzer (SR785, Stanford Research Systems) was used to acquire and analyze the noise signal from the working electrode (Channel B of SR785) (i.e., bare SPE, MWCNT/SPE, MWCNT-βCD/Phys/SPE and MWCNT-βCD(SE)/SPE electrodes). The reference and the counter electrodes were connected to a low-noise voltage amplifier (SR560, Stanford Research Systems) to its negative and positive input terminals, respectively. A low noise current amplifier (SR570) was used to amplify the output current of the working electrode. A custom-made battery cell with potentiostat was used as the bias voltage source. Three multimeters were used to monitor the voltages at the biasing battery source, the reference electrode and the output of the current amplifier which corresponds to current from the working electrode by the factor of the amplifier gain ranging from 0.5 to 10 µA/V.

D. Noise data processing and analysis procedure

The noise data processing and analysis procedure requires several steps. In step 1, the raw data of noise signal from the Dynamic Signal Analyzer (SR785) is extracted to a personal computer in *.csv file format using VEE Pro 9.33 software developed by Keysight Technologies. The extracted noise spectra contained 60 Hz harmonics signal, which was due to interference from electrical power networks. In step 2, the 60 Hz harmonics from the signals were removed from the spectra using a 2nd order Butterworth notch filter implemented in MATLAB. In step 3, the filtered signals were then scaled with the gain of the current amplifier (SR570). Finally, in step 4, the current voltage (I-V) characteristics were obtained along with other noise related parameters.

III. Results and Discussions

The noise measurement setup for the electrochemical sensing electrodes was based on the potentiostat three-electrode configuration, since the actual electrochemical measurements also use the same. In this potentiostat configuration (Figure 1), we analyzed the current noise fluctuations at the working electrode. In electrochemical sensing measurements, the applied bias voltages are scanned between two voltages, which is usually in the range of few hundred millivolts. Therefore, we also used multiple bias voltages from 0 V to 0.6 V to analyze the effect of the bias voltages on the sensor noise (Table I). Figure 2 shows the power spectral density (PSD) spectra of the noise current when the electrodes were immersed into K₃[Fe(CN)₆] (a-d) and PBS (e-h) solutions. In each type of electrode, the bias voltage applied between the reference and counter electrodes is set at 0, 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 V. The frequency range of the PSD spectra is from 2 Hz to 1.6 kHz and each spectrum is...
obtained by averaging 64 acquisitions.

TABLE I. THE NOISE MEASUREMENT SETUP WITH DIFFERENT ELECTROLYTES, SENSING ELECTRODES AND BIASING VOLTAGES.

<table>
<thead>
<tr>
<th>Type of Electrolyte</th>
<th>Type of Electrode</th>
<th>Bias Voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>K3[Fe(CN)]6 (5mM)</td>
<td>SPE</td>
<td>0.0 V</td>
</tr>
<tr>
<td></td>
<td>MWCNT-SPE</td>
<td>0.1 V</td>
</tr>
<tr>
<td></td>
<td>MWCNT-[βCD]/SPE</td>
<td>0.2 V</td>
</tr>
<tr>
<td></td>
<td>MWCNT-[βCD]/SE</td>
<td>0.3 V</td>
</tr>
<tr>
<td>PBS (pH 7.4, 0.01 M)</td>
<td>SPE</td>
<td>0.4 V</td>
</tr>
<tr>
<td></td>
<td>MWCNT-SPE</td>
<td>0.5 V</td>
</tr>
<tr>
<td></td>
<td>MWCNT-[βCD]/SPE</td>
<td>0.6 V</td>
</tr>
<tr>
<td>pH Buffer</td>
<td>PBS/PDD based pH</td>
<td></td>
</tr>
<tr>
<td>PBS (pH 7.4, 0.01 M)</td>
<td>Amine modified</td>
<td>0.0 V</td>
</tr>
<tr>
<td></td>
<td>carbon electrode</td>
<td>0.1 V</td>
</tr>
<tr>
<td>Tap Water</td>
<td>Si and PEDOT/PSS</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2. The power spectral density (PSD) spectra of the noise current (at the working electrode) with K3[Fe(CN)]6 solution for (a) SPE, (b) MWCNT/SPE, (c) MWCNT-[βCD]/Phys/SPE, and (d) MWCNT-[βCD]/SE/SPE electrodes.

A. Noise in the pharmaceutical/heavy metal sensor

To investigate noise correlation between the working electrode and the reference/counter electrode, we performed correlation/coherence measurement with the Dynamic Signal Analyzer (SR785). First, we performed correlation between the voltages and currents of all electrode terminals. However, there was no significant correlation between them. This signifies that the noise originating from the electrode terminals was based on independent processes that only happens within the vicinity of the corresponding electrode and does not affect the other two electrodes. After that, we performed coherence measurement which showed coherence value of less than 30%, again signifying little coherence between the noise sources of all electrode terminals.

The effects of biases/materials/solutions on noise characteristics were studied. The current noise PSD spectra in Figure 2(a-d) and Figure 3(a-d) shows seven noise PSD spectra which were recorded with seven different applied bias voltages ranging from 0 V to 0.6 V. There is no specific pattern or variation of the seven different noise PSD spectra showing very weak dependence on the applied bias. However, there is an increase in the noise PSD level from ~10^-18 A^2/Hz to ~10^-17 A^2/Hz in lower frequency range (2-10 Hz) when the bare SPE electrode is modified with MWCNT and MWCNT-[βCD]/Phys/SE, as shown in Figure 2(a-d) and Figure 3(a-d). The increase in the noise level with the addition of MWCNT on the SPE electrode may indicate that the noise behavior are due to the inherent materials properties of the electrodes. Also, the noise may be related to the concentrations of ions of the solutions going into the working electrode. However, when the electrodes are changed from K3[Fe(CN)]6 solution to PBS buffer solution, the noise level remains almost the same, which means that the noise does not depend on the type or concentration of ions that are present in the electrolyte solution. Thus, the noise behavior that are observed in Figure 2(a-d) and Figure 3(a-d) are simply due to characteristics of each electrode itself. The noise PSD spectra shown in Figure 2(a-d) for K3[Fe(CN)]6 solution shows 1/f noise characteristics since the level of the noise spectra decreases by approximately one decade per decade of frequency. The 1/f noise refers to the phenomenon of the spectral density, S(f), of a stochastic process having the form [26]:

\[ S(f) = K_f (I_{avg} or V_{avg})^\alpha = f^{\alpha} \]

where \( f \) is frequency, \( I_{avg} \) or \( V_{avg} \) are the average current/voltage of the sensor, \( K_f \) is the flicker noise coefficient, and the exponent \( \alpha \) is in the range of 0.8 < \( \alpha < 1.2 \) and is usually close to 1, and this noise is known as flicker noise. 1/f fluctuations are widely found in nature. The noise PSD spectra shown in Figure 3(a-d) for PBS solution are, however, not purely pink noise. In this case, the value of \( \alpha \) is somewhere between 0.5 and 1. Moreover, the noise PSD spectra in Figure 3(a-d) also show very weak Lorentzian humps at around 100 Hz, which could be due to defect-assisted generation-recombination (GR) processes [27] or other bistable random fluctuation [28]. Figure 3(e-h) shows the noise PSD spectra multiplied with frequency to identify the level of 1/f noise and the frequency at which the Lorentzian humps are observed. The initial flat region of the curves in Figure 3(e-h) in the lower frequency range provides the level of 1/f noise. It is observed from Figure 3(e-h) that the 1/f noise increases by only few times when the electrodes are modified with MWCNT and βCD. Also, the Lorentzian humps are observed at around 100 Hz for all the electrodes.

The applied bias influenced the electrochemical potential and noise level. Figure 4(a) shows the “\( V_{ele} - V_{ele}^\alpha \)” vs \( I_e \), where \( V_{ele} \) is the voltage in the reference electrode, \( V_{ele}^\alpha \) is the voltage at the working electrode, and \( I_e \) is the current in the working electrode. This current-voltage relationship of the working electrode with respect to the reference electrode (i.e., \( V_{ele} - V_{ele}^\alpha \)) shows two different types of characteristic curves of the set of electrodes due to the two electrolyte solutions (i.e., K3[Fe(CN)]6 and PBS solutions, separated by a dashed gray line).

In case of PBS, the \( I_e \) ranges from ~0.1 μA to ~8 μA, whereas in case of K3[Fe(CN)]6, the current varies in a narrow range between ~9 μA to ~11 μA. If we extrapolate the I-V curves, their intercepts on the vertical axis will provide corresponding electrochemical potentials of the electrodes. The electrochemical potential of the SPE and modified SPE
electrodes in PBS solution can be estimated to be close to 0 mV. However, the electrochemical potential can be extrapolated to at least below ~200 mV when K$_3$[Fe(CN)$_6$] solution was used. It is also observed in Figure 4(a) that the electrochemical potential reduces with the functionalization of the SPE electrode with MWCNT and MWCNT–βCD(Phys)/SE.

Similar to the case as shown in Figure 4(a), the $I_w$ has a wider range when PBS solution is used as the electrolyte compared to that of K$_3$[Fe(CN)$_6$] solution. However, the LFN noise in band 8-20 Hz remains almost constant at different current of the working electrode ($I_w$, which is proportional to the bias voltages), signifying negligible bias dependence of the LFN noise level. However, the level of LFN noise increases around three times when the SPE electrode is modified with MWCNT and MWCNT–βCD(Phys)/SE.

![Fig. 4. (a) Current voltage characteristics of SPE and modified SPE working electrodes with PBS and K$_3$[Fe(CN)$_6$] solutions measured in potentiostatic configuration, (b) Low frequency noise (LFN) of SPE and modified SPE working electrodes with PBS and K$_3$[Fe(CN)$_6$] solutions in frequency band of 8-20 Hz with respect to the current at the working electrodes.](image)

![Fig. 3. The power spectral density (PSD) spectra of the noise current (at the working electrode) with PBS solution for (a) SPE, (b) MWCNT/SPE, (c) MWCNT–βCD(Phys)/SPE, and (d) MWCNT–βCD(Phys)/SPE electrodes. The noise PSD spectra multiplied with frequency for (e) SPE, (f) MWCNT/SPE, (g) MWCNT–βCD(Phys)/SPE, and (h) MWCNT–βCD(Phys)/SPE electrodes.](image)

Based on the abovementioned observations, the noise level of the SPE and modified SPE electrodes have negligible bias dependence and a very small dependence on the electrode materials. In fact, the noise level only increases by ~3 times when MWCNT or MWCNT–βCD(Phys)/SE is attached to the SPE electrode. The increase of the noise level is not significant compared to the increase of the electrochemical sensing signals when MWCNT based modification are done. Since the noise level has a negligible bias dependence, the signal-to-noise ratio of the electrodes during actual sensing experiments will not depend much upon the range of the scanning potential. Therefore, these results signifies that the modification of SPE with MWCNT and functionalized MWCNT will not introduce any significant noise to the electrochemical sensing signals.

The influence of the analyte concentration on the noise of the pharmaceutical sensor was investigated by using three different concentrations of acetaminophen (APAP) with the MWCNT–βCD(Phys)/SPE electrode. The three concentrations of APAP were 1, 3 and 6 µM, which were prepared in PBS (7.4) buffer solution. The bias voltage used in the noise measurement was 0.3 V that is close to the APAP oxidation potential. Applying this bias voltage mimics the actual conditions during the sensing experiment so that the noise behavior can be estimated and related to the experimental conditions. The APAP concentration dependent noise spectra is shown in Figure 5(a).

It is observed that the low frequency noise PSD increases by around one to two orders of magnitude when the concentration of APAP is increased. However, the peak current at the oxidation potential is at least few orders of magnitude higher than the base-line current. Therefore, the increase in the noise due to higher APAP concentration does not significantly deteriorate the overall noise, as shown in Table II. Also, the increase of noise at very low concentration of APAP (<1µM) is
also insignificant compared to the noise at high APAP concentration (e.g., 6 µM). Another noise characteristic is the value of α corresponding to 1/f noise parameter. In the case of blank PBS solution, the value of α is 0.7 which increases to 2 with high concentrations of APAP (i.e., 6 µM). Higher value of α corresponds to sharper decrease of slope of the noise PSD spectra.

B. Noise in the pH sensor

The noise in Pd/PdO based pH sensors was studied by using three different pH buffers (i.e., 4, 7 and 10). As the pH sensors are operated in potentiometric configurations, where the pH of the solution is transduced into corresponding output potential, the noise PSD of the pH sensor was measured by connecting the pH sensor and a reference electrode to the positive and negative terminals of a low-noise amplifier, respectively. As shown in Figure 5(b), the value of α in the noise PSD is 2 for all the pH buffers in the frequency range of 4-100 Hz. Also, the noise power increases by one order of magnitude in pH buffer 4 and 10 compared to that of pH 7 in lower frequency range. The increase in the noise power at pH 4 and 10 may be attributed to the higher concentrations of H\textsuperscript{+} and OH\textsuperscript{−} ions, respectively, compared to that of neutral pH 7. However, the noise power levels (i.e., nanovolts square) for different pH are significantly smaller than the signal of the pH sensors, which are in the range of few tens to hundreds millivolts. Thus, the noise power of the pH sensor does not introduce any significant experimental uncertainty during the pH measurement.

C. Noise in the free chlorine sensor

The noise in the amine-modified carbon electrode-based free chlorine sensor was studied by using the electrode in a three-electrode voltammetric configuration along with an Ag/AgCl reference electrode and a platinum counter electrode, as shown in Figure 5(c). The bias voltages used in the noise were 0 V and 0.1 V, since the free chlorine sensor was operated only in these two biasing conditions in chronoamperometry measurement. It is observed that the noise behavior does not change with the biasing voltages. Also, the value of α in 1/f parameter is 2 in both biasing conditions. Moreover, the noise power is in the range of less than few nanovolts square per Hz, whereas, the minimum current detected in the free chlorine sensing is in the range of few tens of nanovolts. Therefore, the noise behavior of the free chlorine sensor does not introduce additional degradation in the sensitivity and limit-of-detection of the sensing signal.

D. Noise in the temperature sensor

The temperature sensor used in the integrated water quality monitoring system was based on a Wheatstone-bridge based temperature sensor made of PEDOT:PSS and silicon (Si) thermistors. Therefore, the output signal of the temperature sensor provides a potential difference. The overall noise behavior of the temperature sensor depends on the individual PEDOT:PSS and Si thermistors. Thus, we studied the voltage noise of the individual thermistors, as shown in Figure 5(d). The noise power of the PEDOT:PSS thermistor was ~1 to 3 orders of magnitude higher than that of the Si thermistor. This could be due to polymeric nature of the PEDOT:PSS thermistor which possesses higher molecular irregularity/defects compared to that of silicon, where the atoms are arranged in an almost perfect crystalline structure. Also, the 1/f parameters shows that the value of α were 0.45 and 1.3 for PEDOT:PSS and Si, respectively. This difference in the α value could be related to the transport mechanism of the two types of semiconductors. The noise power levels for both of the thermistors were in the range of few nanovolts square, which is much below the temperature induced voltage difference in the range of few hundreds of millivolts.

### Table II. Noise $K_r$ and LFN in different sensors compared to the noises during actual measurements.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Analyte</th>
<th>$\alpha$</th>
<th>Signal (in actual measurement)</th>
<th>Noise $K_r$</th>
<th>Noise (in actual measurement)</th>
<th>LFN in band 8-20 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWCNT-βCD(Phys)</td>
<td>0 µM APAP</td>
<td>0.7</td>
<td>0.6 µA</td>
<td>1.56x10^{-5}</td>
<td>&lt; 10 nA</td>
<td>3.64 nA</td>
</tr>
<tr>
<td></td>
<td>1 µM APAP</td>
<td>1.2</td>
<td>~3 µA</td>
<td>1.16x10^{-5}</td>
<td>&lt; 0.1 µA</td>
<td>9.14 nA</td>
</tr>
<tr>
<td></td>
<td>3 µM APAP</td>
<td>2.0</td>
<td>~5 µA</td>
<td>2.26x10{-5}</td>
<td>&lt; 0.5 µA</td>
<td>0.11 µA</td>
</tr>
<tr>
<td></td>
<td>6 µM APAP</td>
<td>2.0</td>
<td>~8 µA</td>
<td>1.77x10^{-5}</td>
<td>&lt; 2 µA</td>
<td>1.55 µA</td>
</tr>
<tr>
<td>Pd/PdO</td>
<td>pH 4</td>
<td>2.0</td>
<td>~60 mV</td>
<td>2.08x10^{-14}</td>
<td>&lt; 0.1 mV</td>
<td>2.12 nV</td>
</tr>
<tr>
<td></td>
<td>pH 7</td>
<td>2.0</td>
<td>~200 mV</td>
<td>1.92x10^{-14}</td>
<td>&lt; 0.1 mV</td>
<td>0.74 nV</td>
</tr>
<tr>
<td></td>
<td>pH 10</td>
<td>2.0</td>
<td>~400 mV</td>
<td>4.7x10^{-13}</td>
<td>&lt; 0.1 mV</td>
<td>2.14 nV</td>
</tr>
<tr>
<td>Free chlorine</td>
<td>PBS 7.4</td>
<td>2.0</td>
<td>~100 to 1500 nA</td>
<td>&lt; 1.84x10^{-1}</td>
<td>&lt; 20 nA</td>
<td>11 nA</td>
</tr>
<tr>
<td>Temperature</td>
<td>Si</td>
<td>1.3</td>
<td>~100 to 800 mV</td>
<td>&lt; 3.97x10^{-14}</td>
<td>&lt; 0.1 mV</td>
<td>10 nV</td>
</tr>
<tr>
<td></td>
<td>PEDOT-PSS</td>
<td>0.45</td>
<td>~100 to 800 mV</td>
<td>&lt; 1.86x10^{-14}</td>
<td>&lt; 0.1 mV</td>
<td>55 nV</td>
</tr>
</tbody>
</table>
IV. CONCLUSIONS
In this study on noise in electrochemical sensors, several experiments were reported in an effort to understand the dependence of noise level on the type of electrolyte solutions, bias voltages and electrode materials. This would allow us to identify possible effect of noise on electrochemical sensing performances. Analyte types and electrode modifications, as well as bias voltages dependent 1/f noise were studied in screen printed electrode (SPE) and MWCNT/MWCNT-βCD(Phys/SE) modified SPE. The experimental results showed that the noise level remains almost constant, independent on the type of electrolyte solutions or bias voltages. However, a slight increase of the noise level (about three times) was observed when SPE electrode was modified with MWCNT and MWCNT-βCD(Phys/SE). The PBS solution showed wider current range of the working electrode to that of K₂[Fe(CN)₆] solution. Also, the electrochemical potential of the electrodes in PBS solution was close to 0 mV, whereas it was below −200 mV for K₂[Fe(CN)₆] solution. The increase of the noise level due to MWCNT and MWCNT-βCD(Phys/SE) modifications has negligible impact on the signal-to-noise ratio in the electrochemical sensing measurements. In fact, SNR improves when surfaces are functionalized with MWCNT. The noise in pH, free chlorine and temperature sensors are also analyzed. From the noise measurements, it is shown that there was negligible influence on the sensing signal, and the noise is lower than the limit-of-detection reported in previous chapters.

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REFERENCES
Nanoimprint Technology for Liquid-Gated Si Nanowire FET Biosensors: Noise Spectroscopy Analysis

Volodymyr Handziuk, Yurii Kutovyi, Hanna Hlukhova, Ihor Zadorozhnyi, Nazarii Boichuk, and Svetlana Vitusevich*
Bioelectronics (ICS-8)
Forschungszentrum Jülich
D-52425 Jülich, Germany
Email: *s.vitusevich@fz-juelich.de

Mykhailo Petrychuk
Faculty of Radiophysics, Electronics and Computer Systems
Taras Shevchenko National University of Kyiv
03127 Kyiv, Ukraine

Abstract—Silicon nanowire (Si NW) field-effect transistors (FETs) are currently attracting considerable attention in both theoretical and practical studies. Their remarkable properties, in particular their high sensitivity to surface charges, open up prospects for various sensing applications. Devices based on Si NW FETs are usually fabricated by relatively expensive electron-beam lithography (EBL). Here, we report on the results of studies involving accumulation mode p-type liquid-gated Si NW FETs fabricated making use of optimized, cost-efficient, and CMOS-compatible nanoimprint technology. Electrical transport in the fabricated structures is investigated using DC current-voltage measurements and low-frequency noise spectroscopy. The results obtained indicate the high quality of the Si NW FET structures and their stable operation in the electrolyte liquid environment without leakage currents. Noise studies reveal that the main origin of noise is the mobility fluctuation of the charge carriers in the nanowire channel. The nanowire FET performance parameters are extracted by data analysis and compared to those reported in the literature.

Index Terms—Silicon nanowires, nanoimprint technology, noise spectroscopy, biosensors.

I. INTRODUCTION

Continuous quality monitoring of silicon nanowire field-effect transistor (FET) structures is an essential prerequisite for the successful development of robust and reliable biosensors based on silicon nanowires (Si NWs). Thus, appropriate performance characterization tests have to be performed at every development stage from device fabrication up to encapsulation and calibration.

Fabrication is usually performed on the basis of commercially available silicon-on-insulator (SOI) wafers. Choosing either p- or n-type SOI wafers with the defined level of impurities determines the doping level of nanowires and their further performance as FET biosensors. The source and drain areas should have small resistances to be able to form ohmic contacts. They are usually defined by photolithography and a subsequent heavy doping procedure. Then the mesa structure, including the pattern for the source and drain areas as well as for the areas reserved for nanowires, is defined using reactive ion etching (RIE). Nanosized nanowire structures are patterned by means of electron-beam lithography (EBL), followed either by RIE or wet chemical etching procedures. Finally, passivation and surface modification steps are performed to make the fabricated Si NW FET devices suitable for selective and sensitive measurements of bioliquids.

One of the key advantages of the “top-down” fabrication technology is its high complementary metal oxide semiconductor (CMOS) compatibility. It also provides precise control over the orientation of nanowires. State-of-the-art electron beam and focused ion beam lithographies allow patterning of nanoscale objects less than 10 nm in size. However, such techniques are relatively expensive and not well suited for large-scale serial production. Recent advances in fabrication techniques, such as nanoimprint lithography (NIL) permit reliable and cost-efficient nanostructure patterning, which facilitates rapid large-scale fabrication of nanowires with a high yield rate.

The fabrication of nanowire biosensors using nanoimprint lithography consists of two main steps: fabrication of the mold (a reusable master for patterning the nanowire structures) and subsequent fabrication of Si NW FETs – active transducers of the sensor. Optimization of the technological processes involved in fabrication is crucial for the performance of the devices, and thus has to be performed carefully. The master structure should allow low-roughness high-resolution nanopatterning. It should also have good mechanical properties, e.g., stiffness, to be suitable for multiple use. In this respect, silicon can be considered a perfect substrate material for mold fabrication. It possesses high mechanical stability, and it can be processed using well-developed CMOS-compatible technologies. Anisotropic wet etching techniques allow very smooth etching profiles to be achieved, which is crucial for device performance.

Despite the tremendous sensitivity of biosensors based on Si NW FETs, they still have certain disadvantages. One of the most fundamental problems is the degeneration of their electrical characteristics while working in electrolyte liquid environments [1]. This unwanted effect is most often caused by the degradation of the gate insulator, which is exposed to the bioliquids under study in the sensing experiments.
Therefore, the quality of the gate dielectric often defines the performance of Si NW FET sensors [2]. In this regard, using techniques to constantly control device performance and establishing novel nondestructive sensing approaches is crucial for the development of reliable Si NW biosensors.

Ultimate downscaling of the electronic devices substantially restricts the use of traditional characterization techniques, e.g., capacitance-voltage measurements. Thus, an alternative approach has to be utilized to investigate the conducting channel/insulator and insulator/electrolyte interfaces in nanoscale liquid-gated Si NW FETs. Low-frequency noise spectroscopy can therefore provide information about the quality, electrical performance, and operating regimes of silicon nanowire structures [3]. This is a powerful, highly sensitive, and nondestructive characterization approach. Thus, it can replace capacitance-voltage measurements and complement DC current-voltage characterization.

In this contribution, we report on studies of transport and low-frequency noise behavior of accumulation mode $n$-type liquid-gated silicon nanowire field-effect transistors, fabricated using advanced CMOS-compatible nanoimprint technology. We estimated the important characteristics of our NW FET devices, i.e., the threshold voltage $V_{Th}$, the subthreshold swing $SS$, and the carrier mobility $\mu$. Investigation of the noise behavior revealed that the excess noise originates from the mobility fluctuations in the NW channel. We also estimated the dimensionless Hooge parameter, the measure of device “noisiness”, and compared it with the values reported in the literature. Our findings indicate the high quality of the fabricated Si NW FET devices, and they are important for the development of highly-sensitive Si NW-based biosensors.

II. RESULTS AND DISCUSSION

A. Device fabrication

We designed and fabricated the nanoimprint mold with decreased roughness on the basis of Si(110) substrates using electron-beam lithography and wet KOH etching. It was utilized as a reusable mask to pattern the nanowires. The high quality of the mold for NW structures was confirmed by atomic force microscopy (AFM) imaging (see Fig. 1 (a)). It should be noted that the fabricated mold allows simultaneous patterning of the tiny nanowire structures as well as the large feed lines and contact pads.

The silicon nanowire biosensors were fabricated on the basis of 100 mm $p$-type SOI wafers, purchased from Soitec. The substrates had 70 nm of a $p$-type active Si(100) layer, separated from the highly doped base silicon by a 145 nm thick buried oxide (BOX) layer. To transfer the pattern from the mold to the SOI substrate, the latter was coated with mr-I 7020R resist and thermal nanoimprint lithography was performed. After the nanowires had been defined in the resist, dry and wet etching procedures were used to transfer the pattern first to the hard mask and then to the active silicon layer. Wet etching was performed in 5% tetramethylammonium hydroxide (TMAH) solution at 60°C in order to implement a smooth etching profile of the nanowire sidewalls. The SEM micrograph of a typical single silicon nanowire structure after TMAH etching is shown in Fig. 1 (b). This approach allowed a significant reduction of the NW surface roughness, and therefore a considerable improvement of the performance of the devices. Nanowire etching was followed by ion implantation, metallization, and thermal annealing of the feed lines to implement ohmic contacts to NW structures. The uniform 8 nm thin gate dielectric was formed using a dry thermal oxidation process. Lastly, passivation and encapsulation were performed to make the devices suitable for measurements in electrolyte liquid environments.

B. Transport characterization

DC electrical transport in the fabricated Si NW FET devices was investigated using the current-voltage (I-V) approach. The NW structures, covered by silicon dioxide, were exposed to 10 mM PBS pH = 7.4 solution. An external Ag/AgCl reference electrode was used to apply the gate potential while the drain–source bias $V_{DS}$ was set at 100 mV in all the measurements performed, thus implementing the linear operation regime for the transistor structures. The current-
Fig. 2: Set of transfer curves, measured on the accumulation mode p-type single Si NW FET structures with a width of 250 nm and lengths in the range of 6 µm to 22 µm. The corresponding transconductance curves are shown in the inset.

Voltage characteristics were measured using a Keithley 2602A two-channel source-measurement unit. Typical transfer curves measured on the accumulation mode p-type single silicon nanowire FETs with a width of 250 nm and lengths in the range of 6 µm to 22 µm are shown in Fig. 2. The fabricated devices show typical transistor-type behavior of the electrical characteristics and stable operation in the electrolyte-liquid environment. It should be noted that the registered leakage current $I_L \leq 1$ nA was at least three orders of magnitude lower than the drain current $I_D$ in the entire range of the liquid gate voltages applied. This confirms the high quality of the fabricated nanowire FET structures.

Analysis of the NW FET transfer curves allows the quantitative determination of such important parameters as threshold voltage $V_{Th}$, subthreshold swing $SS$, and carrier mobility $\mu$. The first two parameters can be extracted directly from the transfer curve while the remaining parameter can be estimated using the following expression [4]:

$$\mu = \frac{g_m L}{W C_{ox} V_{DS}}.$$  \hspace{1cm} (1)

Here, $C_{ox}$ is the gate oxide capacitance per unit area, $g_m = \partial I_D/\partial V_G |_{V_{DS}=const}$, is the transconductance, $W$ and $L$ are the width and length of the NW channel, respectively. In the first approximation, the $C_{ox}$ value can be estimated using the expression for the parallel plate capacitor:

$$C_{ox} = \varepsilon_0 \varepsilon,$$  \hspace{1cm} (2)

where $\varepsilon_0 \approx 8.85 \times 10^{-12}$ F m$^{-1}$ is the dielectric permittivity of vacuum, $\varepsilon = 3.9$ is the relative permittivity of the gate oxide, and $t_{ox} = 8$ nm is the gate oxide thickness. The parameters of the studied Si NW FET structure, extracted from the transfer curves, are summarized in Table I. The obtained values are in good agreement with the data reported in the literature [5].

C. Noise studies

The low-frequency noise spectra (from 1 Hz to 10 kHz) were measured using our advanced noise measurement setup developed in house [6]. Typical normalized current noise spectra, measured on the single silicon nanowire FET structure with a width of 250 nm and length of 6 µm, acquired at different values of the liquid gate voltage are shown in Fig. 3. The measured spectra demonstrate a clear $1/f$ dependence in the entire range of the gate voltages applied.

To analyze the origin of the noise, we calculated the input-referred noise $S_U$ using the current noise spectral density $S_I$ and the transconductance value $g_m$, obtained as the first derivative of the NW transfer curve:

$$S_U = S_I \frac{1}{g_m}.$$  \hspace{1cm} (3)

The input-referred noise level, taken at the frequency of 100 Hz, is plotted vs. the overdrive gate voltage $V_{OV} = V_G - V_{Th}$ in Fig. 4. At increased gate voltages $V_{OV} > -0.5$ V, the $S_U$ increases with increasing liquid gate overdrive. Such noise behavior indicates that the main noise source is the mobility fluctuation of the charge carriers in the nanowire channel, which can be described by the Hooge model [7].

To compare the noise level of the fabricated devices with that reported in the literature we estimated the dimensionless

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold voltage $V_{Th}$</td>
<td>-0.8 V</td>
</tr>
<tr>
<td>Subthreshold swing $SS$</td>
<td>130 mV dec$^{-1}$</td>
</tr>
<tr>
<td>Carrier mobility $\mu$</td>
<td>$95 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$</td>
</tr>
</tbody>
</table>
The fabricated nanowire FET structures is related to the mobility fluctuations of the charge carriers in the nanowire channel. The estimated values of the Hooge parameter were close to $\alpha_H \approx 1 \times 10^{-3}$, which is comparable with or even lower than those reported for most traditional fabrication technologies including electron-beam lithography.

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**III. CONCLUSIONS**

To conclude, we fabricated silicon nanowire field-effect transistor devices with enhanced characteristics using cost-efficient nanoimprint technology. The high quality of the Si NW structures was confirmed by AFM and SEM imaging. The fabricated nanowire FETs were characterized using the DC current-voltage approach and low-frequency noise spectroscopy. Important performance characteristics including the threshold voltage, subthreshold swing, and carrier mobility were estimated by analyzing the NW FET transfer curves and summarized in Table I. The obtained values are in good agreement with those reported in the literature. Analysis of the noise spectra revealed that the main noise source in the fabricated Si NW FET structures is related to the mobility fluctuations of the charge carriers in the nanowire channel.

**Fig. 4.** Input referred noise taken at the frequency of 100 Hz as a function of the overdrive liquid gate voltage.

**Fig. 5.** Hooge parameter as a function of the liquid gate overdrive voltage.

Here, $q = 1.602 \times 10^{-19}$ C is the electron charge. The Hooge parameter obtained vs. the overdrive gate voltage dependence is shown in Fig. 5. The $S_{IH}$ and $\alpha_H$ values in the liquid-gated NW structures fabricated using the optimized nanoimprint technology are comparable with those obtained for structures fabricated using electron-beam lithography [10] for the same range of overdrive gate voltages. The results demonstrate the high quality of the liquid-gated NW FET structures, which can be used for biosensing and the biomedical monitoring of biological liquids.

$$\alpha_H = \frac{fS_I}{I_D} \frac{L^2}{q\mu V_{DS}}.$$  

(4)
Ascorbate Detection Using Single-Trap Phenomena in Two-Layer Si NW FETs

Yuriy Kutovyi
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
y.kutovyi@fz-juelich.de

Ihor Zadorozhniy
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
i.zadorozhniy@fz-juelich.de

Natalia Naumova
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
naumova.kandaurova@gmail.com

Nazarii Boichuk
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
n.boichuk@fz-juelich.de

Mykhaylo Petrychuk
Taras Shevchenko National University of Kyiv, 03127 Kyiv, Ukraine
m.petrychuk@gmail.com

Svetlana Vitusevich
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
s.vitusevich@fz-juelich.de

Abstract—Biosensing and detection of various biological and chemical species are extremely important for the diagnosis of different diseases. Nowadays, liquid-gated silicon nanowire field-effect transistors (Si NW FETs) are attracting considerable attention as powerful sensing transducers. We have recently designed and fabricated novel silicon nanowire structures that consist of two silicon layers with different doping concentrations and applied them for the detection of ascorbate biomolecules in electrolyte solutions. The response of the sensor to different concentrations of target molecules was revealed by monitoring the changes in both the drain current of the transistor and the capture time as a characteristic of a single trap responsible for a random telegraph signal (RTS) noise of the device. The results demonstrated that changes in the surface potential introduced by ascorbate molecules are registered with enhanced sensitivity in the case of a single-trap-based biosensing approach compared to the standard drain current approach. Here we report that single-trap phenomena in fabricated two-layer structures are favorable for biosensing applications and can be effectively used as a highly sensitive tool for monitoring such antioxidant molecules as ascorbate in order to measure oxidative stress in chemical, biochemical, and biological systems.

Keywords—Nanowire biosensors, single trap, low-frequency noise, ascorbate molecules, oxidative stress.

I. INTRODUCTION

Research into and development of new devices, as well as new methods enabling a direct, sensitive, and rapid analysis of biological and chemical substances, are essential for both theoretical studies and practical applications. Nowadays, among a large variety of biosensing platforms, silicon nanowire field-effect transistors (NW FETs) are considered as excellent candidates for biosensing from the viewpoint of the development of label-free, high-speed and ultrasensitive FET-based biosensors [1-3]. Although the utilization of NW FETs as biochemical transducers offers considerable advantages such as mass fabrication and biocompatibility, there is still considerable debate concerning both the best transducer configuration and approach for the detection of different target bio-objects, e.g. ions, proteins, DNA molecules, viruses. To address these problems, the noise level of the devices has to be considered, which usually affects the sensitivity of the sensors and defines their detection limit.

Typically, the physical sources of noise in large-area devices originate from fluctuations of microscopic entities, such as mobility or number of charge carriers, due to their interactions with different traps randomly distributed in a gate insulator layer [4, 5]. However, with a scaling down of small devices the noise might be determined by a single carrier phenomenon [5, 6]. In particular, a small device can be fabricated with characteristic sizes below 100 nm, where a point defect (a single trap in a nanotransistor gate dielectric layer) can exchange charge carriers with the conductive channel of a nanotransistor. In this case, a two-level discretized fluctuation random signal known as random telegraph signal (RTS) noise can be observed [5-7]. Being a fully stochastic process, RTS noise can be characterized by its main parameters, i.e., capture (τc) and emission (τe) time constants as well as RTS amplitude, e.g. a change of the signal when a transition event occurs. At the same time, the capture time constant indicates the average time during which the trap is empty, while the emission time constant reflects the time during which the trap is occupied by a charge carrier.

Usually, an RTS phenomenon in a nanoscale device is treated as noise affecting its performance. However, due to the discrete nature of the phenomenon and its specific properties, the phenomenon provides a significant opportunity for practical applications including biosensing [6, 7]. Recently, in order to achieve a better single-trap switching kinetic (more readily controllable capture/emission time characteristics of a single trap) we fabricated a new type of silicon nanowire consisting of two silicon layers with different concentrations of dopants: the first layer is low-doped silicon with an impurity concentration of $10^{13}$ cm$^{-3}$, while the second layer is highly-doped silicon with an impurity concentration of $10^{17}$ cm$^{-3}$.

In this study, we show that the fabricated devices indeed display pronounced RTS noise with advanced characteristics favorable for biosensing. We apply such two-layer (TL) nanowire sensors for the detection of ascorbate molecules in buffer solutions and demonstrate that nanotransistor sensors exploiting single-trap phenomena allow different concentrations of target molecules, i.e. ascorbate molecules, to be detected with enhanced sensitivity. It should be noted that no attempts to measure changes in ascorbate concentration using nanowire-based sensors have yet been published.

II. EXPERIMENTAL METHODS

A. Fabrication procedure

The structures under study are silicon two-layer nanowire FETs with p-type conductivity configured as accumulation mode transistors. Nanowire structures were fabricated using a CMOS-compatible top-down approach. The fabrication process was based on silicon-on-insulator (SOI) wafers with a 50 nm thick p-type silicon layer and a 145 nm thick buried oxide layer (BOX). A second p-type silicon layer with a boron concentration of $10^{17}$ cm$^{-3}$ was grown epitaxially using the
chemical vapor deposition method on top of the active silicon layer of SOI wafers with a boron concentration of $10^{15}$ cm$^{-3}$. Single nanowire structures were defined using e-beam lithography and patterned using tetramethylammonium hydroxide (TMAH) wet chemical etching in order to provide atomically flat surfaces as well as reduced defect density of the nanowires. Source and drain contacts were highly implanted with boron dopants resulting in p'-p-p' transistor structures. An 8 nm thick SiO$_2$ layer was grown thermally in order to protect the silicon nanowires and to serve as a gate dielectric layer. Low-resistive 200 nm thick contact feedlines were formed by thermal evaporation followed by a lift-off process and high-temperature annealing in order to achieve an ohmic contact. Finally, samples were passivated with a polyimide layer in order to prevent current flow between metal leads and liquid during measurements. Small windows allowing a gating solution to exclusively access the nanowire area were opened by means of photolithography. All fabrication processes were performed at the Helmholtz Nano Facility of Forschungszentrum Jülich.

**B. Experimental setup**

A schematic illustration of the fabricated structure including the measurement configuration is shown in Fig. 1a. The PDMS microchannel constructed in our laboratory was used for liquid delivery. An Ag/AgCl reference electrode was inserted into the outlet tube close to the microchannel in order to avoid any pollution of the gating solutions flowing through the microfluidic channel. Both current-voltage characterization and noise measurements were performed in order to characterize the nanowire FETs.

**III. RESULTS AND DISCUSSION**

**A. I-V and noise characterization**

In the present study, we discuss the sensing results obtained at room temperature for a 70 nm wide and 400 nm long two-layer silicon nanowire with a triangular cross-section as shown in Fig. 1b. Transfer and output curves measured for the same liquid-gated nanowire transistor are shown in Fig. 2a and Fig. 2b, respectively. A phosphate-buffered saline (PBS) solution with pH = 7.4 was used as the gating solution in this case. It should be noted that the nanowire demonstrated typical p-type FET behavior with a subthreshold swing of 100 mV/decade (see Fig. 2a). At the same time, the leakage current through the 8 nm thick SiO$_2$ gate dielectric layer was negligibly small during the current-voltage and noise measurements.

As can be seen from Fig. 2b, at liquid-gate voltages higher than $V_{LG} = -1.4$ V, noticeable fluctuations in the drain current were observed. Such stepwise behavior of the drain current implies the presence of a single trap in the front-gate dielectric layer exchanging the carriers with the conductive channel of the nanowire transistor. In order to investigate the observed fluctuation phenomena, noise spectroscopy was used as a powerful tool for the analysis of device performance.

**B. Noise spectroscopy analysis**

Fig. 3a illustrates the noise measurement results obtained for the same 70 nm wide and 400 nm long two-layer silicon nanowire structure at different liquid-gate voltages. The drain-source bias was kept constant at -100 mV, which corresponds to a linear working regime of the transistor, and in this case the back-gate electrode was grounded. As can be seen from Fig. 3a, a drain-source voltage noise PSD demonstrates Lorentzian-shaped noise behavior with the characteristic frequency shifting to higher frequencies with increasing liquid-gate voltage. Such noise behavior is typical for nanowire transistors yielding RTS noise [7-9]. At the same time, as can be seen in the inset of Fig. 3a, the drain current of the transistor is indeed affected by RTS noise thus demonstrating switching kinetics between two distinct states (levels): the low level, which corresponds to the case when the trap is occupied, and the high level of the drain current, which corresponds to the state when the trap is empty (a carrier is in the channel).

Current-voltage characteristics of the nanostructures were measured using Keithley 2400 and 2430 current/voltage measurement units providing an accurate measurement of both the drain-source and the gate-source currents with a resolution of less than 10 pA. Low-frequency noise characteristics were measured using a fully automated ultralow noise measurement setup. A detailed description of our measurement setup is presented elsewhere [7]. The noise data collection was performed using a U2542A data acquisition module. A fast Fourier transform method was then applied to the measured data in order to translate time-dependent source-drain voltage fluctuations into a voltage power spectral density (PSD) $S_V$. In order to achieve reliable noise characteristics of the samples under study, all electrical measurements were performed in a custom-built Faraday cage to protect against any external electromagnetic fields.

![Fig. 1.](image-url) (a) A schematic view of a Si TL NW FET with a microfluidic channel (cross section along the nanowire). (b) SEM micrograph of the fabricated single silicon nanowire cross section obtained using focused ion beam milling.
As was mentioned above, RTS noise is a fully stochastic process that can only be characterized by the average capture and emission time constants. Therefore, in order to estimate RTS characteristic times, a statistical analysis using the amplitude histogram method was performed \cite{7,10}. The histograms of RTS time traces measured at different liquid-gate voltages are shown in Fig. 3b. Two Gaussian peaks corresponding to the capture and emission states can be clearly resolved. It should be noted that the amplitude of the peaks reflecting a probability of the trap being in a certain state (occupied by a carrier or empty) strongly depends on the V_{LG} applied to the sample. The calculated average capture $\tau_c$ and emission time $\tau_e$ constants are presented in Fig. 4a as a function of the drain current controlled by the liquid gate. It can be seen that both time constants depend on the drain current, i.e. density of carriers, and demonstrate behavior which deviates from that predicted by the classical Shockley-Read-Hall (SRH) theory. It should be emphasized that similar dependences of RTS noise time constants were observed for different devices fabricated in the same technological run. The similar characteristics of RTS noise observed for the inversion mode TL NW structures are published in \cite{7}. In the case of the inversion regime, such strong dependences of time constants on the density of carriers can be explained in the framework of the Coulomb blockade energy model \cite{7,11}. At the same time, an additional energy barrier should be considered in order to explain the dynamics of capture and emission processes in the accumulation working regime \cite{11}.

The results of the sensing experiment are presented in Fig. 5 in term of changes in both the drain current (the standard I-V approach) and the capture time (the single-trap approach) as a characteristic parameter of the single trap. As can be seen from Fig. 5, changes in both parameters were registered with increasing concentration of ascorbate molecules. However, the average capture time constant changes much more strongly in response to the change of the concentration of target molecules compared to the drain current. This result demonstrates that the single-trap approach is indeed favorable for biosensing with enhanced sensitivity using the fabricated TL NW-based sensors.

Fig. 2. (a) Transfer curves of a 400 nm long and 70 nm wide Si TL NW FET measured with a liquid gate at a constant V_{GS} of -100 mV (left axis, linear scale; right axis, semi-logarithmic scale). (b) Output characteristic of the same Si TL NW FET measured at different liquid-gate voltages that were varied in the range from -1.0 V to -1.5 V with a step size of -0.1 V.

Fig. 3. (a) Drain-source voltage noise PSD measured at different liquid-gate voltages and at a constant drain-source bias of -100 mV (inset: two-level RTS fluctuations measured at V_{LG} = -1.70 V). (b) Histograms of measured time traces confirming two-level fluctuations.

Fig. 4b illustrates the probability of the trap being occupied by a single carrier (a hole for the p-type FET) $g = \tau_e / (\tau_e + \tau_c)$ at different liquid-gate voltages. It can be seen that trap occupancy probability is indeed effectively controlled by the liquid-gate implying that the single trap characteristics are also very sensitive to the changes of surface potential. As was mentioned above, RTS noise is usually considered an undesirable effect that affects transistor performance. However, in this work we applied the nanowire transistor which demonstrated single-trap phenomena with advanced characteristics for the detection of different concentrations of ascorbate molecules diluted in the 180 mM PBS solution with initial pH = 7.4. In this respect, the current-voltage as well as the low-frequency noise measurements at different concentrations of ascorbate molecules were performed at a defined working point (V_{DS} = -100 mV, V_{LG} = -1.8 V).

The results of the sensing experiment are presented in Fig. 5 in term of changes in both the drain current (the standard I-V approach) and the capture time (the single-trap approach) as a characteristic parameter of the single trap. As can be seen from Fig. 5, changes in both parameters were registered with increasing concentration of ascorbate molecules. However, the average capture time constant changes much more strongly in response to the change of the concentration of target molecules compared to the drain current. This result demonstrates that the single-trap approach is indeed favorable for biosensing with enhanced sensitivity using the fabricated TL NW-based sensors.
Fig. 4. (a) Capture and emission characteristic time constants as a function of the drain current controlled by the liquid-gate. The dashed red line with slope -8.2 reflects a strong dependence of capture time on the drain current. (b) Trap occupancy probability as a function of the liquid-gate voltage applied. The dashed red line here represents a guide for the eye.

IV. CONCLUSIONS

To conclude, we fabricated liquid-gated silicon two-layer nanowire structures and applied them for the detection of different concentrations of ascorbate molecules. Electrical and transport properties as well as noise phenomena in Si TL NW FETs were investigated utilizing both the current-voltage characterization technique and noise spectroscopy. We demonstrated that applying a single-electron trapping-detrapping process in the fabricated sensors allows the detection of ascorbate molecules with enhanced sensitivity. The results shed light on the development of highly sensitive tools for monitoring such important antioxidants molecules as ascorbate.

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REFERENCES

Noise in Single-Trap Punctual Nanobiosensors

Ihor Zadorozhnyi
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
I.zadorozhnyi@fz-juelich.de

Yuriy Kutovyi
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
y.kutovyi@fz-juelich.de

Soo Hyeon Kim
Institute of Industrial Science (IIS), The University of Tokyo, Tokyo 153–8505, Japan
shkim@iis.u-tokyo.ac.jp

Tero Fujii
Institute of Industrial Science (IIS), The University of Tokyo, Tokyo 153–8505, Japan
tfujii@iis.u-tokyo.ac.jp

Andreas Offenhauser
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
a.offenhauser@fz-juelich.de

Svetlana Vitusevich
Bioelectronics (ICS-8), Forschungszentrum Jülich, 52425 Jülich, Germany
s.vitusevich@fz-juelich.de

Nicolas Clément
LIMMS/CNR-S-IIS, The University of Tokyo, Tokyo 153–8505, Japan
nclement@iis.u-tokyo.ac.jp

Abstract—Punctual nanobiosensors based on the trapping/detraping of a single electron in a defect near the channel of a nanotransistor have been proposed for ultimate scaling and high sensitivity [1]. Unlike the usual nanotransistor-based biosensors, where the threshold voltage shift is the signal and voltage fluctuations are the noise, the signal in these devices is a trap occupancy probability. The fluctuations of this parameter become the noise. Therefore, the signal-to-noise (S/N) ratio needs to be quantitatively studied theoretically in order to compare the performance of sensors and to optimize experimental conditions. Here we show that under optimized conditions for the background noise amplitude and the averaging filter the S/N ratio can be substantially increased, above the level expected for devices monitoring threshold voltage shift.

Keywords—Nanowire sensors, single trap, low-frequency noise, signal-to-noise ratio.

I. INTRODUCTION

A transfer of a single charge carrier between a process-induced defect (e.g. a single trap) located in a gate dielectric layer and a conductive channel of a nanotransistor results in a two-level discretized fluctuation signal known as random telegraph signal (RTS) noise [1]. Usually, such RTS fluctuations in nanoscale devices are treated as a noise source that degrades nanodevice performance. However, due to the discrete nature of the phenomenon and the possibility of monitoring its response with surface potential, RTS noise provides a significant opportunity for practical applications including biosensing [1–3]. In particular, monitoring the time constants of a single trap can be used as a new way to sense pH or biomolecules interacting with a nanotransistor. Such an approach permits a substantial increase in sensitivity in comparison to standard approaches that use the threshold voltage or drain current shift as the signal. However, as the signal-to-noise (S/N) ratio is a key parameter for any sensor, it should also be carefully introduced and investigated for nanotransistor sensors exploiting single-trap phenomena. This would allow sensor performance to be compared and experimental conditions to be optimized. In the case of transistor-based sensors, the S/N ratio is usually defined as the ratio of threshold voltage or drain current shift against low-frequency noise. This noise mainly originates from electron trapping/detraping events [4, 5]. Therefore, it is now well accepted that the S/N ratio for transistor-based sensors can be

optimized by modifying device dimensions, the quality of device fabrication, and the nature of noise sources [5–7]. However, nanoscale transistors statistically have a single trap, and when sensing based on single-trap phenomena is exploited (see Fig. 1), RTS is the signal and not a noise source. Therefore, the investigation of the nature of the noise is of fundamental importance in this particular case where the biosensor becomes a single and punctual single-electron trap.

Fig. 1. Schematic representation of a nanotransistor with a punctual defect.

In this work, we demonstrate that for punctual nanobiosensors exploiting single-trap phenomena, noise can be defined as a fluctuation of trap occupancy probability (g-factor) affecting the distribution of RTS time constants. Moreover, in this study we present a method enabling us to calculate g-factor noise. In this respect, an S/N ratio was carefully introduced for the RTS-based biosensing approach. Furthermore, we show that under optimized conditions for the background noise amplitude (dielectric polarization noise) and an averaging window filter time lapse, the S/N ratio can indeed be substantially increased in comparison to sensors whose working principle is based on monitoring changes in threshold voltage or drain current.

II. RESULTS AND DISCUSSION

RTS is generated numerically using master equations with the additional consideration of background noise components (dielectric polarization noise and thermal noise). The capture and emission rates were defined using the following formulas:
\[ R_c = R_0 \exp \left[ -\frac{q}{kT} \gamma (E_{\text{trap}} - a V_g) \right] \]  

(1)

\[ R_e = R_0 \exp \left[ -\frac{q}{kT} (1 - \gamma) (E_{\text{trap}} - a V_g) \right] \]  

(2)

where \( q \) is the elementary charge, \( T \) is an absolute temperature, \( k \) is the Boltzmann constant, \( \gamma \) is the charge transfer coefficient, \( E_{\text{trap}} \) is the energy of the trap, \( V_g \) is the gate voltage applied, and \( a \) is the ratio between the trap depth \( d_{\text{trap}} \) and the dielectric thickness \( t_{\text{ox}} \). For simulation purposes, the following values were used: \( T = 300 \, K \), \( E_{\text{trap}} = 100 \, meV \), \( d_{\text{trap}} = 2 \, nm \), \( t_{\text{ox}} = 20 \, nm \), thus \( \alpha \approx 0.1 \). The charge transfer coefficient \( \gamma \) was set to 1 for all simulation results presented in this study, so that the emission rate does not depend on the gate voltage applied. The prefactor \( R_0 \) was set to a constant value of 100.

RTS timetraces generated for different liquid-gate voltages are presented in Fig. 2A. The averaged capture and emission times characterizing the RTS process are plotted in Fig. 2B against the applied liquid-gate voltage and reflect the typical behavior of capture (\( \tau_c \)) and emission (\( \tau_e \)) times for nanowire FET devices [2, 8].

As can be seen from Fig. 3B, the trap occupancy factor \( g \) follows a widened Fermi-type distribution (due to the partial potential drop at the trap level). At low \( V_{LG} \), the trap is unoccupied and at high voltages, it is fully occupied.

We assume that \( g \) is the signal and that it fluctuates in the noise. We expect that the sources of g-factor fluctuations have the same origin as in any nanotransistor including flicker noise and thermal noise. However, the impact of these noise sources on trap occupancy fluctuations has to be estimated. Therefore, we suggest converting RTS voltage fluctuations into fluctuations of the trap occupancy factor. More precisely, we calculate g-factor probability over a given time window \( T \) and then by sliding the window along the generated timetrace one can obtain timetrace with the trap occupancy factor fluctuations. Typical g-factor fluctuations calculated for different time windows are presented in Fig. 4A. It should be noted that g-factor fluctuations decrease with increasing \( T \) while keeping the value of \( g \) unchanged (here \( g = 0.5 \) is obtained at \( V_{LG} = 1 \, V \)).
Finally, the power spectral density (PSD) of the trap occupancy factor g (Fig. 4B) is calculated from g-factor timetraces shown in Fig. 4A. It should be emphasized that in the frequency domain, the g-factor PSD demonstrates a clear $1/f^2$ dependence in the frequency range 1 Hz – 1 kHz for the calculated time windows. As can be seen from Fig. 4B, the g-factor noise amplitude decreases with increasing time window T confirming the data in Fig. 4A. The result can be explained considering the fact that a larger time window T corresponds to a larger number of trapping/detrapping events occurring over time T, and therefore the trap occupancy factor can be estimated with higher accuracy for larger windows. It should be noted that such an approach for g-factor noise calculation is similar to the averaging filter in a stochastic process. Note that such noise suppression cannot be obtained by averaging drain-source voltage fluctuations.

In order to compare the signal-to-noise ratio obtained for the single-trap phenomena approach with the usual approaches, we introduced the input-referred trap occupancy factor noise $S_{9g}$ in a similar manner to the way in which one can calculate the equivalent input-referred noise for voltage fluctuations $S_{1g}$ as defined by [8-11]:

$$S_{9g} = S_{1g} / g_m^2$$

where $S_{1g}$ denotes the current noise and $g_m = \partial V_c / \partial V_b$ stands for the conductance. Similarly, we define the input-referred trap occupancy factor noise $S_{9g}$ as:

$$S_{9g} = S_{9g} / g_m^2$$

where $S_{9g}$ is g-factor noise and $g_m = \partial g / \partial V_b$ stands for a trap occupancy derivative (slope) (Fig. 3B).

Fig. 5. S/N ratio estimated for different conditions. The dashed blue line represent an aid for the eye.

As a last step, we estimate an S/N ratio for nanosensors whose working principle is based on single-trap phenomena as follows:

$$S/N = \frac{5.9 \text{mV}}{\sqrt{S_{9g}}}$$

where $S_{9g}$ is an input-referred trap occupancy factor noise at a given frequency and 5.9 mV is selected as a signal which corresponds to a threshold voltage shift of the liquid-gated nanosensor caused by 0.1 pH change in the gating solution. The S/N ratio calculated for different time windows T at $S_{9g}$ taken at 10 Hz is shown in Fig. 5. As can be clearly seen, the S/N ratio strongly increases with increasing aggregation time T, showing that under optimized conditions it can indeed be substantially increased, even above the level expected for devices monitoring threshold voltage shift. This suggests that, as long as the RTS amplitude remains well above the other noise sources, the single-trap phenomena approach can be used as a noise filter when sufficiently large averaging windows are used.

III. CONCLUSIONS

In order to demonstrate the full potential of the single-trap approach for biosensing, we generated RTS noise numerically and proposed a method which permits the calculation of trap occupancy factor noise. Under these conditions, the S/N ratio was estimated for nanosensors exploiting RTS noise as a signal. We revealed that the S/N ratio can indeed be substantially increased. The results shed light on relevant parameters required to optimize nanosensors based on single-trap phenomena.

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Nonstationary Low Frequency Noise in
Switched MOSFET Circuits & Circuit Simulation

A. Gokcen Mahmutoglu Alper Demir

Koç University
Istanbul, Turkey
aldemir@ku.edu.tr

Abstract—Modeling and analysis of low frequency noise under strongly time-varying bias conditions is a long-standing open problem in circuit simulation. In this paper, we present the background on the nonstationary low frequency noise modeling problem in circuit simulation, the legacy noise models, and our solution based on a recently proposed computational modeling and analysis framework. Our model and simulation techniques were developed based on an analogy that relates low frequency noise in transistors and electronic circuits to stochastic behavior of ion channels in biological neurons and stochastic chemical kinetics. Results on electronic circuit examples, namely switched MOSFET circuits and oscillators, show that our computational models implemented in an electronic circuit simulator correctly predict the impact of nonstationary low frequency noise that match experimental measurement data reported in the literature, whereas the legacy noise models produce erroneous results.

I. INTRODUCTION

Noise in electronic circuits sets the limit on the minimum detectable signal and adversely affects performance. Intrinsic noise in electronic devices, such as thermal, shot and 1/f noise, is usually characterized with a spectrum. Thermal and shot noise have a white spectrum, whereas there are noise sources where the power is concentrated at low frequencies. For such colored noise sources, samples in time are correlated, whereas for white noise there is no correlation. Low frequency noise has an impact on circuit performance even if operating at high frequencies, due to frequency translation, i.e., up-conversion, via circuits such as oscillators and mixers. Low frequency noise phenomena are not unique to electronic circuits, in fact, ubiquitous [1].

Noise modeling involves the representation of relevant physical properties of noise with mathematical and computational models, e.g., power spectral density (PSD). In noise simulation, one tries to mimic the behavior of a noisy circuit on a computer using random number generators, e.g. via transient, time-domain stochastic simulation. Noise analysis [2], on the other hand, is the semi-analytical assessment of the impact of noise on a circuit through computational models and numerical analysis, in a non Monte Carlo manner, without the use of random number generators, e.g., as in SPICE AC noise analysis [3], [4]. Noise simulation and analysis are crucial tools for analog, RF and mixed-signal circuits, in designing them to be robust against noise effects and in meeting stringent noise specifications with few design iterations.

In conventional noise models and AC noise analysis, it is assumed that the circuit operates in small-signal mode (no significant change in bias point), and small-signal equivalent device models are used. This essentially means that stationary noise statistics is assumed, and Linearized Time Invariant (LTI) models are used [2]. Noise sources are modeled with PSDs in the frequency domain. Noise analysis then entails computing transfer functions from the noise sources to the output, and the output noise spectrum [3], [4].

Noise modeling and analysis for circuits under time-varying bias conditions, such as mixers and oscillators, requires a more elaborate approach. Time-varying bias implies time-varying, i.e., nonstationary, statistics for the noise sources. The nonstationary nature of thermal and shot noise can be captured by modulating a white and stationary noise source [2], believed to be correct and accurate. This is due to the fact that a white noise generating mechanism has no internal dynamics with memory (un correlated samples), and hence instantaneous modulation is appropriate. On the other hand, it is not quite obvious how one would model low frequency, colored noise, such as 1/f noise, with time-varying bias conditions. The legacy model used in this case is based on, again, modulating a stationary colored noise source modeled with a PSD [2]. However, this type of model was shown to be incorrect and inaccurate, overestimating the effect of low frequency noise on circuits [5]–[7]. This is partially due to the fact that the internal dynamics of low frequency, colored noise sources are not captured correctly with an LTI model. A proper model has to take into account the fact that low frequency noise exhibits correlation in time, therefore internal dynamics (not LTI) with memory.

Considerable amount of work was done and published in the devices and circuits literature, on the modeling and characterization of nonstationary low frequency noise, e.g., [5]–[12] and many others that we cannot list here. We have greatly benefited from this body of literature in order to develop a circuit simulator compatible nonstationary low frequency
noise model, that can be used in conjunction with advanced noise analysis algorithms which can handle circuits with time-varying bias conditions.

II. MODELING OF NONSTATIONARY LOW FREQUENCY NOISE FOR CIRCUIT SIMULATORS

The computational modeling and simulation framework we review in this paper [13]–[19] is founded on techniques from stochastic chemical kinetics and computational neuroscience [20], [21]. The model we have developed captures the internal random dynamics of noise sources with mathematical models, that are nonstationary, nonlinear, coupled with the device and circuit models, and compatible with circuit simulators. Based on this model, we have developed carefully crafted, stochastically correct Monte Carlo simulation algorithms [17], which are needed for the verification of the noise analysis algorithms, and that can be used for simulating transient noise phenomena. We have also implemented the models into non Monte Carlo noise analysis techniques [19].

Charge carrier traps in the gate oxides of MOSFETs are a source of low frequency noise [10], [22]. These traps randomly capture and emit charge carriers, with an impact on the current flow through the device. On the other hand, rates of the capture and emission events depend on the voltages and currents of the device. The noise source associated with such a trap can be modeled with a random telegraph signal (RTS), hence referred to as RTS noise [10], [22]. The noise associated with a single, stationary trap (with time-invariant capture and emission rates) can be modeled in the frequency domain with a Lorentzian PSD. It can be shown that independent multiple traps with a certain distribution of capture/emission rates can collectively generate power law noise, such as $1/f$ noise over a frequency range [7]. However, the PSD description for trap noise is appropriate only when the traps are stationary with constant capture/emission rates.

We model nonstationary low frequency noise through “traps”, that are either actual or abstract. A notional trap can be thought of as a pseudonym for an elemental noise generation mechanism with time-constant(s) that depend on the device bias point. We capture the nonstationary trap dynamics with a Markov Chain (MC) based model, that is discrete and fine-grained, with state transition rates as functions of device voltages/currents [13], [14], [16]. Then, starting with the MC model, we derive a continuous model in the form of Stochastic Differential Equations (SDEs). The SDE based trap model is approximate and coarse-grained. In the fine-grained MC model, the trap is either full or empty, modeled with a 0-1 variable. On the other hand, in the coarse-grained SDE model, trap state can assume fractional values. At first thought, the link between these two models may be puzzling. However, one can reach a resolution by noting that the correspondence between them is not on a sample-path basis, but rather in a second-order statistical sense. That is, the models concur with each other for means, variances, correlation functions and spectral densities, but not for individual sample paths. We have verified with practical circuit examples that the two models indeed agree with each other for these second-order stochastic characterizations [17]. In almost all circuit design scenarios, such a second-order stochastic characterization is adequate. However, in the case of very sensitive circuits where individual trap events that involve single charge carriers are of significance, one has to use the fine-grained MC based model.

The fine-grained model can be used in only transient, stochastic noise simulations, whereas the coarse-grained model can be used in non Monte Carlo noise analyses as well [17], [19]. With the discrete, fine-grained model, MC based traps are coupled with the circuit. If simulated with stochastically correct algorithms, this model yields accurate statistics. We have expended quite a bit of effort in developing numerical techniques for hybrid discrete-continuous simulations that do not result in any subtle, statistical bias [13], [17]. However, the MC model is computationally expensive due to the need to handle jump events in an otherwise continuous stochastic simulation. Hence, it can be used only for small circuits. Moreover, it is not easily integrated into existing simulation frameworks and circuit simulators. In deriving the coarse-grained SDE model from the fine-grained MC model, we use techniques that were developed in stochastic chemical kinetics [20], [21]. This derivation involves essentially the approximation of (differences of) Poisson random variables with corresponding Gaussian random variables, but has other nontrivial steps [14], [17]. The coarse-grained SDE model can also serve as the basis for stochastic transient simulations. However, there are several complications one has to deal with. We have developed a technique we call smooth diffusion for running robust transient noise simulations based on the SDE model [17].

The SDE based model is compatible, by construction, with circuit simulators. We can easily embed trap dynamics into circuit equations by augmenting modified nodal analysis (MNA) equations with the trap SDEs [19]. The circuit variables (e.g., node voltages, branch currents) are also augmented with the trap variables. These trap variables can be considered as additional, internal “pseudo” nodes in the device. There is of course an overhead associated with the introduction of these extra variables in a circuit simulator. However, the interactions of the trap variables with the other circuit variables are local, confined to within a device. The device model equations need to be augmented with equations that model the two-way coupling between the traps and the charges, currents and voltages in the device. On one hand, the trap event rates are a function of circuit variables. That is why the traps exhibit nonstationary behavior with time-varying bias conditions. On the other hand, the trap events have an impact on the state of the device, causing noise in voltages and currents. We have augmented a MOSFET model [23] with these extensions [15].
The trap enhanced MOSFET model can be used in conjunction with previously developed noise analysis techniques, such as cyclo-stationary noise analysis [24], time-domain non Monte Carlo noise analysis [2], phase noise analysis for oscillators [25]. We have implemented all of these techniques with a trap enhanced MOSFET model [19] in a MATLAB® [26] based circuit simulator called CIRSIUM [15].

In the case when the trap variables represent actual, physical charge carrier traps in a transistor, the physics-based or empirical compact model equations need to be augmented with equations that model the behaviors and interactions of the charge carrier traps with other dynamics in the device. This is a challenging task and requires extensions and enhancements in compact model development. We have addressed this issue only in a rudimentary manner [15]. The difficulty is compounded by the fact that low frequency noise arising from fabrication defects and imperfections has a statistical nature [14]. That is, the trap configuration and placement in a certain device is randomly determined at fabrication, and in a different manner in every device. Thus, the stochasticity of low frequency noise is both temporal and spatial. The modeling and analysis framework reviewed in this paper does not address the spatial statistical aspect of the problem.

III. Results

To summarize, the low frequency modeling, simulation and analysis framework we have developed features:

- Transient, stochastic simulation based on the fine-grained, discrete, MC based trap model, that can capture the effect of individual charge carrier events.
- Transient, stochastic simulation based on the coarse-grained, continuous-state, SDE based trap model.
- Non Monte Carlo noise analysis, both in time and frequency domain, based on the SDE based trap model augmented with circuit equations.

We have extensively tested the techniques summarized above on a variety of circuits in a number of scenarios, comparing them against each other, as well as with standard AC noise analysis for time-invariant bias, and with the legacy, simplistic low frequency noise models for time-varying bias, e.g., for switching MOSFET circuits [13]–[19]. The test circuits and scenarios are as follows:

- MOSFET transistor in common source configuration, with both constant and switching input, with a single trap and also with multiple traps that produce 1/f noise.
- Ring-oscillator (where the transistors experience large time-varying bias) phase noise and timing jitter characterization, both in time and frequency domain.
- Digital memory circuits.
- RF mixer circuit.
- Sawtooth oscillator circuit [8], [9].

We have arrived at the following conclusions:

- The SDE based continuous-state model agrees with the MC based fine-grained model in the sense of second-order stochastic characterizations, i.e., correlation functions and spectral densities [17]. Except for rare scenarios where individual charge carrier events are of significance, the SDE based model is adequate for modeling nonstationary low frequency noise.
- While the use of the SDE based model in non Monte Carlo, analysis-based noise characterizations involves further approximations (in addition to the discrete-to-continuous conversion in moving from the MC model to the SDE model), results obtained with non Monte Carlo schemes match the ones obtained from (computationally expensive) transient stochastic simulations with the SDE based model [19]. Except for rare scenarios where noise magnitude is large enough to invalidate the approximations the non Monte Carlo schemes rely on, efficient near-linear-complexity analysis schemes for cyclo-stationary noise [24], phase noise [25], time-domain noise [2] may be used in conjunction with the SDE based low frequency noise model. However, there is an overhead involved, since one needs to introduce the trap states as additional circuit variables (as pseudo-nodes) into the device model.
- The legacy nonstationary low frequency model, based on the simple modulation of a stationary noise source, overestimates the noise impact in switching MOSFET circuits. On the other hand, the results obtained with the proposed model are in accordance with the measurement results reported in the literature [19].

IV. Conclusions

We believe that the computational modeling and analysis framework outlined and reviewed in this paper solves an open problem in circuit simulation, namely modeling and analysis of low frequency noise under strongly time-varying bias conditions. This modeling framework was built on, and subsumes, the immensely useful work that has been done in the devices and circuits communities over many years. The circuit simulator compatible low frequency noise model, and the analysis and simulation techniques were developed by exploiting an analogy that relates low frequency noise in transistors and electronic circuits to stochastic behavior of ion channels in biological neurons and stochastic chemical kinetics. The adaptation of the stochastic chemical kinetics formalism made it possible to develop a systematic methodology for both fine-grained and coarse-grained modeling of low frequency noise. The techniques developed were applied to practical circuits, and were shown to predict intricate, nonstationary noise effects in switching MOSFET circuits that match measurement data reported in the literature. A complete exposition of the work reviewed in this paper can be found in [13]–[19].

Furthermore, by building on the work reviewed in this
paper, we have developed a noise simulator for biological neurons and neuronal circuits [27], [28], which may be used in understanding the noise control and exploitation mechanisms in neuronal circuits [29], and in designing the neuro-morphic and hybrid neuro-electronic circuits of the future.

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Abstract—Negative photoconductivity and reduction of noise under illumination are found in HgTe/CdHgTe quantum wells with electrons dominated conductivity. These effects are explained by electron-holes' pairs generation outside the quantum well. Electrons and holes are separated by the built in electric field. Holes, which get inside the quantum well, recombine with electrons there and reduce the conductivity. A fraction of holes is captured by those traps which cause the noise. Change of occupancy function of these traps provides the noise reduction.

Keywords—HgTe, noise, negative photoconductivity, noise reduction.

I. INTRODUCTION

The band gap in the meV range and high electron mobility make HgTe/CdHgTe quantum wells (QWs) structures promising for the terahertz applications. Detection by different mechanisms and emission in the terahertz frequency range were already demonstrated in several publications [1-11]. The low-frequency noise presents in all kinds of electronic materials and devices [12-25]. The knowledge of noise properties of HgTe QWs is important for the understanding of the current flow mechanisms and judgement about possible applications.

II. RESULTS AND DISCUSSIONS

The HgTe/CdHgTe QW structures were grown by molecular beam epitaxy (MBE) on GaAs substrate (013) with the thickness of $d = 7 - 7.5 \text{ nm}$, corresponding to the inverted band structure at 4 K and normal band ordering at 300 K [26]. Figure 1 shows the schematic view of the quantum well structures. Composition profile of the studied structures is shown in Fig.2. Hall measurements at 4K indicated that all structures had electrons dominated conductivity with the mobility and concentration $\mu = (4 - 70) \cdot 10^3 \text{ cm}^2/\text{Vs}$ and $n = (2 - 10) \cdot 10^{11} \text{ cm}^{-2}$, respectively.

At 300 K, all structures demonstrated negative photoconductivity under ambient visible light illumination, i.e. the resistance increased under illumination. In order to be sure that the contacts are not involved in the effect, the resistance was measured in 4-probe configuration. The characteristic times of the transient processes with light on and off were in the range of hundreds of seconds (Fig. 3). As seen from Fig. 3, the fast change in the resistance is followed by a long non exponential tail. Existence of the long relaxation times is a sign of the deep traps with small capture cross section. This trap also should contribute to the low frequency noise. This kind of negative “persistent” photoconductivity is known for other types of heterostructures [27-29] but to the best of our knowledge it has not been yet reported for HgTe QWs.
Several mechanisms were discussed for negative photoconductivity. In quantum well structures the most realistic one is the separation of electrons and holes generated in the barrier layer (Cd$_{0.7}$Hg$_{0.3}$Te in our case) by built in electric field. Electrons go to the surface and are trapped by the deep states. Holes go inside the quantum well where they recombine with electron reducing their concentration and, therefore the conductivity.

Figure 4 shows the spectral dependence of the negative photoconductivity. Although the amplitude of the resistance change under illumination increases slightly with wavelength decrease, the dependence is weak owing to small thicknesses of the layers in the nanometer range (Fig.2).

The low frequency noise was measured at room temperature in the dark and under illumination. Figure 5 shows the dependence of noise on current for several samples. As seen the spectral noise density of the current fluctuation is proportional to the current squared. This confirms that the noise is due to the resistance fluctuations and the current does not affect these fluctuations.

Figure 6 shows the noise spectra at low voltage $V = 10$ mV. As seen, the spectra in the dark has the form of the 1/f noise. Illumination leads to the change of the noise spectra shape and to overall reduction of the noise at frequencies $1 \text{ Hz} < f < 100 \text{ Hz}$. 

Fig. 2. Composition profile of the studied structures. Quantum well width $d = 7$ nm.

Fig. 3. Transient processes in HgTe/CdHgTe quantum well as a result of illumination.

Fig. 4. Transient processes of the resistance change under illumination (a) and spectral dependence of the negative photoconductivity (b).
In order to characterize the amplitude of noise, the effective trap density responsible for noise can be calculated based on the McWhorter model [30,31]:

$$\frac{S_i}{f^2} = \frac{kTN_i}{\gamma fWNL_i^2},$$

where $k$ is the Boltzmann constant, $T$ is the temperature, $N_i$ is the effective trap density, $f$ is the frequency, $WL$ is the channel area, $n_i$ is the concentration and $\gamma$ is the attenuation coefficient of the electron wave function under the barrier:

$$\gamma = \frac{4\pi\sqrt{2m^* \Phi}}{h}.$$

Here $m^*$ is the electron effective mass, $h$ is Planck’s constant, and $\Phi$ is the tunneling barrier height seen by the carriers at the interface. In Si MOSFET’s the $\gamma$ value is usually taken to be $\gamma=10^6 \text{cm}^{-1}$. In the studied system the effective mass of electrons and tunneling barrier height are both about one order of magnitude smaller than in Si MOSFET. Therefore, the estimate for the attenuation coefficient in the studied system yields $\gamma\approx10^2\text{cm}^{-1}$. With known amplitude of noise and electron concentration from the Hall measurements trap density estimated based on McWhorter model for different samples is $N_i = 10^{18} - 10^{20} \text{cm}^{-3}\text{eV}^{-1}$.

Reduction of noise under illumination in semiconductors is usually explained by the model which includes the capture of the minority carriers by the traps responsible for the carriers’ number fluctuations [32-40]. As a result, the occupancy of these levels changes. It is well known that the generation-recombination (g-r) noise amplitude is proportional to $F (1-F)$, where $F$ is the noisy level occupancy function. It is clear that depending on $F$ value in the dark, illumination can either increase or decrease the noise. The strong enough illumination always leads to the noise reduction [32-40]. If we assume that the $1/f$ noise is a superposition of several g-r processes, this scenario explains the noise reduction and change of the noise spectra. It is important to note that in HgTe QWs, contrary to the previous studies, the noise reduction under illumination is accompanied by the negative photocconductivity.

CONCLUSIONS

In conclusion, the amplitude of the noise in HgTe quantum wells is within the range found for Si MOSFETs. Similar noise level is reported for other low dimensional structures, like graphene and MoS$_2$. Noise reduction under illumination and negative photocconductivity is explained by electrons and holes generation in the Cd$_{0.7}$Hg$_{0.3}$Te layer followed by holes’ recombination in quantum wells. These results are in favor of the number of carriers fluctuations mechanism of noise.

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Micromagnetic Modeling of Telegraphic Mode Jumping in Microwave Spin Torque Oscillators

B. Gunnar Malm  
School of EECS  
KTH Royal Institute of Technology  
Kista, Sweden  
gunta@kth.se

Anders Eklund  
School of EECS  
KTH Royal Institute of Technology  
Kista, Sweden  
ajeklund@kth.se

Mykola Dvornik  
Department of Physics  
Gothenburg University  
Gothenburg, Sweden  
mykola.dvornik@physics.gu.se

Abstract—The time domain stability of microwave spin torque oscillators (STOs) has been investigated by systematic micromagnetic simulations. A model based on internal spin wave reflection at grain boundaries with reduced exchange coupling was implemented and used to study the oscillator under quasi-stable operating conditions. Telegraphic mode jumping between two operating frequencies (23.3 and 24.1 GHz) was observed in the time domain with characteristic dwell times in the range of 10-100 ns. The oscillating volume was shown to have a different shape at the distinct operating frequencies. The shape difference is governed by spin wave reflections at the grain boundaries. The resulting non-linear behavior of the oscillator was shown to be a collective effect of spin wave scattering at different locations within a few spin wavelengths from the nano-contact.

Keywords—telegraphic mode jumping, micromagnetic simulations, microwave oscillators, spin transfer torque

I. INTRODUCTION

Spintronic devices are interesting alternatives for novel memory and logic functionality, and can also be used for microwave generation and detection. These device are based on a stack of thin magnetic layers and have a very small footprint, e.g. compared to advanced CMOS devices and hence they can be integrated into the back-end-of-line processing. In spin torque oscillators (STOs) for microwave application the frequency is tuned by a DC current flowing into a nano-contact (NC) with diameter less than 100 nm. The current-frequency relation, or slope $df/dI$, typically shows prominent low-frequency 1/f noise [1]. In our previous work [2] a model based on internal spin wave reflection, due to reduced exchange coupling at grain boundaries [3, 4], was proposed and could successfully reproduce a wide body of experimental observations. In this work, for the first time, we use systematic simulations in the GPU accelerated micromagnetic software MuMax [5, 6] to study the time domain stability of the different frequency modes. The simulations included a stochastic thermal field ($T = 300$ K) that acts on the individual micromagnetic spins. A characteristic telegraphic switching was observed for DC currents that correspond to abrupt frequency jumps.

II. MICROMAGNETIC SIMULATION APPROACH

Micromagnetic simulations are based on a finite-difference time domain numerical solution of the well-known LLGS equation including a spin transfer torque (STT) term. The equation is discretized onto a mesh where each node or cell of the mesh represents a single micro spin quantity. For typical simulation domain sizes GPU accelerated codes provide significant simulation time reduction over CPU based codes. In our study, since we are considering propagation and reflection of spin waves in an extended magnetic film that is significantly larger than the actual oscillating volume located underneath the nano-contact (100 nm NC diameter vs. 640 nm square), see Fig. 1.

Fig. 1. Grain structures realized by a tessellation algorithm, average grain size 30 nm. The nominal placement and size of the nanocontact (NC) are indicated by the circle.

The simulation domain was divided into 256 by 256 cells in the x-y plane and 5 layers in the $z$-direction to represent the different layers of the stack. The layers correspond to our experimental devices with 8 nm Co (fixed layer), 8 nm Cu non-magnetic spacer, NiFe 4.5 nm (free layer). In this study we used a fixed NC size of 100 nm, an applied external field of 10 kOe, out-of-plane angle 70° and currents in the range 20.5 mA to 32.5 mA in order to excite the propagating mode. For these particular conditions the propagating mode becomes stable around 18 mA. As shown in Fig. 1 different grain structures can be implemented by providing a random seed to a tessellation algorithm. In addition to this the average exchange coupling at each individual grain boundary can also be randomized. For actual device processing the placement of the nano-contact with respect to the grain boundaries is arbitrary. In order to replicate this situation, in an eScience approach, we performed a set of simulations where the seed for grain generation was fixed while the nano-contact itself was displaced a few unit cells corresponding to either 5 or 12.5 nm in both x- and y-directions. The chosen displacement is smaller than the average grain size, 30 nm, and also smaller than the estimated spin wavelength (70-100 nm).
All simulations were performed for a duration of at least 100 ns and the solution, magnetization state $m = (m_x, m_y, m_z)$ as well as the total energy density, $E_{\text{tot}} \propto m \cdot B_{\text{eff}}$, was stored for each increment of 5 ps. The internal time step, used by the adaptive solver, was observed to be in the range of $0.5 - 1 \times 10^{-13}$ s. Running simulations with a fixed internal time step of less than $1 \times 10^{-13}$ s yielded virtually identical results. Still the adaptive time stepping allows a reduction of total simulation time for cases with a finite temperature, random thermal field, turned on. The 100 ns time traces give a frequency resolution of 10 MHz. In order to get higher frequency resolution (1 MHz) and in particular to study long-term time domain stability we also performed many simulations of 1 μs duration. These finished in about 48 h on high performance GPUs (GTX-1080i, Tesla K80 or similar).

### III. RESULTS AND DISCUSSION

As described in Sec. II we systematically varied the NC placement and the results of this set of simulations are summarized in Fig. 2. The power spectral density (PSD) of the $y$-component of the magnetization $m_y$ was obtained using the signal processing capability of MATLAB® and the frequency at peak power was extracted vs. DC current. Most of the placements result in a varying degree of continuous non-linear behavior while there is one case with a large mode jump (~1 GHz) and several with smaller jumps of a few 100 MHz. A reference case with full exchange coupling at all grain boundaries, replicating an ideal homogeneous material, indicates that the slope $df/dI$ is inherently constant for these oscillators. Such behavior is seldom observed in actual experimental devices, at least not for such a wide current sweep. In the following analysis we will focus on the time domain behavior of the mode transition at 27.75 mA for the (5, 5) case.

![Fig. 2. Oscillation frequency at peak power for +/- 5 unit cell (+/- 12.5 nm) displacements of the nano-contact, a reference case with no grains is included.](image1)

In Fig. 3 a) we illustrate the full PSD, both power and frequency, for the nominal case that shows a non-linearity and b) the RTN case that has both a non-linearity at lower current around 26 mA and clear jump at 27.75 mA. In the nominal case the spectrum clearly shows a broader linewidth in the regions with steeper slope $df/dI$ around 28 mA. The time domain data still shows a relatively slow frequency instability that does not exhibit any preferred low and high energy states that would yield a telegraphic behavior.

![Fig. 3. Simulated output power spectral density (dB/Hz) vs. driving DC current. a) nominal placement of NC resulting in continuous but non-linear behavior b) 12.5 nm offset in x- and y-direction showing two active frequency modes at 27.75 mA.](image2)

By plotting the total oscillator energy density vs. simulation time, shown in Fig. 5, it was found that the two frequency modes are excited in a telegraphic manner and hence do not co-exist in time. A moving average window of 0.5 ns was used to filter out faster energy fluctuations. Typical dwell times in the different modes are in the range of ten to hundred ns. For example the top panel of Fig. 5 shows two initial segments of 50 ns at the low and high energy states of...
the oscillator respectively. The longest observed stable segments are more than 200 ns in duration and are found at both energy levels. In our previous experimental study, we mainly focused on higher current where dwell times in the range tens of nanosecond and as high as milliseconds could be observed [7].

The telegraphic behavior was relatively robust, changing the applied field in the range 9.75 – 11.0 kOe, as well as using a lower temperature of 200 K still yielded two separated frequencies. In Fig. 6 the energy states are illustrated for three different applied fields, the current corresponding to the location of the frequency jump varies slightly. For the 9.75 kOe case the higher energy dominates, for the 10.25 kOe the lower energy dominates, while the 10.50 kOe case exhibits a clear lower energy and an indication of multiple possible higher energy states, with relatively short dwell times.

Any physical system that presents a telegraphic behavior must have some inherent energy barrier between the two states. To analyze this we performed a FFT over the micromagnetic data ($m_i$) in each mesh node. Similar results are obtained by analyzing the other magnetization components or the total energy. For example, the time trace with the first thermal seed showed that the oscillator operated at 23.3 GHz for the first 50 ns and at 24.1 GHz for the next 50 ns. The FFTs showed that the shape of the oscillating volume was distinctly different at these two frequencies, see Fig. 7 a) and b). The resulting shape of the oscillating volume is influenced by the interference of spin waves that are scattered or reflected at multiple grain boundaries, in particular at so called grain boundary triple junctions (corners). The oscillating volumes do not have a smooth spatial extent and there are regions (locations) indicated by the arrows where the oscillation is showing a standing wave pattern, to the left of the NC in Fig. 7 a), or damped out, towards to lower left corner in Fig. 7 b).

Fig. 6. Energy density (moving averages over 0.5 ns) for different applied fields and currents corresponding to the mode transitions between low and high oscillation frequencies.
In an effort to isolate the dominating reflections in our particular grain tessellation configuration, we identified boundaries or junctions with significantly reduced exchange coupling in close vicinity of the NC area, see Fig. 8. Simulations were done for cases, where the exchange was restored to 100% at the grain boundaries given by the points indicated by letters A-F.

The results is Fig. 9 indicate that the non-linear behavior is governed by the collective effect of spin wave scattering at points that are in within several wavelengths of the NC including points inside the NC or at its perimeter. While progressively restoring the exchange in the grain landscape non-linear effects are still visible, albeit becoming increasingly continuous and shifted towards higher drive currents. The strong reflection at point “E” seems to be mainly responsible for the occurrence of the RTN behavior, since by setting the exchange coupling to 100% at this point the device goes to a continuous non-linear behavior, similar to the nominal case.

IV. CONCLUSIONS

The time domain stability of microwave spin torque oscillators has been investigated by systematic micromagnetic simulations including a randomized grain configuration with reduced exchange coupling at grain boundaries and a stochastic thermal field. Telegraphic mode jumping between frequencies was observed in the time domain with characteristic dwell times in the range of 10-100 ns. The frequency separation between quasi-stable energy states of the oscillator was as large as 1 GHz. The oscillating volume was shown to have a different shape governed by spin wave reflections at grain boundaries with reduced exchange coupling. The resulting non-linear behavior and telegraphic frequency mode jumping of the oscillator were shown to be a collective effect of spin wave scattering at different locations within a few spin wavelengths from the nano-contact.

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Measurement of the cyclostationary third moment in the noise of a tunnel junction

Pierre Février
Université de Sherbrooke
Dépt. de Physique, Institut Quantique
Sherbrooke, QC, Canada
pierre.fevrier@usherbrooke.ca

Christian Lupien
Université de Sherbrooke
Dépt. de Physique, Institut Quantique
Sherbrooke, QC, Canada
christian.lupien@usherbrooke.ca

Bertrand Reulet
Université de Sherbrooke
Dépt. de Physique, Institut Quantique
Sherbrooke, QC, Canada
bertrand.reulet@usherbrooke.ca

Abstract—We measure the cyclostationary third moment in the photo-assisted noise of a tunnel junction. We use an homodyne detection where the junction bias is modulated at 14.55GHz, and the noise is demodulated at 4.85GHz. We identify contributions due to the presence of the measurement setup.

Index Terms—Cyclostationary noise, non-Gaussian noise, third moment, tunnel junction

In electronics, the term ”current noise” often refers to the variance of current fluctuations, defining an effective temperature for a system. However, since fluctuations are in general not Gaussian, a lot of information can be extracted from the measurement of higher order moments. For example, in mesoscopic conductors, the existence of a third moment stems from the binomial statistics of charge transfer [1], and correlations between current and emitted photons [2]. It can also probe internal timescales [3] or electronic interactions [4]. In this work we present a new tool to study high frequency fluctuations in conductors. We measure the cyclostationary third moment of voltage fluctuations, i.e. the dynamical response of the third moment to an external drive.

In stationary regime, the third moment of current fluctuations is given by the correlator:

\[ M_3(\tau, \tau') = \langle i(t) i(t-\tau) i(t-\tau') \rangle \]  

(1)

which is invariant by translation in time \( t \), and depends on the two delays \( \tau \) and \( \tau' \). The brackets designate an ensemble or time average. With the Fourier transform of Eq.(1), we can show that, for a given bandwidth \( |f - f|, f + \Delta f| \), the measurement of \( M_3 \) requires a minimum width of \( \Delta f > f/3 \). Smaller bandwidth leads to \( M_3 = 0 \). This stringent condition on bandwidth makes the measurement of the third moment difficult at high frequency. However, in a cyclostationary regime, i.e. when the third moment is periodically modulated at \( nf_0 \) with \( n \in \mathbb{Z} \), we can measure the cyclostationary third moment defined by the bispectrum:

\[ K_n(f, f') = \langle i(f) i(f') i(-f + f' + n f_0) \rangle \]

(2)

This correlator can be non-zero when integrated over a small bandwidth \( 2\Delta f \ll f \) by choosing \( f_0 = f \) and \( n = \pm 1 \) or \( n = \pm 3 \):

\[ K_{\pm 1}(f) = \langle i(\pm f)^2 i(f) \rangle, \quad K_{\pm 3}(f) = \langle i(\pm f)^3 \rangle \]

(3)

These two correlators are measured with the static third order moment \( K_3 \) of the signal after a phase sensitive demodulation at frequency \( f \) of current fluctuations. In this work, we demonstrate the measurement of \( K_3 \), i.e. the measurement of the third moment of current fluctuations, while the system (here a tunnel junction) is driven at 3f. This communication is organized as follows: we first describe our experimental setup, and the method used to extract the non-Gaussian contribution to the noise attributed to the tunnel junction. In a second part, we show how the presence of the measuring circuit modifies the statistics of current fluctuations in the junction.

I. MEASUREMENT OF THE CYCLOSTATIONARY THIRD MOMENT

A. Non-Gaussian noise generation using a tunnel junction

To generate the cyclostationary non-Gaussian-noise, we used a tunnel junction with metallic electrodes. The junction have been made using usual lithography technique and evaporation of aluminum electrodes on a silicon substrate. The insulating tunnel barrier is obtained by controlled oxidation of the first electrode under oxygen atmosphere. The dc resistance of the junction is \( R = 130 \Omega \) at 3.7K, with a RC cutoff frequency estimated around 8GHz. Because electronic transport in a tunnel junction is a Poisson process, all moments of current fluctuations are linear with the average current: \( \langle (I - \langle I \rangle)^2 \rangle = e \langle I \rangle \) and \( \langle (I - \langle I \rangle)^3 \rangle = e^2 \langle I \rangle \). By modulating the bias current of the junction at 3f with an amplitude \( I_{ac} \), we expect to measure a linear modulation of the third moment: \( K_3 \propto e^2 I_{ac} \). The junction is current biased through a bias-Tee (see Fig. 1), allowing the separation of high frequency signal from DC bias.

B. Homodyne detection

The experiment has been performed in an helium-free cryostat with a base temperature of 3.7K. The microwave signal from the junction propagates through an isolator before being amplified by a low-noise cryogenic amplifier with a bandwidth of 4-8GHz and a nominal noise temperature of 2.5K. The role of the isolator is to make the temperature and the impedance of the environment seen by the junction well defined. A room-temperature amplifier provides extra-amplification, before demodulation. In order to measure \( K_3 \),
we used an homodyne detection where the noise is demodulated at \( f = 4.85 \text{GHz} \). The junction is photoassisted (i.e., ac driven) by a single tone at \( 3f = 14.55 \text{GHz} \) using a directional coupler. Driving at \( 3f \) instead of \( f \) avoids dealing with the excitation reflected back in the amplification chain.

We use an IQ mixer with high linearity to separate the signal into two quadratures \( X \) and \( P \). Each quadrature is amplified and low-pass filtered. The final measurement bandwidth \( \Delta f \) is \( \pm 225\text{MHz} \). The low frequency signal is digitized with a 2-channel fast acquisition card (14bits, 400MS/s), to compute \( X \) and \( P \) and low-pass filtered. The final measurement bandwidth we used an homodyne detection where the noise is demodulated at \( f = 4.85 \text{GHz} \). The junction is photoassisted (i.e., ac driven) by a single tone at \( 3f = 14.55 \text{GHz} \) using a directional coupler. Driving at \( 3f \) instead of \( f \) avoids dealing with the excitation reflected back in the amplification chain.

The linear dependence at high bias is characteristic of shot-noise. The rounding appearing at low bias is a direct consequence of the noise modulation by the ac drive and can be used to extract \( I_{dc} \). At \( I_{dc} = 0 \), the variance is proportional to the voltage dependent current noise spectral density \( S_i(V) \) in \( \lambda^2\text{Hz}^{-1} \) in addition to a constant contribution from the amplifier (dash-dotted line). The rounding is the cross-over with junction’s thermal noise at 3.7K.

We use an IQ mixer with high linearity to separate the signal into two quadratures \( X \) and \( P \). Each quadrature is amplified and low-pass filtered. The final measurement bandwidth \( \Delta f \) is \( \pm 225\text{MHz} \). The low frequency signal is digitized with a 2-channel fast acquisition card (14bits, 400MS/s), to compute the probability density \( P(X, P) \).

After demodulation the skewness on each quadrature \( \langle X^3 \rangle \) and \( \langle P^3 \rangle \) is given by the real and imaginary part of:

\[
\langle X^3 \rangle + i\langle P^3 \rangle = \iint df_1df_2G(f_1)G(f_2)G(f_3)\times
\left[ \frac{1}{4}K_{e,3}(f_1, f_2)e^{i\phi_0} + \frac{3}{4}K_{e,1}(f_1, f_2)e^{i\phi_0} \right]
\]

\[
f_3 = 3f - f_1 - f_2
\]

where \( K_{e,3} \) and \( K_{e,1} \) are the third order cyclic moments of voltage fluctuations at the input of the amplifier. \( G \) is the voltage gain of the amplification chain accounting for the finite bandwidth and losses. \( \phi_0 \) is a global phase due to the delay between excitation and detection. For frequency independent gain, in a narrow bandwidth \( |f - \Delta f, f + \Delta f| \) with \( \Delta f \ll f \) Eq.(4) simply gives:

\[
\langle X^3 \rangle + i\langle P^3 \rangle = \frac{1}{4}G^3 \left[ K_{e,3}(f) e^{i\phi_0} + 3K_{e,1}(f) e^{i\phi_0} \right] \frac{3}{4} \Delta f^2
\]

The 3/4 correction to the bandwidth comes from the convolution appearing in Eq.(4) due to the condition Eq.(5). The amplitude of the ac excitation \( I_{ac} \), the gain and the noise of the amplification chain, are determined by measuring the variance of the photo-assisted noise \( \langle X^3 \rangle \) and \( \langle P^3 \rangle \) [5] (see Fig. 2).

We find that the added noise temperature is around 3K which is close to the noise of the cryogenic amplifier. Signal to noise ratio is limited by the junction intrinsic noise at high bias. The estimated total power gain is \( \sim 78 \text{dB} \).

C. Third moment from the symmetry of the histograms

The phase coherence between the detection and excitation can be switched off by a slight detuning of one of the sources. This averages to zero the contributions that depend on \( \phi_0 \). Contributions to \( P(X, P) \) that are independent of the phase of the excitation are removed by measuring the difference in histograms obtained with and without phase coherence giving the differential probability \( \Delta P(X, P) \). These contributions include, non-Gaussian noise added by the non-linearity of the amplification chain, the non-linearity of the acquisition card and the almost Gaussian photoassisted noise showed in Fig. 2.

\( \Delta P(X, P) \) shows clear rotational symmetry (see Fig. 3). This is the direct consequence of the homodyne demodulation of the noise at a fraction of the noise modulation frequency. It is then natural to use polar coordinates: \( X = r \cos(\theta) \) and \( P = r \sin(\theta) \) and express \( P(r, \theta) \) as the Fourier series:
\[ P(r, \theta) = \sum_n P_n(r)e^{-in\theta} \]  

We define:

\[ W_{\alpha,n} = \frac{\pi}{2} \int_{-\infty}^{\infty} P_n(r)r^{\alpha+1}dr \]  

Moments of the distribution \( P(r, \theta) \) are related to the functions \( W_{\alpha,n} \). Simple algebra shows that third order moments (skewnesses and coskewnesses) are given by the first and third order rotational symmetries:

\[ \langle X^3 \rangle = \text{Re}(W_{3,3} + 3W_{3,1}), \quad \langle XP^2 \rangle = \text{Re}(W_{3,1} - W_{3,3}) \]
\[ \langle P^3 \rangle = \text{Im}(W_{3,3} - 3W_{3,1}), \quad \langle PX^2 \rangle = \text{Im}(W_{3,1} + W_{3,3}) \]  

The measurement of the joint probability \( P(X, P) \) is necessary to separate the two contributions \( W_{3,1} \) and \( W_{3,3} \). In our experiment, it appears that \( W_{3,1} \simeq 0 \) (see Fig. 4). This corresponds to \( K_{\nu,1} = 0 \) as expected for a noise modulation at 3\( f \). We consider in the following \( \langle X^3 \rangle = \text{Re}(W_{3,3}) \) and \( \langle P^3 \rangle = \text{Im}(W_{3,3}) \), as it improves signal to noise ratio. We also choose a phase \( \phi_0 \) that maximizes \( \langle X^3 \rangle \) and minimizes \( \langle P^3 \rangle \) at high bias.

\[ \langle X^3 \rangle \] at high bias shows a plateau with an amplitude proportional to \( I_{ac} \). This is qualitatively close to what is expected for a tunnel junction as discussed before. We however observe a deviation from this intrinsic contribution at low bias, a dip with a width of the order of \( I_{ac} \) on both quadrature. In the following, we show that this deviation can be attributed to contributions induced by the presence of the electromagnetic environment of the setup.

FIG. 5. MODEL FOR THE IMPEDANCE MISMATCH BETWEEN THE JUNCTION AND THE TRANSMISSION LINE

II. EFFECT OF THE MEASURING CIRCUIT

Ideally, currents fluctuations in a conductor are measured using a perfect ammeter with zero input impedance. This is usually not the case for radio-frequency measurements, since transmission lines have a 50\( \Omega \) characteristic impedance. Additionally, the junction itself has a geometric capacitance in parallel with \( R_J \). The wire bond between the junction and the 50\( \Omega \) adapted microstrip launch pad also adds an inductance in series with some delay and dissipation. The circuit between \( R_J \) and the transmission line (TL) can be represented by a reciprocal two port network (see Fig. 5). This circuit shows an input impedance \( Z_{in}(f) \), the impedance seen by the junction, and an output impedance \( Z_{out}(f) \), causing an impedance mismatch with the TL. The voltage transmission coefficients \( t(f) \) and \( t'(f) \) characterize the transmission from the junction to the TL and from the TL to the junction. Because the circuit is reciprocal, we have \( t' = t \frac{R_J}{Z_0} \). The intrinsic contribution to \( K_{\nu,3} \) from the junction is:

\[ K_{\nu,3} = \frac{Z_{eff}(f)^3}{(3^3)^3} c^2 I_{ac} \]  

where \( Z_{eff}(f) = (R_J Z_{in}(f))/(R_J + Z_{in}(f)) \).

Due to the finite impedance \( Z_{in} \) at finite frequency, the voltage drop \( V(t) \) at the junction shows fluctuations \( \delta V(f) \). Indeed, \( V(t) \) is influenced by the current noise \( \delta i(t) \) generated by the sample as well as by an external noise source \( v_{env}(t) \), the noise coming from the measurement setup:

\[ \delta V(f) = -i(f) \frac{R_J Z_{in}(f)}{R_{J} + Z_{in}(f)} + t'(f) v_{env}(f) \]  

Because photo-assisted noise variance depends on bias voltage, this leads to environmental corrections to the statistics of fluctuations. [6] [7] [8]. Similar corrections are expected in the measurement of the cyclostationary third moment [9].

Due to the impedance mismatch between the junction and the transmission line, part of the noise coming from the environment is reflected back to the measurement setup. In our case the environment is the 50\( \Omega \) load on the circulator at 3.7K, and \( \langle v_{env}^2 \rangle = \frac{1}{2} k_B T_{env} Z_0 \). This reflected signal is correlated with the fluctuations of junction noise variance through the fluctuation of bias (Eq.(11)). This adds a contribution to \( K_{\nu,3} \):

\[ K_{\nu,env}(f) = 3\sigma(f)t(f)^2 Z_{eff}(f)v_{env}(f)i(f)^2 \]  

If the modulation \( \delta V \) is small, it has been showed [9] that \( i(f)^2 \) can be rewritten as :
\[ i(f)i(f') = \sum_\alpha S_\alpha (\alpha f_0 - f - f') + D_\alpha \delta V(f + f' - \alpha f_0) \]

(13)

Where the \( S_\alpha \) and \( D_\alpha \) are the complex Fourier coefficients of the noise spectral density \( S(V(t)) \) (see Fig. 2), and its derivative \( dS(V(t))/dV \) averaged over bias fluctuations \( \delta V \).

From Eq.(11) and Eq.(13) we obtain a new expression for Eq.(12):

\[ K_{v,env}(f) = Z_{eff}(f)^3 \langle i(f) \rangle^3 2D_3k_B T_{env} \alpha_r(f) \frac{Z_{in}}{Z_{in} + R_4 e^{i\phi}(f)} \]

(14)

The reflection coefficient \( r \) is given by \( (Z_{out} - Z_0)/(Z_{out} + Z_0) \). We also used the fact that \( t^*(f) = t(-f) = |t| e^{-i\phi} \) where the phase \( \phi \) accounts for the propagation between the junction and the transmission line. Only \( D_3 \), the modulation at \( \alpha f_0 = 3 \) of \( dS_i/dV \) contributes to the environmental effect. \( D_3 \) is finite near zero bias where \( dS_i/dV \) is almost linear in \( I_{dc} \) and zero at high bias where \( dS_i/dV \) is constant, with a characteristic width given by \( I_{ac} \). Because of the phase \( \phi \) and the reflection that can be complex, we expect the contribution of the environment not to be in phase with the intrinsic contribution. To check the validity of Eq.(14), we increased the temperature of the environment by sending a Gaussian noise on the junction via the same coupler used for the ac excitation. Part of this noise is reflected on the junction and collected by the amplification chain. The effective noise temperature seen on the junction via the same coupler used for the ac excitation.

III. CONCLUSION

We report the measurement of the cyclostationary third moment in the photoassisted noise of a tunnel junction. The signal is consistent with the Poissonian statistics of current fluctuations in a tunnel junction. We also observe feedback and environmental contributions due to the presence of the measurement setup. This experiments opens the possibility to measure the frequency dependence of non-Gaussian noise due to the presence of an intrinsic dynamics. It could be for example, the classical dynamics of diffusion of electrons in a wire [10], or the quantum dynamics associated with the timescale \( h/eV \) [11]. We acknowledge fruitful discussions with Edouard Pinsolse and Charles Marseille, and the technical help of G. Laliberté. This work was supported by the Canada Excellence Research Chair program, the NSERC the MDEIE, the FRQMT via the INTRIQ, the Université de Sherbrooke via the Institut Quantique and the Canada Foundation for Innovation.

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Shot-noise of high-impedance quantum devices using impedance matching

1st C. Schönenberger
Department of Physics
University of Basel
Basel, Switzerland
Christian.Schoenenerberger@unibas.ch

2nd T. Hasler
Department of Physics
University of Basel
Basel, Switzerland
Thomas.Hasler@unibas.ch

3rd G. Puebla-Hellmann
Department of Physics
University of Basel
Basel, Switzerland
Gabriel.Hellmann@unibas.ch

4th V. Ranjan
Department of Physics
University of Basel
Basel, Switzerland
Vishal.Ranjan@unibas.ch

5th M.-C. Harabula
Department of Physics
University of Basel
Basel, Switzerland
Cezar.Harabula@unibas.ch

6th R. Haller
Department of Physics
University of Basel
Basel, Switzerland
Roy.Haller@unibas.ch

7th M. Jung
Department of Physics
University of Basel
Basel, Switzerland
Minkyung.Jung@unibas.ch

8th G. Fülöp
Department of Physics
University of Basel
Basel, Switzerland
Gergo.Fulop@unibas.ch

Abstract—High-impedance devices, such as quantum devices, for example quantum wires and quantum dots, are difficult to measure accurately due to the large impedance mismatch between the quantum device and the high frequency (RF) interface given by the typical coaxial cable wave impedance of 50Ω. Fast and reliable read-out therefore requires impedance matching. [1], [2] As compared to wideband detection without impedance matching, the signal-to-noise ratio can be enhanced by as much as a factor of ~ 103 for a device with an impedance of 100 kΩ. [3] Impedance matching is achieved by coupling the device to an on-chip resonant RF circuit, typically resonating at values in between 2–8 GHz. This is a range for which low-noise cryogenic amplifiers and quantum-limited parametric amplification schemes have been developed in recent years.

We compare two approaches: a) the use of an on-chip coil (spiral) inductor made from superconducting Nb (NbTiN) [4] whose inductance defines an effective lumped-circuit LC resonator with the stray capacitance, and b), the use of transmission line resonators, [3] specifically so-called stub-tuners. [1] The coil approach is more compact and has a wider bandwidth than resonators based on transmission lines and potentially a better signal-to-noise ratios. In particular for measurements of radiation emitted by the device, such as shot noise, the ~ 50 x larger bandwidth reduces the time to acquire the spectral density. In contrast, the advantage of the stub resonator concept is the ease with which the response of the circuit can be predicted, designed, and fabricated. Due to the smaller bandwidth, it is easier to obtain all relevant matching circuit parameters in a reliable way from power-reflectance measurements. This is due to a reflectance background which is not flat, but displays spurious resonances that often cannot be eliminated.

We have tested the two approaches on quantum dots (QD) defined in a single carbon nanotube. [2]–[5] We typically find suppressed shot-noise as expected for and in agreement with sequential tunneling through a QD. However, we also find regions of enhanced shot noise (super-Poissonian) within and outside of Coulomb-blockade (CB). This is compared with full-counting model calculations [5], [6] that consider (in the minimal case) two charge states $N$, $N+1$, and takes at least one excited state into account, e.g. $N^\ast$. It is then possible that inelastic cotunneling within CB can be enhanced, if the QD is residing in its excited state $N^\ast$. This then gives rise to electron bunching and super-Poissonian noise, which originates from random switches between the strongly and weakly conducting states of the QD. For sequential tunneling, the charge cycle $N \to N+1 \to N^\ast$ is allowed for a smaller bias-voltage as compared to the sequence $N \to N + 1 \to N$. If the two processes have very different coupling strengths, super-Poissonian noise emerges in a similar fashion as before. The weakly-coupled state is called blocking-state. We find blocking-states in all measurements, sometimes they appear very often, sometimes they are rarer. No systematics on the appearance of blocking-states could be found. One should also emphasize that blocking-states can also be recognized by a negative $\partial I/\partial V$ in bias-dependent conductance measurements.

Index Terms—

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Fluctuation Analysis of Repetitive Writing Motion by Using DFA

1st Masafumi Uchida
Graduate School of Informatic and Engineering
The University of Electro-Communications
Cho-fu, Japan
uchida@ee.uec.ac.jp

Abstract—Control of voluntary movements is a dual structure of cognitive and the physical controls. Cognitive control involves attentional resources, whereas physical control does not. Various voluntary movements can be performed by combining cognitive and physical controls. If these two controls could be separated, we would be able to predict the level of acquisition of physical exercise skills from the expenditure of attentional resources. The relationship between repetitive body movements and attentional resources has already been investigated using synchronous tapping methods. Body movements that depend on attentional resources are performed by cognitive control, and synchronous errors fluctuate with white noise. The fluctuations of the synchronous errors gradually shift to the one-over-f fluctuation, as the dependence on the cognitive control in the body movement decreases. We are able to estimate the control system related to the body movements of synchronous tapping using the fluctuation features of the synchronous error time series. In our previous study, we reported that a time series of the handwriting time element (HTE) contains self-similarity features, and the detrended fluctuation analysis (DFA) was applied to the analysis of the self-similarity at the time. As a result of examination, if the time series of the HTE had self-similarity, it implied that this time series did not have a fixed timescale. Some previous studies reported a crossover phenomenon associated with a change in the short- and long-range self-similarity features during the DFA. We investigated the relationship between the local self-similarity on the timescale and the differences in the difficulty levels for writing Kanji characters by hand. Therefore, in this study, we focus on the differences of fluctuations for body movements and attentional resources.

Index Terms—one-over-f fluctuation, handwriting, Kanji character, attentional resource, coping strategy

I. INTRODUCTION

Control of voluntary movements is a dual structure of cognitive and physical controls. Cognitive control involves attentional resources, whereas physical control does not. Various voluntary movements can be performed by combining cognitive and physical controls. If these two controls could be separated, we would be able to predict the level of acquisition of physical exercise skills from the expenditure of attentional resources. The relationship between repetitive body movements and attentional resources has already been investigated using synchronous tapping methods [1] [2]. Action timing must be anticipated for the synchronous tapping task. It is possible to estimate a control system related to the body movements of synchronous tapping using the fluctuation features of the synchronous error time series. Body movements that depend on attentional resources are performed using cognitive control. Synchronous errors fluctuate with white noise. Fluctuations of the synchronous errors time series were found to gradually shift to the one-over-f fluctuation, as the dependence on the attentional resources in the body movement decreases. Detrended fluctuation analysis (DFA) is an effective method used for analyzing physiological signal fluctuations because it needs only a few rigid assumptions regarding signal stationarity [3]. If the signal time series has self-similarity, it is interpreted as covering all timescale. However, in many cases, the same self-similarity is not considered to cover the entire timescale in the time series of actual systems. Some previous studies reported a crossover phenomenon associated with a change in the short- and long-range self-similarity features during the DFA [4].

In our previous study, we reported that a time series of the handwriting time element (HTE) contains self-similarity features [5] [6] [7]. It was difficult to ensure the correct stationarity of the HTE time series; therefore, we used the DFA method. Furthermore, we investigated the relationship between the local self-similarity on the timescale and the differences in the difficulty levels for writing Kanji characters by hand. In this study, we focus on the differences of fluctuations for the local process constituting synchronous writing one Kanji character. For the purpose of investigation, we evaluate the difference of coping strategies when writing one Kanji character repeatedly, based on the relationship between fluctuations of body movements and attentional resources. The coping strategy here means something like that subject’s peculiarity when writing one letter. The specific task of repetitive body movements is to write specific Kanji character by hand. That is, the task is repetitive writing of each of five kinds of Kanji characters; ‘den’, ‘tsu’, ‘ki’, ‘dai’, and ‘ichi’. These five Kanji characters have slightly different difficulty levels; from “Very hard” to “Very easy”. Depending on the difficulty of handwriting, coping strategies for handwriting have been expected to change.

II. EXPERIMENT METHOD

Fig. 1 shows the experimental protocol. First, we performed
tests comprising the profile of mood states (POMS) and the state-trait anxiety inventory (STAI); second, the subjects were required to close their eyes and rest for 150 s before being asked to perform the assigned handwriting tasks; third, the tests were repeated after the 150 s rest. These steps were repeated by them with enough breaks and comprised one set of the experimental process. A dual-task method was applied in this experiment. This method involved synchronization tapping and handwriting tasks.

Fig. 2 shows the configuration of the experimental system. The handwriting field was set on a pen tablet. This field was

obtained in each HTE. Each time series of the HTE consisted of 250 samples. The status of the subject’s autonomic nervous system activation during the experiment was monitored by three contact temperature sensors worn on the trunk (near the umbilicus) and both index fingers. At the present time, we do not know how to extract information related to writing a Kanji character from activation of autonomic nervous system, so we do not discuss it in this report.

TABLE I shows the handwriting task. Five Chinese Kanji characters were selected for the handwriting task. These characters; 'den', 'tsu', 'ki', 'dai', and 'ichi', has slightly different difficulty levels; “Very hard”, “Hard”, “Middle”, “Easy”, and “Very easy”, and their stroke counts are different. Then, the stroke count was chosen as the index of task difficulty. The slight differences between character shapes were regarded as the difference of difficulty between handwriting tasks; the stroke count is an index of the handwriting difficulty.

III. ANALYSIS METHOD

The time series of the HTE is represented by \( x_i (i = 1, 2, 3, \ldots, N - 1, N) \); then, \( x_i \) is the intended signal of the DFA where \( N = 250 \) samples of the HTE, that is, \( N = 250 \). The time series \( y_k (k = 1, 2, 3, \ldots, N - 1, N) \) of the cumulative sum for \( x_i \) was computed as \( \sum_{i=1}^{k} (x_i - \bar{x}) \), where \( \bar{x} \) was a mean value of \( x_i \), \( y^*_k (k) \) is the regression line of the time series \( \{y_k, y_{k+1}, y_{k+2}, \ldots, y_{k+n-1}, y_{k+n-1}\} \) where \( n \) was the timescale and \( n = 3, 4, 5, \ldots, N - 1, N \). The slope of the regression line on the log-log scale field of the \( F(n) - n \)
TABLE I

FIVE KINDS OF HANDWRITING TASKS

<table>
<thead>
<tr>
<th>Pronunciation</th>
<th>Character shape</th>
<th>Stroke count</th>
<th>Handwriting difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>den</td>
<td>電</td>
<td>13</td>
<td>Very hard</td>
</tr>
<tr>
<td>tsu</td>
<td>通</td>
<td>9</td>
<td>Hard</td>
</tr>
<tr>
<td>ki</td>
<td>6</td>
<td>Middle</td>
<td></td>
</tr>
<tr>
<td>dai</td>
<td>3</td>
<td>Easy</td>
<td></td>
</tr>
<tr>
<td>ichi</td>
<td>1</td>
<td>Very easy</td>
<td></td>
</tr>
</tbody>
</table>

The fluctuation characteristics of the six HTEs are similar on respectively. Furthermore, the standard deviation of the six components in the HTEs indicating the maximum and minimum of the characteristics was very noisy in this study. So, we have proposed the characteristics of the modified local trends. That is, the $L_{\beta m} - m$ characteristics where $L_{\beta m}$ represents the maximum value of LeT is smaller than the other. Moreover, the vector $L_{\beta m}^{knj}$ is a set of $hte$ $L_{\beta m}^{knj}$ values when the timescale is $m$ in a Kanji character $knj$ as shown (3).

$$L_{\beta m}^{knj} = \{ L_{\beta m}^{knj}, S_{Ld} L_{\beta m}^{knj}, S_{Ls} L_{\beta m}^{knj}, S_{St} L_{\beta m}^{knj}, S_{St} L_{\beta m}^{knj}, S_{St} L_{\beta m}^{knj} \}$$

The HTEs indicating the maximum and minimum of the six components in $L_{\beta m}^{knj}$ are extracted as $\max hte_{m}^{knj}$ and $\min hte_{m}^{knj}$ (knj = den, tsu, ki, dai, ichi), respectively. Furthermore, the standard deviation of the six components in $L_{\beta m}^{knj}$ is calculated as $\sigma_{knj}$. If the $\sigma_{knj}$ value is large, the fluctuation characteristics of the two HTEs $\max hte_{m}^{knj}$ and $\min hte_{m}^{knj}$ will be more different on this timescale $m$. On the other hand, if the $\sigma_{knj}$ value is small, the fluctuation characteristics of the six HTEs are similar on $m$. Based on the two HTEs $\max hte_{m}^{knj}$ and $\min hte_{m}^{knj}$ to be extracted and $\sigma_{knj}$ to be calculated, a coping strategy common to the subjects for the handwriting of that Kanji character $knj$ is estimated. If it is interpreted that the ordinary scaling index $\beta$ is an evaluation index of fluctuation covering the entire timescale, the proposed $L_{\beta m}$ is able to interpret as an evaluation index of fluctuation in the local timescale. Since $\max hte_{m}^{knj}$ and $\min hte_{m}^{knj}$ are $\min hte_{m}^{knj}$ according to the definition, $\min hte_{m}^{knj}$ tends to be relatively white noise than $\max hte_{m}^{knj}$. That is, $\min hte_{m}^{knj}$ is able to interpret as a more cognitive element than $\max hte_{m}^{knj}$. Furthermore, if $\max hte_{m}^{knj}$ or $\min hte_{m}^{knj}$ common to subjects is found on a specific timescale, the tendency of the fluctuation is able to interpret as specific coping strategy in the handwriting of that $knj$.

IV. RESULTS AND DISCUSSION

Fig. 4 shows six types of HTE values measured from five subjects; sub. A, B, C, D, and E. The columns of graphs arranged in $5 \times 5$ are five Kanji characters, the rows are five subjects. The horizontal axis of each graph is a time (ms), bins of width 100 ms are set, and the number of occurrences of HTE values in each bin is the vertical axis.

Fig. 5 shows (a) $F(n)$-$n$ characteristics (upper figure) and (b) $L_{\beta m} - m$ characteristics (lower figure) of the six HTE time series measured by subject A (sub. A in Fig. 4), respectively. Although the $F(n)$ value of LeT is smaller than the other five types of HTE, the point to be noticed here is the slope of the $F(n)$-$n$ characteristics. The slope of the regression line of the $F(n)$-$n$ characteristic is the scaling index in the DFA, and the plot value of $m = 250$ on the $L_{\beta m} - m$ characteristic represents this index. In the $F(n)$-$n$ characteristics, the gradient differences between the six HTEs are confirmed. On the other hand, it is obvious from this figure that the scaling index is not constant at any HTE. It is possible to
calculate the difference value of $F(n)$-m characteristic in order to capture the trend of the local scaling index with respect to $m$, but this trend is very noisy and confuses our interpretation. The $L_{\beta_m}$-m characteristic proposed by this research seems to visualize local trends of the scaling index along the $F(n)$-m characteristic.

In Fig. 6, the upper graphs are the $\sigma_{knj}^{m} - m$ characteristics and the lower graphs are the $\max_{m} hte_{m}^{knj} \& \min_{m} hte_{m}^{knj}$-m characteristics. Graphs of the $\sigma_{knj}^{m} - m$ characteristics are arranged for each subject and for each $knj$. The horizontal axis is $m$, and the vertical axis is $\sigma_{knj}^{m}$. The lower graphs in this figure focuses on the variety of $\max_{m} hte_{m}^{knj}$ or $\min_{m} hte_{m}^{knj}$ extracted from five subjects. This graph was plotted with a vertical bar or an asterisk on $m$ in which five HTE types measured from five subjects were common to five subjects or limited to two types. A vertical bar represents $\max_{m} hte_{m}^{knj}$ and an asterisk represents $\min_{m} hte_{m}^{knj}$. The horizontal axis is $m$, and the vertical axis is six kinds of the HTE. All $\sigma_{knj}^{m} - m$ characteristics seem to be flat as the difficulty level of $knj$ goes down. This implies that, on lower difficulty levels, the six components of $LB_{m}^{kn}$ have nearly identical values on any timescale. There is no room to include various coping strategies in the handwriting of simple Kanji character. It is interpreted that handwritten execution of simple Kanji character is executed by very simple coping. A very simple coping may be interpreted as a physical process. That is, the coping strategy is associated with attention resources assignment.

V. Conclusion

In this report, we evaluated the difference of coping strategies when writing Kanji characters repeatedly, using relationship between fluctuations of body movements and attentional resources.

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Making the eavesdropper's life harder

Gergely Vadai*
Department of Technical Informatics, Interdisciplinary Excellence Centre, University of Szeged
Szeged, H-6720, Hungary
vadaig@inf.u-szeged.hu

Laszlo B. Kish
Department of Electrical and Computer Engineering, Texas A&M University
TAMUS 3128, College Station, Texas, 77843-3128, USA
Laszlo.kish@tamu.edu

Abstract — The unconditionally (that is, information theoretically) secure Kirchhoff-law-Johnson-noise (KLJN) key exchanger operates with two pairs of resistors, one at Alice’s side and another one at Bob’s end. Whenever the cable resistance is not negligible, the original scheme leaks information to the eavesdropper (Eve). The typical way to reduce this leak is treating the exchanged key by privacy amplification. In the present work our goal is to reduce the leak earlier, during the bit exchange level. So far, the common belief has been that the resistance values in Alice's/Bob’s pairs should be very different for an easy separation of the bit values to yield low bit error probability in the key exchange. However, such situation is helping Eve during all the currently known types of attacks owing to higher information leak. We explore the possibility of enhancing the security by narrowing the difference between the resistance values. The impact on security and the cost of reduced communication speed are demonstrated for the Bergou-Scheuer-Yariv attack.

Keywords—information-theoretic security, KLJN key exchange, Bergou-Scheuer-Yariv attack, enhanced Johnson noise, unconditional security via wire channels

I. INTRODUCTION: THE KLJN SECURE KEY EXCHANGE

The unconditionally (that is, information theoretically) secure Kirchhoff-law-Johnson-noise (KLJN) hardware key exchanger [1-16] operates with two pairs of resistors, one at Alice's side and another one at Bob's end, see Fig. 1. During the secure bit exchange operation, one of the resistors (RL or RH) are independently and randomly selected and connected to the wire by units A (Alice) and B (Bob), respectively. The noise generators represent either the Johnson noises of the resistors or external generators mimicking much higher effective noise temperature Teff. In the secure state (mixed density spectrum) can measure the noise voltage and current there and evaluate their power density spectra RS and RH, respectively, at effective temperature Teff. Eve has access to the wire and can measure the noise voltage and current there and evaluate their power density spectrum S(f) and S(f), respectively.

Fig. 1. The core KLJN scheme [1, 2]. The "thermal" noise voltage generators of the resistors RL and RH have power density spectra SL and SH, respectively, at effective temperature Teff. Eve has access to the wire and can measure the noise voltage and currents injected into the wire, which will naturally yield the value of the loop resistance RL+RH from which Alice and Bob know their own connected resistance, can calculate the resistance value at the other side by subtraction.

II. ASYMMETRY BASED ATTACKS

The asymmetry of the resistor values in the secure bit exchange (LH/HL) situations allows information leak to the eavesdropper (Eve). Such attacks have two major classes:

- Non-ideality based passive attacks [1]. When the system deviates from the core scheme, the security is not perfect with a passively listening (measuring) Eve. Examples are the non-zero wire resistance (Bergou-Scheuer-Yariv, BSchY) attack [9-11]; the temperature inaccuracy (Hao) attack [12]; cable capacitance attack [13]; transient attack [14]; etc. Making the difference greater between the RL and RH resistors increases not only Alice's and Bob's fidelity but also Eve's signal-to-noise-ratio and the information leak to her, provided the bit exchange duration (communication speed) is kept fixed.

- Active attacks [2] are present when Eve modifies the system. Relevant examples are the current injection attack [15]; the man-in-the-middle attack [16]; etc. The current injection attack is based on the distribution of currents injected into the wire and that will naturally increase Eve's signal-to-noise-ratio and information leak when the difference is greater between the RL and RH resistance values provided the bit exchange duration (communication speed) is kept fixed. On the other hand, the man-in-the-middle attack [16] is not utilizing these differences thus it is immune against the asymmetry of the resistors.

* Corresponding Author. On leave at the Electrical and Computer Engineering Department, Texas A&M University, January 18-31, 2019.
The typical ways to reduce the information leak, out of reducing non-idealities, are simulating the communication channel and Eve’s attack, and then dropping high-risk bits \([4, 15]\) (with a proper combination with other dropped bits), and/or treating the exchanged key by privacy amplification (hash functions). In the present work our goal is to reduce the leak also at an earlier stage, during the bit exchange level, by narrowing the difference between the resistance values, thus reducing the asymmetry. The cost, similarly to the above methods, is a reduced communication speed. The impact on security is demonstrated for the Bergou-Scheuer-Yariv cable resistance attack \([9-11]\). Note, for that attack, compensation techniques exist \([e.g. 1,10,11]\) however they increase the vulnerability against several other attack types.

III. IMPACTS OF REDUCING THE RESISTANCE DIFFERENCE

Reducing the resistance difference increases the \(\text{BEP}\) of Alice and Bob at a given communication speed. In the following, we show its impact on the communication speed while we keep the \(\text{BEP}\) at a fixed value, and we examine the effect on the information leak. Note, the communication speed is inversely proportional to the number of statistically independent measurement samples \(N\) during the exchange of a single bit.

A. The bit error probability of Alice and Bob

Alice and Bob must distinguish the insecure (HH and LL) and secure (HL and LH) bits. The parallel resultant resistance between the wire and the ground is \(R_{\text{HH}} = R_i/2\) in the HH state and \(R_{\text{LL}} = R_i/2\) in the LL state, and in the secure bit exchange situation it is

\[
R_i = R_{\text{HH,LL}} = \frac{R_H R_i}{R_H + R_i}. \quad (2)
\]

The mean-square voltage and the current on the wire are proportional to the parallel resultant resistance between the wire and the ground. Alice and Bob measure the noise voltage and the current on the channel for a \(t\) time window involving \(N\) statistically independent measurement points for the exchanged bit. They separate the secure and insecure bits by evaluating the mean-square voltage (and/or current) on the wire \([1,2,7,8]\). For the sake of simplicity, we examine the case of the noise voltage only.

The standard deviation of the mean-square voltage \(\Delta \text{HH}\) of the HH state scales inversely with the root mean square of bit length \([1,7]\):

\[
\Delta \text{HH} \propto \frac{1}{\sqrt{T}} \propto \frac{1}{\sqrt{N}}. \quad (3)
\]

For studying the impact of narrowing resistance difference, we fix the value of \(R_H\) — therefore \(R_{\text{HH}}\) is also fixed — and vary the value of \(R_i\). The gap \(\kappa\) between \(R_i\) and \(R_{\text{HH}}\) is given as:

\[
\kappa = \frac{R_{\text{HH}}}{2} - R_i = \frac{R_H^2 - R_H R_i}{2(R_H + R_i)}. \quad (4)
\]

The dependence of \(R_i\) and \(\kappa\) versus \(R_L\) are demonstrated in Fig. 2.

![Fig. 2. The increase of loop resistance \(R_l\) defined by \(2\) and the decrease of the gap \(\kappa\) while the lower resistance value \((R_L)\) value is approaching the resistance of the higher resistor value \((R_H=10\, \Omega)\).](image)

For a given \(N\), reducing the gap between \(R_H\) and \(R_H\) implies increasing \(\text{BEP}\) for Alice and Bob because, due to statistical errors, it makes harder to separate the mean square voltage values \([1,7]\). If we want to keep the \(\text{BEP}\) at a fixed value while \(\kappa\) is reduced, we must keep the ratio of \(\Delta \text{HH}\) and the gap \(\kappa\) at a fixed value. From \(3\), \(N\) must be changed as:

\[
N \propto \frac{1}{\kappa^2} \quad (5)
\]

In order to examine the \(\text{BEP}\) of the key exchange and the information leak we have carried out numerical simulations based on a code written in the LabVIEW environment. In the analyzed system, \(R_i = 10\, \Omega\), the rms voltage of the related external noise generator was 1 V and the resistance of the cable was 200 \(\Omega\). We varied the value of \(R_L\) from 330 \(\Omega\) to 9 \(\Omega\), while the rms voltage followed the Johnson formula \([1]\).

The exchange of 50000 secure bits was simulated for each value of \(R_L\). The value of \(N\) was 30 for the lowest \(R_L\) and it had been increased by following \(5\), see Fig. 3.

Fig. 4 shows that — in accordance with our goal — the ratio of \(\Delta \text{HH}\) and \(\kappa\) remains constant; therefore the \(\text{BEP}\) of Alice and Bob remains the same for every value of \(R_L\).
The required number of independent samples $N$ for a single secure bit exchange for a steady BEP value for Alice and Bob, see (5). The lower resistance ($R_L$) value is approaching the resistance of the other resistor ($R_H = 10k\Omega$).

Fig. 4. The ratio of $\Delta HH$ and $\kappa$ while the lower resistance ($R_L$) value is approaching the resistance of the other resistor ($R_H = 10k\Omega$) and $N$ has been following (5) as shown Fig. 3. The wire resistance was 200 $\Omega$.

B. Information leak during the BSchY attack

We examined the information leak caused by the non-zero wire for the Bergou-Scheuer-Yariv attack [1,9,10]. Note, at known other cable resistance based attacks (such as 10) Eve’s BEP is proportional to the BEP in the present work.

As it is shown in Fig. 5, our new tool is favorable for Alice and Bob: while their BEP remains the same, Eve’s BEP of her eavesdropping on the bit exchange is dramatically increasing and it is approaching the limit of 0.5, the limit of zero information entropy.

The obvious cost of the reduced information leak is the reduced communication speed, which is significant in the case of small difference between the resistances. However, the BEP of Eve is saturating when the relative resistance gap is small. For example at $R_L = 6k\Omega$ her BEP is nearly as large as at $R_L = 9k\Omega$, see Fig. 5, but the speed is much higher in the former case, see Fig. 3.

Fig. 5. Eve’s bit error probability $= 1 - p$ where $p$ is the probability of successful guessing of the bit value) while the lower resistance ($R_L$) value is approaching the resistance of the other resistor ($R_H = 10k\Omega$). Compare with $N$ increase in Fig. 3. The wire resistance was 200 $\Omega$.

IV. CONCLUSION

We have pointed out that reducing the gap between the resistor values in the KLJN secure key exchange protocol reduces Eve’s signal-to-noise-ratio and the information leak during various types of attacks. We demonstrated this claim with the BSchY wire resistance attack where we could significantly reduce Eve’s information. The price of the methods is slowdown. In practical situations, the gain and price of this method should be compared with that of the privacy amplification and the discarding of high-risk bits.

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Kirchhoff-Loop-Johnson (-Like)-Noise Cipher and Expansion by
Randomness Assists in Wireless Connectivity

Sumit Kumar1, IEEE, Member, Taniya Singh1, Mohammad Akhlaqur Rahman2, Rajib Kumar Jha2, IEEE, Senior Member

1 Indian Institute of Information Technology Una
2 Indian Institute of Technology Patna

Email: (sumitphd13, singhtaniya1998, rahman.iitp, jharajib)@gmail.com

Abstract—In a larger area with less number of deployed nodes, to localize the event such as the location of the fire in a forest, the transmission range (TR) of different nodes plays a crucial role. Here, this study demonstrates a method to increase the transmission range (TR) using the benefits of randomness. With the mathematical expression, we have shown how randomness can benefit us. The increment in $\sigma$ causes an average increase in the transmission range. This increment in transmission range causes a high probability of connection $C(r)$ between two remote nodes. According to the event, some of the nodes can be placed in an area for the detection of the exact location of the fire. It can be possible that either some of the nodes positioned at the precise location or can be some meters away. We are assuming the case of nodes placed at a distance from the location of the fire. The reflected signal which will be received by the transmitting node from the destination will of lower strength and hence it won’t be possible for us to detect the exact location. For increasing the exactness of the location, we can introduce disturbance between the two distinct nodes which in turn creates more randomness due to the multiple reflections.

Index Terms—Wireless network, Link connectivity, Transmission range.

I. INTRODUCTION

With the rapid development of the internet, energy consumption plays a crucial problem in the wireless industry [1]. The data volume of wireless networks is expected to increase by a factor of ten every five years, which is associated with an increase in energy consumption of 16-20%, according to statistics. By applying this rate to mobile communications, it will contribute 15-20% of the total energy consumption of Information and Communications Technologies (ICT) [2]. With the advancement in multimedia technology, video or image [3]–[7] also needs to be communicated over wireless network.

The wireless industry faces a sustainable development in these coming years. However, the issues of connectivity decide the performance of the wireless network system when the channel is dynamic. In the context of wireless communications, universal accepted service is the connectivity i.e., nodes must have high coverage range. In the remote or hilly area, the coverage plays a significant role to give the internet service to the customers. The deployment of nodes for many different applications such as forest fire monitoring and detection of faulty nodes, agriculture etc., is done to have a higher probability of connection ($C$) [8].

At the same time, the study of noise or random behavior of channel has to be taken care of. We always assume that the noise deteriorates the performance of the system. However, this statement is not valid in general. Benzi et al. [9] originally suggested the concept of stochastic resonance (SR), which describes a curious nonlinear physical phenomenon in a bistable system that is subjected to periodic and random forcing: an increase in input noise can lead to an improvement in the signal-to-noise ratio of output. Different applications such as weak signal detection using the use of external noise have been shown in the papers [5], [10]–[14].

The primary application nowadays is a point to point communication. The wired connection costs much higher than wireless transmission. There are important applications where sensor nodes are used to find out the location of the moving nodes. For example, we need to find out the exact location of the fire in a forest; we need to deploy the sensor nodes in the forest.

Wireless sensor networks may comprise of numerous different types of sensors like low sampling rate, seismic, magnetic, thermal, visual, infrared, radar, and acoustic, which are intelligent to monitor a wide range of ambient situations. Sensor nodes are used for constant sensing, event ID, event detection & local control of actuators [15]. The applications of wireless sensor network mainly include health, military, environmental, home, & other commercial areas. In Fig. 1, we have shown how the nodes are distributed in a channel. The channel keeps on varying as the nodes move on. Therefore, it is quite challenging to get high connectivity in the wireless sensor network.

Fig. 1: Wireless connectivity with different nodes. The connectivity is a random process. Node mobility represents those nodes which can move in any direction. With respect to a sink node, we have encircled a sink node coverage.
Structure of the paper: Section II discusses the basic concepts of communication models. In Section III, we describe the outcomes of the proposed study. In Section IV, we describe the conclusion of the proposed study.

II. DIFFERENT COMMUNICATION MODELS

A link is a transmitting medium between a sender and transmitter node [16]. There are different models with which communication between sender and transmitter takes place. In the absence of any disturbance or interference, we always assume that the receiver node is connected with the transmitter. Let us assume that the transmitter transmits the power \( P_t \). Assume the \( L \) as channel attenuation or power loss. Then, the received power can be written as \( P_r = P_t - L \). If the \( L < L_{th} \), then a direct radio link between nodes can exist, where \( L_{th} = P_t - P_{t_{min}} \) with \( P_{t_{min}} \) stands for minimum power received by the receiver. Generally, for moving channel between a sender and a receiver, we use a different model for the study of the link. These are classified into different groups which are as follows.

A. Deterministic Model

This model is described by the following equation

\[
L_0(r) = k_0 + k_1 \ln r,
\]

where \( L_0(r) \) stands for power loss at a distance \( r \). \( k_0 \) and \( k_1 \) stand for the characteristics of the channel. Here, we assume that sender is located at the center of the disk of radius \( r_0 \). The transmission range (TR) is a constant and is given by

\[
TR = r_0 = e^{\frac{L_{th} - k_0}{k_1}}
\]

In the wireless channel, generally, the nodes keep on moving. Therefore, the channel variation can easily be seen. Thus, we need a variable transmission range rather than a constant transmission range.

B. Statistical Model

The randomness or stochastic nature of the wireless channel enforces us to have variable transmission range to have proper connectivity. Here, we discuss the benefits of the channel randomness and how does this improve the connectivity.

\[
L = k_0 + k_1 \ln r + S,
\]

where \( S \) is called a shadowing sample which is assumed as Gaussian distributed with zero mean and standard deviation \( \sigma \). The corresponding transmission range (TR) can be given as

\[
TR = e^{\frac{L_{th} - k_0 - \sigma}{k_1}},
\]

where \( \sigma \) denotes the Gaussian distributed random variable.

Fig. 2: (Red) shows the deterministic channel where (Green) shows the stochastic channel.

C. Probability of link connection

Here, we derive the expression for the probability of link connection \( C(r) \). The probability that the two nodes are connected when the path loss is lesser than a predefined \( L_{th} \), i.e., \( L \leq L_{th} \) [17].

\[
C(r) = P ( L \leq L_{th} ) = P ( S \leq L_{th} - L_0(r) )
\]

\[
C(r) = 1 - \frac{1}{2} \text{erfc} \left( \frac{L_{th} - k_0 - k_1 \ln (r)}{\sqrt{2} \sigma_{new}} \right),
\]

where \( \text{erfc}(.) \) is complementary error function. After simplification, it can be written as follows.

\[
C(r) = Q \left( \frac{L_{th} - k_0 - k_1 \ln (r)}{\sigma_{new}} \right),
\]

where \( \sigma_{new} \) can be given as follows.

\[
\sigma_{new} = \sqrt{\sigma^2 + \sigma_{ext}^2},
\]

where \( \sigma_{ext}^2 \) is the variance of externally added noise. \( k_0 \) and \( k_1 \) show the parameters of the medium or channel. \( Q \) represents the complementary cumulative distribution function (CCDF). Using this model, we can derive the expression for the disk model when \( \sigma = 0 \) i.e., when \( \sigma = 0 \), the random variable \( s \) becomes 0. In that case,

\[
C(r) = \begin{cases} 1, & r \leq r_0 \\ 0, & \text{otherwise} \end{cases}
\]
Poisson point process (PPP) with density $\rho$. Let us assume that $\mu$ represents the average number of neighbors the sink.

\[
\mu_{\text{new}} = \begin{cases} 
2\rho r_0 e^{\frac{\sigma^2 r_0^2}{2\pi^2}} & \text{for } d = 1 \\
\pi\rho r_0^2 e^{\frac{\sigma^2 r_0^2}{\pi^2}} & \text{for } d = 2 \\
4\pi\rho r_0^3 e^{\frac{\sigma^2 r_0^2}{8\pi^3}} & \text{for } d = 3 
\end{cases}
\]  

(10)

where $r_0$ can be given by $r_0 = e^{\frac{L_0-L_0}{L}}$ and $d$ represents the dimension. The $\mu_{\text{new}}$ becomes $\mu$ when $\sigma_{\text{new}}$ becomes $\sigma$. $\sigma_{\text{new}}$ increases as $\sigma_{\text{ext}}$ increases. The expression of $\mu$ can be written as follows.

\[
\mu = \begin{cases} 
2\rho r e^{\frac{\sigma^2 r^2}{2\pi^2}} & \text{for } d = 1 \\
\pi\rho r^2 e^{\frac{\sigma^2 r^2}{\pi^2}} & \text{for } d = 2 \\
4\pi\rho r^3 e^{\frac{\sigma^2 r^2}{8\pi^3}} & \text{for } d = 3 
\end{cases}
\]  

(11)

It suggests that noise is actually beneficial to the wireless system in terms of its connectivity. The percentage change in the $\mu$ is given by

\[
I = \frac{\mu_{\text{new}} - \mu}{\mu}.
\]  

(12)

Solving for $I$

\[
I = \begin{cases} 
\frac{\sigma^2 r}{2\pi^2} - 1 & \text{for } d = 1 \\
\frac{\sigma^2 r^2}{\pi^2} - 1 & \text{for } d = 2 \\
\frac{\sigma^2 r^3}{8\pi^3} - 1 & \text{for } d = 3 
\end{cases}
\]  

(13)

When $\sigma_{\text{ext}} = 0$, then $I = 0$. However, Eq. 13 shows the increment in $\mu$.

### III. Discussion

Here, in this section, we discuss the result and effects of randomness in the connectivity. When $\sigma = 0$, the statistical model behaves as the deterministic model. However, the major drawback about the deterministic model is that its transmission range. The effect of randomness produces a high transmission range. It gives a high probability of connection, $C$. Fig. 3 shows the probability of connection as $\sigma$ is increased.

Also, an increase in $\mu$ causes the increment in the average number of the neighboring sink. It has been shown in Fig. 4.

### IV. Conclusion and Future Work

This report gives a brief idea about the utilization of external noise or randomness in the wireless sensor network to increase the probability of connection. It also helps in detecting the nodes which are not audible in case of the deterministic model. Furthermore, the results suggest that increasing the variance by adding $\sigma_{\text{ext}}$ gives exponential increment in the audible nodes as far as $d = 3$ is concerned. It opens a gateway to the researchers for the deployment of the sensor nodes for increased connectivity.

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Abstract—Internet connectivity (IoT) of ever increasing numbers of physical sensing devices, results in vast amounts of sensing (measurement) data which have to be transmitted and processed in order to extract information. Data exchange and manipulation raise several concerns regarding privacy, security and integrity of the measurement data (i.e electricity smart meters). Noise injection into the measurement time-series data is proposed in order to enhance data resilience. This noise injection in combination with statistical analysis tests while operating in a machine learning framework enables the detection of malicious data (fraud detection) in the receiver. Statistical analysis is essential in the case of time-series data collection, since the statistical properties of any time-series can give a strong indication of whether the data is being maliciously changed or not.

Index Terms—noise injection, white noise, Poisson distribution, fraud detection, machine learning, time-series statistical analysis

I. INTRODUCTION

The number of IoT devices has increased exponentially during the last few years. Nowadays, terabytes of data are exchanged on a daily basis between several smart devices [1] and according to [2] by the end of 2020 there will be between 20 and 50 billion devices connected. The high frequency of transactions and the huge amount of data transferred causes an increased vulnerability regarding private data sensitivity as well as transactions security and integrity. Between the applications that are benefited from the IoT development is remote smart metering; a possible loss of measured data integrity and privacy would result in costly defects. For example, in the case of electrical energy application at the Utility level, data integrity loss might imply energy theft and set the whole Utility system stability into danger. Rajagopalan et. al [3] describe the problem of IoT deployment as a trade-off between Utility and privacy. Nevertheless, IoT opens opportunities for new services provision, despite the raising of important data related concerns.

There have been several attempts and many different approaches to leverage the data integrity problem. In [4] an on demand security configuration of the IoT device is being proposed, while in [5] a similar, remote configuration of the aggregator internet server is introduced. In [6] the modification and extra hardware configuration of IoT devices is proposed, and despite the cost and complexity of this solution, there already exist several low-end IoT devices serving this purpose. Given that all of the aforementioned solutions require the usage of either complicated and more expensive devices or at least the modification off the software of the current devices (which is also inefficient on most scenarios). In this work, we consider a different approach, that of injecting noise in the measured time-series data without the need to modify the hardware or the software of the device.

Time-series of measurement data can be regarded as progression of various shapes in time. Noise, that is usually of certain type and color, is inherent in time-series or can be artificially introduced. Artificially injected noise is a viable solution not only for protecting the data from being hacked, but also for enhancing the integrity and security of transactions made between connected devices. Several interesting solutions have been introduced which manage to reverse the negative effects of noise into an adhesive data resilience tool. Noise injection in the measurement data (time-series) is a first order solution for fraud detection if the statistical characteristics are known at the signal receiving part [7]. In [8], a light privacy model with noise injection in the data is presented in order to limit the risk of hacking. Only the masked, noise injected data are transferred outside of the device. The investigation of non-stationarity and the classification of specific events in noisy time series data [9] are used for the analysis of the received signal. If the received data (time-series) are altered to some degree by noise, the investigation of noise characteristics are an effective method for fraud detection. Our method enhances the ability to estimate the degree of noise corruption, by examining specific statistical characteristics, such as non-stationarity.

The proposed methodology is most suitable for smart grid load measurement data, where noise injection in the data series at the smart meter side is combined with statistical analysis and machine learning algorithms at the receiving end of the energy aggregator. This analysis leads to conclusion about data validity. The proposed methodology enhances the secure transaction between IoT devices, while preserving data privacy.

II. PROPOSED METHODOLOGY

The usual approach for studying various structures in time-series analysis is to assume that a certain process with
known statistical/physical characteristics results to a specific, recognizable, temporal trace. Detecting such a trace in the time-series one can conclude regarding the data integrity and recognition. In this work, white noise with known statistical properties and characteristics is the physical injection mechanism for detecting malicious data. The white noise process is one of the usually involved type of noise for direct detection of malicious data. When adding white noise to a signal, the frequency spectrum of the resulting signal remains the same with the original one. A white noise process is defined by the following equation:

\[ S_w(\omega) = \frac{N_0}{2} \]  

where \( N_0 \) is a real constant and called the intensity of the white noise. This exact characteristic of white noise combined with the statistical analysis of the electric load signal enhances the ability to detect any potential fraud or malfunction of the system. The proposed methodology is shown in Fig.1.

At first, the smart meter emits an encrypted signal which is a superposition of the measurement and white noise data (with specific characteristics, so that to achieve an SNR unique for each different smart meter). The encryption process deals only with white noise data injection, thus the fundamental frequency remains unchanged.

The encrypted signal has the shape shown in Fig.2 (encrypted with the addition of white noise).

At the receiver side, a Fourier test over the time series is applied in order to detect any malfunction or intrusion over the received signal. Fourier test enables the detection of any change in the fundamental frequency and leads to a conclusion whether there has been a possible fraud over the signal or not. Failing the Fourier test implies that the measurement data were mixed with false data and for this reason the algorithm classifies this transaction as a fraud. Fig.2 and Fig.3 portray two different noise injections in originally measured time-series data. In Fig.3 the injected noise data follow a Poisson distribution and in Fig.2 the injected data follow a normal distribution resembling white noise.

Fig. 1. The proposed methodology

Fig. 2. White Noise Encrypted Load Time Series Data

The second step of the framework addresses the possibility of having fraud data injection that follows a normal distribution (white noise), thus making harder and more difficult to detect any malfunctions. In this case the statistical property of auto-correlation is employed as the indicative feature en-
Fig. 4. Frequency Analysis Load Time Series Data

Fig. 5. Frequency Analysis of White Noise Encrypted Load Time Series Data

Fig. 6. Frequency Analysis of Poison Noise Encrypted Load Time Series Data

Fig. 7. Auto-correlation Plot of Load Data

Apart from the auto-correlation of the time-series itself, the correlation between different time-series could provide metrics for comparison. If we continue with examining the electric load data we can observe in Fig.8 and Fig.9 that the correlation between electric load and A1 RMS (electric current measurement) or Vthd (Voltage total harmonic distortion) also follows a specific pattern. For this reason, it is almost impossible for someone to inject data by correctly imitating the correlation patterns for all those different measurements.

Fig. 8. Cross Correlation Electric Load and A1 RMS

At this point our decision tree methodology already has strong indications of fraud or not but we still need to confirm the result with a final step which is a machine learning based forecasting tool. If the Mean Absolute Percentage Error (MAPE) between the new time series and our forecasted values is higher than of our model, and at the same time the correlation tests fails, then the data is classified as a fraud. In case of only failing either the forecasting or the correlation test, then the new signal is classified as a possible fraud.
III. CONCLUSIONS

The huge amount of data transferred and the expansion of IoT devices in almost every aspect of our life raises extremely important concerns regarding the security and privacy of several sensitive information (data). Since hacking into the hardware requires specific knowledge, false data intrusion can be seen as a more realistic threat regarding for example the energy data acquired from smart meters. By employing data analysis and novel machine learning techniques, those potential malfunctions can be addressed more efficiently in terms of cost and time. The fact that changing the hardware of IoT devices is not a realistic scenario, the exploitation of data analysis and novel machine learning techniques is the most efficient method to tackle these issues. As examined in this work this is a solution that can provide as with robust results, especially for time-series data which have a physical meaning (the electric load measurements). In this case the statistical analysis and the expected results can be used in order to precisely detect a fraud or not.

REFERENCES


Dynamic Stochastic Resonance Based Blocking Artifacts Removal from Compressed in DCT Domain

Rajib Kumar Jha, Onkar Krishna, Saurabh Kumar Pandey, Sumit Kumar, Vivek Singh Verma

Indian Institute of Technology Patna, PDPM IIITDM Jabalpur

(jharajib, saurabh)@iitp.ac.in, (onkarkris, sumitphd13, viveksv10)@gmail.com

Abstract—In this paper, we have introduced a dynamic stochastic resonance (DSR)-based blocking artifacts removal of compressed images in DCT domain. Blockiness in DCT based compressed images after reconstruction are a very challenging problem. In the proposed technique it has been observed that blockiness is coming due to a few DCT coefficient of the first row of each block, so we focused only on target coefficient and modified them with a weighted average of neighboring block coefficients. To improve the performance of the blocking artifacts reduction methods, we have used the dynamic stochastic resonance (DSR) by exploiting the emor or decadence introduced during the quantization procedure. DSR is an iterative process which modulates the quantized factors so that the impact of the quantization procedure is compressed and image information is enhanced. To find a better outcome and minimize the computational complexity, an adaptive optimization method has been taken for selection of bistable parameters. The main advantage of the proposed method is, it overcomes the blocking artifacts introduced during the DCT based compression. The qualitative and quantitative results are also shown.

Index Terms—Dynamic Stochastic Resonance (DSR); Discrete cosine transform (DCT); Stochastic Resonance (SR).

I. INTRODUCTION

Benzi et al. [1] was the first author who has introduced the term stochastic resonance (SR) in the context of noise enhanced signal processing in 1981; wherein they addressed the problem of periodic recurrence of Earth ice ages. SR phenomenon was also reported by Fauve et al. [2] in 1983. They studied the noise dependence of the spectral line of an ac-driven Schmitt trigger circuit. Besides that, SR is widely used in the image processing applications like image enhancement [3], watermarking [4], signal detection [5]–[8], and segmentation [9] etc.

In 2003, Qinghua et al. [10] used the SR phenomenon for line detection from noisy images based on Radon transform. In 2006, Guangchun et al. [11] investigated the novel watermarking scheme, which is based on the stochastic resonance. Jha et al. used the concept of stochastic resonance in different fields of image and signal processing such as image denoising [12], image segmentation [13], image watermarking [14], contrast enhancement [15]. In these methods, authors utilize the inner noise, which is already present in the input image, as well as they, introduced the external noise to improve the executions.

Image compression is an essential requirement in several areas of communication. The main objective of compression is to reduce the image size for efficient storage and transmission purposes. Numerous of efficient compression methods are used for several applications. From them, a discrete cosine transform (DCT) was adopted in the international standard of JPEG, MPEG, and H.261. The reformed images from JPEG compression produce annoying blocking artifacts near block boundaries, for highly compressed images. The degradation mentioned above is a direct outcome of the coarse quantization of the DCT coefficients and also due to independent processing of the blocks, except the dc coefficients of blocks.

Several techniques have been proposed by researchers; both in spatial and frequency domains for the reduction of blocking artifacts with varying degree of success. Some methods working in the frequency domain are proposed by different authors [16]–[18]. A new index to measure the blocking effects namely the mean squared difference of slope (MSDS) is introduced in [16]. It is shown that the expected value of the MSDS increased with an increase in the quantization of the DCT coefficients. There approaches remove the blocking effect by minimizing the MSDS, while imposing linear constraints corresponding to quantization bounds. Liu et al. [17] proposed a DCT domain method for blind measurement of blocking artifacts, by modeling the artifacts as 2-D step functions in shifted blocks. Zeng [18] proposed another DCT domain method for blocking artifact reduction by applying a zero masking to the DCT coefficients of some shifted image blocks.

Main objective of our algorithm is to maximally reduce the extraneous noise introduced by the quantization process without sacrificing the image quality. For this purpose, we used the concept of dynamic stochastic resonance (DSR). DSR is a phenomenon where a controlled amount of random noise is used for signal and image improvement. Here in this work, we have used internal quantization noise present in the compressed image for blockly artifacts removal.

II. OVERVIEW OF DYNAMIC STOCHASTIC RESONANCE

It has been observed in one-dimensional signals that at an optimum resonant value of noise, the signals cross the threshold and transits into another (enhanced state) [15]. To establish the principles of SR in applications of image processing, the discrete image pixels are incisively treated as discrete particles, whereby the gray value of an image pixel corresponds to a specific kinetic parameter of a physical particle in Brownian motion. The mathematical formulation of the theory of dynamic stochastic resonance can be found in [15]. In this paper, we used DSR technique for blocky artifacts removal purposes. At optimum intrinsic noise density (or the optimum number of oscillations) the particle makes a
transition into the other well. In the proposed analogy, this optimum amount of noise is reached with maximization of PSNR by tuning the quantized DCT coefficients using discrete iterative Eq. 1.

\[ x(n + 1) = x(n) + \Delta t [ax(n) - bx^3(n + 1) + \text{Input}] \quad (1) \]

Here \( n \) is iteration count; \( \Delta t \) is the sampling time, taken as 0.015 experimentally. \( \text{Input} = B \sin(\omega t) + \sqrt{D}\xi(t) \) denotes the sequence of input signal and noise. This denotation can be done keeping in view images affected with blocking artifacts can be viewed as signal consisting of transformed image coefficient and noise corresponding to quantization of the coefficients.

In the paper [15], an analogy to Benzi’s double-well model for the global climate, in the context of blocking artifacts reduction has been developed. The model shows the physical relationship between the proposed work and the stochastic resonance concept. The position of the particle in a double well system is analogous to the state of DCT coefficients responsible for blocking artifacts. A weak periodic forcing is constituted by the DCT coefficients, while the noise is constituted by the noise introduced during the quantization process. Each of the two stable states is represented by a noisy state and enhanced state respectively. The iterative process is used for tuning the DCT coefficients, after a certain number of iterations; coefficients change their state from a weak state to enhanced state. Selection of bistable system parameters is based on the maximization of SNR [15].

III. REPRESENTATION OF BLOCK DISCONTINUITY IN DCT DOMAIN AND PREPROCESSING

We examine two horizontally adjacent blocks \( A \) and \( B \) each of size \( 8 \times 8 \) shown in Fig. 1. Assume block \( A \) has a constant gray value \( a \) and block \( B \) has a constant gray value \( b \), \( a \neq b \), for vertically adjacent blocks, the same assumptions are made. Let the right half of block \( A \) and the left half of block \( B \) form a block denoted as block \( C \). Block \( C \) is an \( 8 \times 8 \) block which contains the boundary pixels of blocks \( A \) and \( B \). Let, \( F_a(u, v) \), \( F_b(u, v) \) and \( F_c(u, v) \) be the DCT coefficients of blocks \( A \), \( B \) and \( C \) respectively. Since the pixels of blocks \( A \) and \( B \) have constant intensity values, so for \( F_a(u, v) \) and \( F_b(u, v) \) has only one nonzero value at \( u = v = 0 \) and all others are zero. Block \( C \) has discontinuity in its middle due to boundary, the same can be observed from the DCT coefficients of block \( C \), which is \( F_c(u, v) = 0 \) for \( v = 2, 4, 6 \) and \( u = 0 \) and \( F_c(u, v) \neq 0 \) for \( v = 0, 1, 3, 5, 7 \) in \( u = 0 \). It implies that the horizontal boundary discontinuity in block \( C \) can be represented only in a few DCT coefficients of the first row. It also can be easily deduced that the heavier the block discontinuity, the larger the ripples in these nonzero values.

We have targeted coefficients responsible for blocky artifacts of an image. As the coefficient adapts to the local information content of the image, altering the values of these coefficients without taking into consideration, the information about the nature of the image pixels in its neighborhood might result in artifacts. This is especially the case for high bit rate applications. Therefore before such filtering is performed, one has to ensure that the edge appearance between blocks \( A \) and \( B \) is not due to a genuine horizontal change in the grey levels of the pictures at that position. Conditions that should be met before modifying the blockiness responsible for blockiness appearance of the smooth regions are in the paper [19].

Now we perform our preprocessing step of blocky artifacts reduction in the DCT domain by modifying the five relevant coefficients of \( F_c(u, v) \). The first row of the DCT coefficient matrix of block \( C \) is modified by the weighted average of the first row of DCT coefficients of blocks \( A \), \( B \), and \( C \). The advantage of using the weighted average of adjacent block coefficients is that it is much adaptive to image contents instead of simply adjusting the value of DCT coefficient of block \( C \). Detailed description of this can be found in the paper [19].

IV. DSR BASED BLOCKING ARTIFACT REMOVAL

Now we apply the DSR concept on pre-processed blocks. We derived the DSR iterative equation as discussed in Section II and calculated double well system parameters by maximization of signal-to-noise ratio (SNR). DSR tries to suppress the noise effect introduced due to quantization error by tuning this internal noise at the different number of iterations, after certain iteration, we get the results in which simultaneously artifacts are invisible and information content enhanced. Therefore, the peak signal-to-noise ratio (PSNR) reached its maximum value. The average number of iterations in our experiment is found to be three. The following algorithm steps using DSR for vertical discontinuities is given below.

(a) From the pre-processed DCT coefficients block, we try to get a new matrix \( T \) of size smaller than the original one. To do so, we remove first and last four columns of the original matrix, and then select various rows leaving eight rows between selected rows as shown in Fig. 2, this is to collect all vertical discontinuity together. \( \sigma_0 \) is the standard deviation of noisy or target coefficients.
Using iterative equation given in Eq. 1, compute tuned set of coefficients of matrix T. Here, $x(n + 1)$ is the DSR enhanced DCT coefficient in $(n + 1)^{th}$ iteration number $a, b$ and $\Delta t$ are the bistable system parameters. $T$ is input matrix of DCT coefficient.

We repeat the iteration till as PSNR gets maximized, due to which artifact gets invisible, the PSNR maximization is because of stochastic resonance process discussed in Section II.

Finally, replace original coefficients with enhanced coefficients obtained after 3-4 DSR iteration.

V. RESULTS AND DISCUSSION

Our experiments are shown on different test images. To justify our results we used PSNR as a quantitative measure. We calculated PSNR value of highly compressed images test images, at different quality factors two of them shown in Fig. 3 and Fig. 4. We also tried to compare our algorithm with two exiting techniques shown in Table II and results after blocky artifacts removal by the proposed algorithm shown in Table I. We tested our proposed algorithm for highly compressed images at different quality factors.

The basic mechanism of DSR for improvement of robustness is attributed to the way DSR modifies the distribution of selected DCT coefficient (only those coefficient which is responsible for blockiness) of blocky image. It is observed that the distribution of DCT coefficients after certain iteration count $n$ which maximizes PSNR value is approximately the same as the distribution of DCT coefficients of the original image. Since the nature of the density function of the coefficients become same after a certain number of iterations using DSR operation. Therefore, it gives the best result at the particular value of iteration.

VI. CONCLUSIONS

A noise-induced dynamic SR-based technique is investigated in this paper. The iterative process on the noisy (DCT) coefficients enhances the image energy. We have applied DSR at the selected DCT coefficient of blocky image. DSR suppress the effect of artifacts by tuning the double well parameters. Our method takes advantage of prior existing techniques of blocking artifacts removal. It can efficiently eliminate blockiness for images coded at different bit rates without producing noticeable artifacts smoothes out the undesired block edges while retaining the sharpness of the decoded image to a noticeable degree. It can highly preserve the high-frequency components while smoothing out the blocking artifacts. This is because only the DCT coefficients related to the blocking artifact are modified while other frequency components remain the same. The most striking feature of our proposed DSR based algorithm is that it reduces the blockiness without producing any other considerable artifacts on the original image.

TABLE I: PSNR for different images at different compression level. (QF-Quality factor)

<table>
<thead>
<tr>
<th>Test Image</th>
<th>QF-8</th>
<th>QF-10</th>
<th>QF-12</th>
<th>QF-14</th>
<th>QF-16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>41.24</td>
<td>38.79</td>
<td>37.05</td>
<td>34.94</td>
<td>32.33</td>
</tr>
<tr>
<td>Barbara</td>
<td>39.32</td>
<td>35.09</td>
<td>35.89</td>
<td>33.21</td>
<td>31.08</td>
</tr>
<tr>
<td>Cameraman</td>
<td>36.74</td>
<td>34.72</td>
<td>33.01</td>
<td>31.72</td>
<td>29.43</td>
</tr>
</tbody>
</table>

TABLE II: Comparison of our proposed DSR based technique with two existing techniques

<table>
<thead>
<tr>
<th>Techniques</th>
<th>Lena</th>
<th>Barbara</th>
<th>Cameraman</th>
</tr>
</thead>
<tbody>
<tr>
<td>QF-12</td>
<td>33.80</td>
<td>29.055</td>
<td>30.484</td>
</tr>
<tr>
<td>QF-14</td>
<td>30.41</td>
<td>28.801</td>
<td>27.701</td>
</tr>
<tr>
<td>QF-16</td>
<td>30.41</td>
<td>28.801</td>
<td>27.701</td>
</tr>
<tr>
<td>Proposed</td>
<td>30.41</td>
<td>28.801</td>
<td>27.701</td>
</tr>
</tbody>
</table>

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Atomistic model of reptation at polymer interfaces

D.G. Luchinsky*† H. Hafiychuk†, M. Barabash∗ V. Hafiychuk†, T. Ozawa‡ and K. R. Wheeler§, P.V.E. McClintock∗

∗Department of Physics Lancaster University, Lancaster, UK, LA1 4YB
†SGT Inc., ARC, Moffett Field, California, 94035, USA
‡Materials Science Section, Engineering Technology Division, JSOL Corporation, JP
§ARC, Moffett Field, California, 94035, USA

Email: d.luchinsky@lancaster.ac.uk

Abstract—We study a molecular dynamics model of a polymer-polymer interface for a polyetherimide/polycarbonate blend, including its thermodynamic properties, its chain reptation, and its corresponding welding characteristics. The strength of the sample is analyzed by measuring strain-stress curves in simulations of uni-axial elongation. The work is motivated by potential applications to 3D manufacturing in space.

Index Terms—polymer interfaces, reptation, welding, strength of the interface

I. Introduction

Understanding the properties of polymer-polymer interfaces is a long-standing problem of fundamental and technological importance [1]. In particular, basic properties such as reptation and entanglement determine the welding dynamics and strength in fused deposition modeling [2]. The atomistic structure of polymers substantially influences these properties. However, earlier research was mostly focused either on bulk properties [3] or on coarse-grained models of interfaces [4].

Here we report the development and analysis of a fully atomistic model of a polymer-polymer interface in a blend of polyetherimide (PEI) polycarbonate (PC) representing material UL TEM 9085, which us currently widely used in aerospace applications. We use molecular dynamics (MD) simulations to investigate the diffusion of polymer chains at the interface and thus estimate the initial time scales for reptation and welding. We assess the thermal and mechanical properties of the blend in the presence of a planar interface and characterise the strength of the interface as a function of the welding time, using strain-stress measurements in uni-axial elongation simulations.

The paper is organized as follows. First, we introduce the model and discuss its validation in Sec. II. Next, in Sec. III, we analyse the diffusion of chains at the interface. The strength of the interface as a function of welding time is discussed in Sec. IV. Finally, conclusions are drawn and presented in Sec. V.

II. Model

We investigate blend composition consisting of 80% PEI and 20% PC, which corresponds to the optimal miscibility of these polymers and is commonly used in ULTEM 9085. Amorphous cells of these blends were prepared and relaxed at temperature $T_0 = 600$ K (or 650K) using the software package J-OCTA [5] and the following procedure: (i) equilibration in the microcanonical (NVE) ensemble; (ii) compression in the isothermal-isobaric (NPT) ensemble to a pressure $P = 100$ MPa; (iii) additional NVE equilibration; (iv) relaxation in the NPT thermostat; and (v) elimination of the translational velocity in the canonical (NVT) Andersen thermostat. Each step was computed during 100 ps with a time step of 1 fs. The simulations used software from LAMMPS [6] and GROMACS [7] as well as from J-OCTA [5].

A nearly atomically flat boundary on each cell was prepared using the Lennard-Jones potential, and the cells were then brought together across this almost flat interface as shown in Fig. 1. The total size of the sample was 41328 atoms, 96 PEI chains and 48 PC chains, with 5 repeating units in each chain. The welding process was simulated during 240 ns at $T_0 = 600$ K and $P_0 = 1$ bar. Fast quenching to 300 K was performed after 60 ns and 240 ns, in 12 steps of 25K and 2 ns, each using the NPT ensemble. Additional thermal cycling between 600 K and 300 K was performed with time steps varying between 12 ns and 25 ns.

The resultant samples at different temperatures were
used to estimate the properties of the PEI/PC blend. The results of the simulations were compared to experimental data available online for pure polyetherimide ULTEM 1000 [8]. Here we provide two examples of such a comparison, as shown in Fig. 2, further details are available in [9]. The first example shows the results of the bulk modulus ($B$) simulations in comparison with experimental data. The MD results were obtained using the fluctuational formula $B = V \langle \sigma_T^2 \rangle / k_B T$, where $V$ is the volume, $\sigma_T^2$ is its variance, and $k_B$ is the Boltzmann constant. The experimental data for $B$ were estimated using measurements of Young’s modulus $E$ and the equation $B = E / 3(1 - 2\nu)$, taking a nominal value of Poisson’s ratio $\nu = 0.36$. The comparison is only available for temperatures below the glass transition temperature $T_g \sim 475$ K. The MD results are in reasonable agreement with the experimental data.

In the second example, the density $\rho$ from the MD simulations is compared to the experimental data as a function of temperature, as shown in the inset of Fig. 2. As obtained from the MD simulations, $\rho$ is $\sim 15\%$ less than in the experiments, which is within acceptable accuracy for MD predictions [10]. In addition, as shown in the figure, the intersection of the straight line sections of $\rho(T)$ below and above the glass transition allows one to estimate $T_g$. The values of $T_g$ obtained in MD and experiment are 451 K and 485 K (ULTEM 1000) respectively. Note, that the experimental value of $T_g$ provided by StratSys for ULTEM 9085 [11] is 459 K, which is closer to our estimate.

Similar results were obtained for the MD estimates of the thermal expansion coefficient [9]. Reasonable agreement with experimental data was found for temperatures below $T_g$. Above $T_g$, experimental data were not available. Overall, we can conclude that MD simulations of the PEI/PC blends provide results consistent with experimental observations. We now consider the results of the welding analysis in these samples.

III. Interface diffusion

The diffusion at an interface is the key process that defines strength of the manufacturing parts [1]. To provide atomistic insight into diffusion dynamics we analyzed reptation in two samples. The first sample was as described in the Sec. II. The second sample was larger and had longer chains: 130 PEI chains each 6 monomer units long and 50 PC chains each 8 monomer long. The total size of this sample was 67912 atoms. The sample was equilibrated using method described in Sec. II at temperature 650 K. The reptation dynamics was qualitatively similar in both samples. Here we describe some results of the analysis performed for the larger sample.

An example of the analysis of chain reptation at the interface for this sample is shown in Fig. 3 for welding at $T = 650$ K.

The motion of the semiflexible chains on a time scale of our simulations $\sim 300$ ns occurs via reptation when the chain remains within a “tube” determined by the intersections with neighboring chains, and the end of the chain moves slowly across the interface in a random fashion. A snapshot of this motion is shown in the inset of Fig. 3 after $\sim 40$ ns of welding. The interface is shown by the transparent blue plane, the atomic structure of the chain is shown by thin gray lines, and the core of the chain is shown by the blue solid line. The gray dots show the locations of the chains that constrain motion of a given chain and shape its reptation “tube” within 10 Å radius of the core. The coarse-grained sub-units of the chain crossing the interface are shown by red dots. Note that initially all sub-units of this chain were on one side of the sample.

The simulations reveal a few time scales of reptation during 300 ns of welding. The first time scale of $\sim 50$ ps corresponds to the so-called “wetting” process when the two surfaces quickly come close to each other [12] attracted by electrostatic and van der Waals forces. Next, we observed a fast diffusion of polymer chains on a time scale of $\sim 20$ ns while the density of atoms at the interface was rising rapidly towards its bulk value. We attribute fast diffusion to the initial existence of relatively “free” chain ends and “vacancies” on the both sides of the interface. Finally, we observed a slow diffusion of the chains across the interface into the bulk of the sample on the other side, cf. [13].

The profiles of the atomic densities on the two sides of the interface, corresponding to these time scales, are shown in Fig. 3. It can be seen from the figure that the two samples are initially well separated, with the half-width of the gap at half bulk density $\sim 5$ Å. After $\sim 0.5$ ns the density at the interface has nearly reached its bulk value. After another 40 ns the density profiles stay almost
the same with only the tails of the distributions extending to the other side by nearly 20 Å.

This extension of the distributions tails has a profound effect on the strength of the interface. Indeed, according to Wool [14] the full strength is obtained when the two polymers filaments are interdiffused at the distance equal to 81% of the radius of gyration ($R_g$). Note, that for PEI/PC blends the interfacial strength is mainly determined by interdiffusion of the PEI chains. The radius of gyration in $z$-direction estimated in our simulations for PEI chain was $R_{gz} \sim 18$ Å and for PC chain $R_{gz} \sim 7.5$ Å. This corresponds to the maximum extension of PEI chains $\sim 50$ Å and for PC chains $\sim 25$ Å.

Therefore, we expect complete healing of the interface when maximum extension of the chains in $z$-direction is $\sim 40$ Å. We observe, however, a substantial slowing down of the tails extension beyond 20 Å. This slowing down is attributed to the structure of the blend samples that has 20% of PC and 80% of PEI chains. PC chains being smaller and more flexible diffuse much faster towards the interface while it takes more time to equilibrate for stiffer and longer PEI chains.

The two different time scales for interdiffusion of PEI and PC chains were directly observed in simulations by following in time the distributions of the center of masses (CMs) of individual chains. It was found that the CMs distribution of PC chains bridges initial gap at the interface and becomes nearly uniform at the time scale of the order of 200 ns. The CMs distribution of the PEI chains tends towards equilibrium but remains nonuniform with the gap at the interface up to 300 ns.

We conclude that in our simulations the strength of the interface is approaching the bulk value as a function of time but the curing process remains incomplete. The interface strength on the time scale of simulations will be mainly determined by the interdiffusion of PC chains and will be lower than the one expected for polyetherimide.

We now provide the details of the MD estimations of the sample strength as a function of welding time.

IV. Interface strength

To test the interface strength as a function of welding time, we performed uni-axial elongation of the samples at constant rate in the $z$-axis direction using the scenario developed by J-OCTA [5]. During deformation, the sample eventually breaks at the interface. The breaking process involves stretching the chains and pulling them out of the bulk of the sample. In addition, the separation process requires the “wetting” potential barrier related to the non-bonding interaction energy between the two samples to be overcome. The thinner the welded layer, the smaller is the strain at which the breakup process is expected to be observed.

The measurements of the elongation have to be performed by resolving the breaking dynamics. Here we demonstrate the results of preliminary analysis obtained for elongation rate 50 m/s with time step 0.5 fs using VSOP solver developed by J-OCTA [5].

The strain-stress curves obtained in the MD simulations for three different welding times are compared with the experimental curve in Fig. 4. The curves deviate from the experimental data due to the high elongation rate used in the MD simulations. However, it can be seen from the figure that both Young’s modulus and the Yield strength increase as the thickness of welded layer increasing. We note that the calculations were performed under assumption of nominal Poisson’s ratio $\nu = 0.36$. This assumption becomes increasingly inaccurate as the breaking of the sample is initiated at the interface. For this reason the strain-stresses curves obtained in MD are shown by the dashed lines for the values of strain larger than 0.1.

![Fig. 4. MD simulations of the strain-stress curves as a function of welding time: (i) 60 ns (diamonds); (ii) 240 ns squares; and (iii) after additional welding during temperature cycling (1200 ns). The experimental data obtained by ULTEM 1000 [8] are shown by teal shaded circles.](image-url)
We also note that we have not observed the expected decrease of the Yield strength (see discussion in the end of the previous section). We attribute this discrepancy to the large elongation rate used in MD simulations.

The value of the Young’s modulus obtained in MD simulations 

\[ E \sim 2 \text{ GPa} \]

is slightly smaller than the value 2 - 2.5 GPa estimated using open data source [11]. We note, however, that the obtained value of Young’s modulus is in good agreement with the data reported for ULTEM 1000 [8] shown in the figure by open teal circles.

V. Conclusions

In summary, we have developed a fully atomistic molecular dynamics model of the polyetherimide/polycarbonate amorphous polymer blends. Two cells were brought together to form a sample with an atomically nearly flat interface and were allowed to equilibrate for 240 ns. The sample was quickly quenched to 300 K after 60 and 240 ns of welding at the interface. Additional thermal cycling between 300 and 600 K was performed after quenching at 240 ns.

The model was validated by comparison of the MD predictions with experimental data for the density, glass transition temperature, bulk modulus, and thermal expansion coefficient below \( T_g \) (where experimental data were available).

The model was used to analyze diffusion of the polymer chains at the interface during the welding process and to estimate the interface strength as a function of welding time. It was shown that, after the initial “wetting” process, the diffusion take place via snakelike motion of the polymer chains. Two characteristic time scales were observed during the first 300 ns of welding: (i) fast diffusion (\( t \lesssim 20 \text{ ns} \)) when chain’s ends can diffuse by filling in vacancies on both sides of the interface; and (ii) slow diffusion (\( t \lesssim 300 \text{ ns} \)) when chain’s ends slowly diffuse through the bulk material on both sides of the interface.

The analysis of the strain-stress curves as a function of time reveals the effect of the thickness of the welded layer on the strength of the interface. It was shown that both Young’s modulus and the Yield strength increase as the thickness of welded layer increasing. The deviations of the strain-stress curves obtained in MD simulations from the experimental data attributed to the large elongation rate and relatively large time step used in the simulations. In the future work the effect of both elongation rate and the time step will be verified.

Overall, the good agreement of the developed model with experimental data paves the way to semi-quantitative predictions of the interface properties of the polymer blends considered.

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References

Determination of activation energies of oxygen ion diffusion in memristor systems from the flicker noise spectrum

Alexey V. Klyuev  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
klyuev@rf.unn.ru

Arkady V. Yakimov  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
yakimov@rf.unn.ru

Oleg N. Gorshkov  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
gorshkov@nifti.unn.ru

Dmitry O. Filatov  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
dmitry_filatov@inbox.ru

Dmitry A. Antonov  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
antonov@phys.unn.ru

Ivan N. Antonov  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
ivani@nifti.unn.ru

Yulia I. Anikina  
Stochastic Multistable Systems Laboratory  
National Research Lobachevsky State University of Nizhni Novgorod  
Nizhny Novgorod, Russia  
 germanovich.julia@mail.ru

Abstract—Noise in memristive systems based on the Yttria stabilized Zirconia (YSZ) thin films is investigated. Measurements are performed with the use of setup Omicron® UHV AFM/STM LF1 at temperature $T = 300$ K. The base pressure of remaining gases in the chamber of Atom Force Microscope (AFM) was at $10^{-18}$ Torr. Current through the cantilever of AFM is analyzed. The pdf and spectra of the noise in this current are treated for two states of conducting filament: “OFF” (high resistance), and “ON” (low resistance). Voltage applied to the cantilever is $V_c = +3$ V in both states. Noise in state “OFF” is caused by the inner noise of experimental setup only. In state “ON” the noise is considerably higher. Its spectrum has the flicker type with exponent $\gamma = 1.3$ in the whole frequency range analyzed. That allows us to determine the range of activation energies of oxygen vacancies diffusing within the filament, $E \in [0.52; 0.68]$ eV. This result coincides qualitatively with result obtained from measurements of electro-physical characteristics of the samples with macroscopic contacts at temperatures 300–500 K.

Keywords—Memristive systems; flicker noise; diffusion; non-destructing analysis

I. INTRODUCTION

Memristive systems are of increasing interest in various areas of science and technology ranging from digital to analog electronics, biologically inspired circuits and learning [1]. These systems have shown a great potential in realizing artificial synapses efficiently for neuromorphic computing [2]. There are a lot of investigations of such systems.

Reproducible bipolar resistive switching has been studied in SiO$_x$ based thin–film structures for the development of nonvolatile memory and memristive systems for future electronics [3], see also [4].

Investigations of the role of noise in resistive switching are of special interest. These investigations cover two problems. The first one deals with the effect of external noise on the switching [5], [6]. The second problem is concerned with the non-destructing analysis of memristive systems, see, e.g. [7], [8].

The aim of this paper is to show that noise measurements may be applied for the determination of activation energies of oxygen vacancies diffusion in memristive systems. That is an additional method for the non-destroying analysis of solid state systems.

Noise in the Yttria Stabilized Zirconia (YSZ) thin films is investigated. The same samples and experimental setup were used as in [9], [10]. The structure and operation principles of such systems are described in [11]. Results of previous investigations on the determination of activation energies of oxygen vacancies diffusion in memristive systems are presented in [12], where electro-physical characteristics of the samples were investigated in the temperature range 300–500 K.

The model of flicker noise in the current through the sample, based on the model by Van der Ziel [13] (see also [14]), is used here. That allows us to estimate the range of activation energies of oxygen vacancies diffusion through the conducting filament in considered memristive system.

This work was supported by the Government of the Russian Federation through Agreement No. 074-02-2018-330 (2). The measurements were carried out using the shared research facility of Research and Educational Center for Physics of Solid State Nanostructures at National Research Lobachevsky State University of Nizhni Novgorod.
II. EXPERIMENTAL SETUP

Measurements were performed with the use of setup Omicron® UHV AFM/STM LF1 at temperature $T = 300$ K. The base pressure of remaining gases in the chamber of Atom Force Microscope (AFM) is at $10^{-10}$ Torr. For more details see [9].

Input part of experimental setup is shown in Fig. 1.

Here the probe (cantilever) is shown schematically. Its tip is in contact with the sample (memristor). The last one may contain the conducting filament. Voltage $V_f$ is applied to the sample through the tip of cantilever. This voltage produces current $I = I(t)$ through the tip.

The sample is the film ZrO$_2$ (12% molar Y), thickness 4 nm (YSZ). This film is deposited on Au sublayer with the thickness 10 nm. Further on Ti sublayer is placed (50 nm) on $n^-$–Si(001) substrate. The Si diamond coated probes NT-MDT® DCP-11 are used. The radius of curvature of the tip is about 70 nm. The probe is brought to the surface of the sample at a distance of about 4 μm.

Current preamplifier of AFM has the range 1 pA–5 nA; frequency band 0–30 kHz; gain of setup noise in state “0” are used. In state “1” the recorded files contain 614 400 digitized samples, which characterize only the preamplifier noise.

Digitized samples $N(t)$ in states “0” and “1” are distributed within 10 ADC bits. Effects of incomplete use of ADC are taken into account [16], [19].

It is found that the noise of AFM operating in states “0” and “1” does not depend on the presence of contact between the probe and the sample.

III. RESULTS OF MEASUREMENTS

Measurements are started from investigation of setup inner noise. The probe is in contact with the sample, $V_f = 0$. Conducting filament is absent (not formed). That is state “1” (in contact). After that the filament is created by applying of voltage $V_f = +6$ V to the sample. Then noise is measured in two states: “OFF” (high resistance), and “ON” (low resistance). Voltage applied to the sample is $V_f = +3$ V in both states. Measurements are finished by additional investigation of the inner noise. The probe is removed from the sample at a distance of about 4 μm. That is state “0” (no contact).

A. Inner noise of the Setup

The analysis shows that the recorded processes represent noise, which can be considered stationary.

The noise in state “1”, in addition to the converter noise, may contain the noise of the probe, and of the sample; recorded files contain 1 843 200 digitized samples. In state “0” the recorded files contain 614 400 digitized samples, which characterize only the preamplifier noise.

Waveforms show the presence of external interference, having almost harmonic character. The pdf is rather close to the Gauss law. The statistical error of obtained estimates is 0.3–0.4 percent in the area of maximum values and reaches 71 percent in the area of minimum values of pdf. The statistical error of spectrum measurement in state “0” is about 4 percent; in state “1” – about 2.4 percent. The spectrum in both states is highly heterogeneous.

The relative statistical error $δS_{ON}(f)$ of spectrum (4) is determined from the following relation:

$$
S_{ON}(f) = S_d(f) - S_0(f).
$$

Relative statistical errors $δS_1$ and $δS_0$ characterizing the accuracy of measurement of the corresponding spectra are determined by the number of processed spectrograms and do
not depend on the analyzing frequency \( f \). Typical error \( \delta S_{ON}(f) \) is about 2 percent; in the frequency range, where quasi-harmonic interference is dominated, it reaches 10–60 percent.

The result of subtraction (4) is shown in Fig. 2. The polyline “ON” is spectrum \( S_{ON}(f) \) of the sample noise. Straight line – approximating spectrum \( S_{fit}(f) \). The subtraction of spectra led to a satisfactory result. The resulting spectrum has a pronounced flicker character. Manual approximation of this spectrum gives:

\[
S_{fit}(f) = A_0 / f^\gamma \, [\text{pA}^2/\text{Hz}].
\]

(6)

Here \( A_0 = 2.3 \times 10^3 \); exponent of this spectrum is \( \gamma = 1.3 \).

[Fig. 2. Result \( S_{fit}(f) \) of subtracting the spectrum \( S_{ON}(f) \) from the total spectrum \( S(f) \) (polyline “ON”). Approximating spectrum \( S_{fit}(f) \) (straight line “Fit”). Bottom line “0” – spectrum \( S(f) \) of setup noise, state “0”.

IV. ANALYSIS OF ACTIVATION ENERGIES

Diffusion of oxygen vacancies along the conducting filament is considered here as the origin of flicker noise in the sample. Model [13] by Van der Ziel is used as the basis, see also [14].

An elementary jump of the vacancy in the filament requires activation energy \( E \). The mean frequency \( f_c \) of these jumps is determined as:

\[
f_c = f_T \exp \left[-E / (kT) \right].
\]

Here \( f_T = 10^{13} \) Hz is the mean frequency of the lattice thermal vibrations.

These jumps yield random telegraph noise (RTN) in the conductivity \( G = G(t) \) of the filament. The spectrum of this noise \( S_{RTN}(f) \) has the Lorentzian type:

\[
S_{RTN}(f) = A_{RTN} \gamma f_c / (f_c^2 + f^2).
\]

(8)

Here \( A_{RTN} \) is the parameter determined by the variance of RTN. Mean frequency (7) has a sense of the corner frequency for spectrum (8).

Vacancies inside the filament have different activation energies \( E \) characterized by probability density function (pdf) \( W_E(E) \). Thus, for corner frequencies we have pdf \( W_c(f_c) \). Both functions are connected by (7).

Total spectrum \( S(f) \) in the conductivity noise is determined by diffusion jumps of all vacancies:

\[
S(f) = N_V S_{RTN}(f) f_c W_c(f_c) df_c.
\]

(9)

Here \( N_V \) is number of vacancies diffusing in the filament. The integral is calculated over the full range of corner frequencies \( f_c \).

Our aim is modeling of spectrum (9) with \( \gamma = 1 \). This is possible if pdf \( W_c(f_c) \) of corner frequencies has the same kind, \( W_c(f_c) \\approx f_c^{-\gamma} \). In common case this distribution takes place in the restricted (but wide enough) limits \([f_i; f_u]\). For the simplicity the finite pdf is used here:

\[
W_c(f_c) = B_c / f_c^\gamma; f_c \in [f_i; f_u].
\]

(10)

If \( \gamma = 1 \) the normalization factor is: \( B_c = 1/ [\ln(f_u/f_i)] \), see [13].

High and low frequencies are determined, in accordance with (7), by limits of activation energies, \( E \in[E_1; E_2] \):}

\[
E_i = kT \ln(f_i/f_0) \quad ; \quad E_s = kT \ln(f_s/f_0).
\]

(11)

The pdf for activation energies is found from (10):

\[
W_E(E) = B_E \exp(\gamma-1)E/kT) \quad \in [E_1; E_2].
\]

(12)

Here \( B_E \) is the normalization factor.

Thus, we have:

\[
S_c(f) = B_E f_c / f^\gamma; B_E = \pi N_V A_{RTN} B_c \sin((\gamma-1)\pi/2) \]

\[
\times \sin(2\gamma\pi) \quad \text{[pA}^2/\text{Hz}] \quad \text{[13].}
\]

Noise in the conductivity \( G(t) \) of the filament is manifested as the noise in current \( I(t) \) with spectrum \( S(f) \).

Relation between these spectra is as follows:

\[
S(f) = V_g^2 S_c(f).
\]

(14)

Thus, knowing spectrum \( S(f) \) we can determine some statistical characteristics of vacancies diffusing within the conducting filament.

The model described above presents no information about low \( f_i \) and high \( f_u \) frequencies of the conductivity noise spectrum \( S(f) \). That is why we use here limits of our experimental data: \( f_i = 15.6 \text{ Hz}; f_u = 7.98 \text{ kHz} \).

[Fig. 3. PdG \( W_E(E) \) obtained from spectrum \( S(f) \). Uniform distribution by van der Ziel [13] is shown as well.

Consider spectrum (6), which is the fit of spectrum \( S(f) \) at \( V_g = +3 \text{ V} \), state “ON”. Using in (11) \( f_t = 10^{13} \) Hz, we can find limits of activation energies, \( E \in [0.52; 0.68] \text{ eV} \). As far as \( \gamma > 1 \), spectrum (6) decreases faster than \( 1/f \). This means,
following to (12), in difference with model [13], that pdf of activation energies is the increasing function, see Fig. 3.

V. CONCLUSIONS

Noise in memristive systems based on the Yttria stabilized Zirconia (YSZ) thin films is investigated. Measurements were performed with the use of setup Omicron® UHV AFM/STM LF1 at temperature \( T = 300 \) K. The base pressure of remaining gases in the chamber of Atom Force Microscope (AFM) was at \( 10^{-10} \) Torr.

Current \( I(t) \) through the tip of AFM probe is analyzed. The pdf, and spectra of the noise in this current are treated. Measurements are made for two states of the conducting filament: “OFF” (high resistance), and “ON” (low resistance). Voltage applied to the cantilever is \( V_f = +3 \) V in both states.

In state “OFF” the only noise of setup is pronounced. In state “ON” the noise is considerably higher. Its spectrum has the flicker type with exponent \( \gamma = 1.3 \) in the whole frequency range analyzed.

That allows us to determine the range of activation energies of oxygen vacancies diffusing within the conducting filament, \( E = [0.52; 0.68] \) eV. This result coincides qualitatively with result [12], \( E = 0.53–0.56 \) eV, obtained from measurements of electro–physical characteristics of the samples with macroscopic contacts at temperatures 300–500 K.

It is worth noting that the analysis presented here cannot reveal the spatial distribution of \( E \) along the filament. The values of \( E \) are averaged over the whole filament length. It is reasonable to assume that these values of \( E \) at the interfaces of the filament with the electrodes may differ from those inside the filaments. However, as the filament interfaces represent a minor part of the total filament length, the contribution of these ones into the averaged value of \( E \) can be neglected.

Thus, noise measurements may be applied for the determination of activation energies of oxygen vacancies diffusion in memristive systems. That is an additional method for the non-destructing analysis of real-world memristors on the basis of circuit theoretic model predictions,” IEEE Circuits and Systems Magazine. Second quarter, pp. 48–76, 2018. DOI: 10.1109/MCAS.2018.2821760.

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Coherent and Conventional Gravidynamic Quantum 1/f Effects

Peter H. Handel

Department of Physics and Astronomy and Center for Nanoscience, University of Missouri - St. Louis USA, handel@umsl.edu

Abstract—The connection between conventional and coherent quantum gravidynamic (QGD) Q1/fE is evaluated by comparing terms in the Hamiltonian. It allows the calculation of QGD quantum 1/f noise from its components, the conventional and coherent Q1/fE. The weight of these components are 1/(1+s") and s"/(1+sa). Here s"=N'r, where r, is the Schwarzschild radius of the particles of mass m in a beam, stream, or jet. N' is the number of particles per unit length of the beam.

Keywords—Fundamental 1/f noise, quantum 1/f noise, conventional quantum 1/f effect, coherent quantum 1/f effect, gravitonic noise, connection between quantum 1/f effects

I. INTRODUCTION

The quantum theory [1]-[9] of fundamental 1/f noise is a new aspect of quantum mechanics, introduced in 1975 as an infrared divergence phenomenon. It is also a decoherence phenomenon, present in many forms. The QED form describes the fundamental 1/f noise in the materials, devices and systems of electro-physics, electronics, microelectronics, nanotechnology, sensors and phase noise in HF or UHF devices and systems, or high stability frequency standards. The quantum gravidynamic (QGD) form replaces photons by gravitons and describes it in macroscopic and mesoscopic streams of matter, with terrestrial and cosmic implications. The quantum lattice-dynamic (QLD) form describes it in electric currents in ferroelectric materials like BaSr(TiO3)2 or bulk GaN, with piezo-phonons replacing the photons as infrquaanta. The Fermi-sphere quantum 1/f noise, present, e.g. in contact noise at a metallic surface, has electron-hole pairs at the Fermi sur-face as infraquaanta. In general, the quantum 1/f noise is always present when a current of any nature has infrared-divergent (IRD) coupling to a system of massless, environment-forming, infra-quaanta. The quantum 1/f noise (Q1/fN) contains in all its forms both, the conventional and coherent Quantum 1/f Effects (Q1/fE).

The conventional Q1/fE (derived for QGD below) is a property of the physical cross sections σ and process rates Γ of quantum mechanics (QM), a new aspect of QM. In QED, e.g., the well-known infrared catastrophe caused by the IRD gives the number of emitted infra-photons as v=(2α/3π)(Δν/c)², and also gives the spectral density of fractional fluctuations in this “physical cross section or process rate” of the process considered, as Sν/σ=2ν/νf. N is the number of particles in the beam section (or length portion) used in order to define the notion of current. The resulting QED conventional fluctuations in current are thus

\[ S_{\nu/j} = (1/Nf)\left(\frac{2\alpha}{\pi}\right)\left(\frac{\Delta \nu}{c}\right)^2. \]  

The derivation assumes that the phases of the bremsstrahlung energy loss components were randomized by decoherence. Here α=e²/hc, c=1/137 is the fine structure constant, Δν/c the vector velocity change (in units of the speed of light) of the electrons in the (e.g., scattering or tunneling, etc.) process considered, and N the number of carriers defining the current whose fluctuations are considered.

The coherent Q1/fE is a property of any current, caused by the coherent state of the (photonic, gravitonic, etc.) field of a physical particle. This is not an energy eigenstate, therefore a nonstationary state, and the ensuing fractional current fluctuations have a universal spectrum

\[ S_{\nu/j} = 2\alpha/\pi Nf \text{ in QED, and} \]

\[ S_{\nu/j} = 2Gm^2/\pi Nf\chi, \text{ in QGD.} \]  

In the usual (QED) case, the resulting Q1/fN is roughly evaluated simply, according to the weights 1/(1+s) and s/(1+s) of the corresponding terms in the Hamiltonian. Combining the conventional and coherent contributions with these weights of 1/(1+s) and s/(1+s), where s is the coherence parameter, is in
fact a very rough and heuristic approximation. However, experiment showed that it worked very well in practice, in the QED case, as we see from early publications of F.N. Hooge and Aldert van der Ziel [8], as well as from many other measurements on larger samples where s is not very small. This coherence parameter is \( s = 2 r_s N' \), i.e., the number of carriers along the current, contained in a slice of thickness \( r_s = e^2 / m c^2 \), known as the classical radius of the electron, perpendicular to the current direction. Here \( N' \) is the number of carriers per unit length and \( m \) the mass of the electron or any current carrier. The QED-Q1/F \( \text{Im} \) is thus

\[
S_{\text{QGD-Q1/F}} = \frac{1}{\text{Im}} \left[ \frac{(4 \alpha / 3 \pi)(\Delta \nu / c)^2 + 2 \alpha c / \pi} \right] \tag{3}
\]

The QGD-Q1/F \( \text{Im} \) is

\[
S_{\text{QGD-Q1/F}} = \frac{1}{\text{Im}} (1 + s^\prime) \left[ (8 G N' e^2 h / c) \mu v^4 \sin^2 \theta + 2 s^\prime G M^2 / \pi c \right], \tag{4}
\]

where \( \theta \) is the scattering angle of the particles of mass \( M \) in the center-of-mass reference system, with relative velocity \( v \) of the particles, and their reduced mass is \( \mu \). Here \( s^\prime = 2 N G M / c^2 = \frac{N' r_c}{r_s} \) evaluated below is the coherence parameter introduced by us like the parameter \( s \) for electromagnetic Q1/\( \text{Im} \) and also like \( s' \) for the piezoelectric Q1/\( \text{Im} \). \( N' \) is again the number of particles per unit length and \( r_s = 2 G M / c^2 \) is here their Schwarzschild radius, with \( G \) the constant of universal gravitation.

II. EVALUATION OF THE QGD COHERENCE PARAMETER \( s^\prime \)

The evaluation of \( s^\prime \) is done for the first time in this paper. The derivation proceeds similar to the earlier derivations of \( s \) and \( s' \). The \( s^\prime \) parameter is defined as the ratio between the "gravitational field's coherent collective contribution to the kinetic energy" of the stream of electrically neutral particles (or bodies) of average mass \( m \), on one hand, and their bare-particle kinetic energy \( m v^2 / 2 \) on the other hand. The former scales with the square number of particles per unit length, like the magnetic field in QED. The latter scales as the number of particles. This ratio is similar to the ratio in the QED case

\[
s_e = \frac{[1 / (8 \pi / r)] / [N' m v^2 / 2]} = \frac{[4 e^2 / 8 \pi c^2] / [2 \pi r^2 / r^2]} / [N' m v^2 / 2] = \frac{[e^2 N' v^2 \sin(\theta / R_o)] / [2 \pi r^2 / r^2] \text{Im}} = 2 N' r_c \ln(r / R_o) \approx 2 N' r_c \tag{5}
\]

Here \( J \) is the electric current, \( B \) its magnetic field, and \( R_o \) is the radius of the electric circuit, a cut-off for the logarithmic divergence. The logarithm is of the order unity and set 1.

In general relativity there is also an analog \( v x g / c \) of the magnetic field vector \( B \), as the field generated from the acceleration of gravity \( g \) by motion of the particle with velocity \( v \). We obtain for the QGD case

\[
s^\prime = s_e = \frac{[1 / (8 \pi / r)] / [N' m v^2 / 2]} = \frac{[4 G N' N_r^2 / (N' m v^2 / 2)] / [2 \pi r^2 / r^2]} / [N' m v^2 / 2] = 2 (G N' m / c^2) \ln(r / R_o) \approx 2 G N' m / c^2 = N' r_c \tag{6}
\]

Here we introduced the Schwarzschild radius \( r_c \) of the particles of mass \( m \) in the beam, stream, or jet.

This completes our evaluation of \( s^\prime \), also denoted by \( s_{\gamma} \) and is very similar with the evaluations of \( s \), and \( s' \).

The QGD Q1/F \( \text{Im} \) can be easily verified in macroscopic streams of matter in any state of aggregation, both on earth and in cosmos.

III. DERIVATION OF THE CONVENTIONAL QGD Q1/F

The derivation of the conventional QGD Q1/F is similar to the derivation [1]-[4] of the conventional Q1/F \( \text{Im} \) in QED. However, in the non-relativistic limit, this time the infrared exponent \( \Box A \) is replaced by the QGD Q1/F infrared exponent

\[
\beta = \beta_{\text{con}} = (8 G / 5 \pi c^2 h) \mu^2 v^4 \sin^2 \theta, \tag{7}
\]

where \( \theta \) is the scattering angle of the particles in the center-of-mass reference system, with \( v \) being the relative velocity of the particles that scatter on each other, and their reduced mass being \( \mu \). \( G \) is the constant of universal gravitation. The rate \( \Delta_{\text{a}} \) of an arbitrary interaction or transition from state \( \alpha \) to \( \beta \), which changes the momentum of a beam of particles or of a stream of matter, is subject to gravitonic infrared radiative corrections. The corrections due to virtual gravitons of small energy \( \lambda < c \ll \Lambda \) that boomerang back, yield a rate [1]

\[
\Delta_{\alpha}(\lambda, \Lambda) = (\lambda / \Lambda)^{\beta \Delta_{\text{a}}}. \tag{8}
\]
Here $\lambda$ is an arbitrarily small cutoff, used to display the infrared divergences, while $\Lambda$ is an energy chosen to be about an order of magnitude below the characteristic center of mass energy of the interaction considered. Note that all virtual photons must be included, so $\lambda$ is actually zero and the rate computed without allowing any energy loss caused by bremsstrahlung of real photons is zero. Eq. (8) was obtained by summing up the series of virtual graviton diagrams defining the matrix element of the process, into an exponential function of a logarithmically divergent integral. Eq (8) gives the squared modulus of this matrix element.

Including also the real gravitons, we notice that the series of diagrams exponentiates again, but only at the level of the process rate [1]. Neglecting the influence of the thermal radiation background, we obtain for the (e.g. scattering) interaction process with gravitonic bremsstrahlung energy losses below a certain limit $\varepsilon_i$, the rate

$$
\Gamma_{\rho\omega}(<\varepsilon_i) = (\varepsilon_i/\lambda)^{\beta} \Gamma_{\rho\omega} = (\varepsilon_i/\Lambda)^{\beta}\Gamma_{\rho\omega}^{\infty} = (\varepsilon_i/\Lambda)^{\beta}\Gamma_{\rho\omega}^{\infty}.
$$

(9)

Here the function $b(\beta) = 1 - \pi\beta^2/12 + ... \approx 1$, since $\beta^2 \ll 1$. The derivation of an equation replacing Eq. (9) is done in [5] in the presence of a thermal radiation background. However, the result is a convolution involving Eq. (9) as one of its factors, which indicates that the spontaneous and induced transitions are statistically independent. This has been used to prove [6]-[7] that the quantum 1/f result remains unaffected in the presence of a thermal radiation background of infraquanta. The background only contributes a statistically independent white noise term, just as it happens in the electromagnetic (QED) case. Since we are interested in the 1/f noise part, this consideration justifies the neglect of the thermal-radiation-background-induced stimulated emission and absorption processes in the final result obtained below.

In Eq. (9) we can use the identity

$$
(\varepsilon_1/\Lambda)^{\beta} = (\varepsilon_0/\Lambda)^{\beta} \left[ 1 + \beta \int_{\varepsilon_0}^{\varepsilon_1} (\varepsilon/\varepsilon_0)^{\beta} \, d\varepsilon/\varepsilon \right].
$$

(10)

We interpret $\varepsilon_c < \varepsilon_1$ as the graviton detection threshold, which is not less than the reciprocal noise measurement time times Planck’s constant. Multiplied by $\Gamma_{\rho\omega}^{\infty}$, the first term in rectangular brackets yields then the elastic scattering rate, while the second term gives the bremsstrahlung scattering rate with energy loss up to $\varepsilon_1$. Corresponding scattering matrix elements could be defined as the square root of these rates. Restoring also the phases, say $\phi$ and $\phi+\gamma(\varepsilon)$, we obtain the scattering amplitude $a=(\varepsilon_c/\Lambda)^{\beta/2}(\beta)\Gamma_{\rho\omega}^{\infty}\exp(i\phi)$, & $a_p\exp(\gamma(\varepsilon))d\varepsilon/\varepsilon^{1/2}$,

$$
\rho_c = (\beta)^{1/2}(\varepsilon_c)^{\beta/2}.
$$

(12)

Here $\gamma(\varepsilon)$ is a random phase of this resulting einselected state, which reflects the uncertainty of the moment of graviton emission and the initial phase of the gravitonic mode of the universe, as well as the interaction with the environment, causing decoherence. Therefore, if the incoming beam of particles or matter is described by a plane wave $\exp[i(\mathbf{p}\mathbf{r})]$ in units with $h^2/c^2 = 1$, the scattered spherical wave will be a stochastic mixture

$$
\psi = a\exp[i(1+\int_{\varepsilon_0}^{\varepsilon_1} \rho_c e^{i\phi}\exp(\gamma(\varepsilon))d\varepsilon/\varepsilon^{1/2})],
$$

(13)

because $\gamma(\varepsilon)$ is a random phase for each $\varepsilon$. Here, considering stationary states, the momentum magnitude loss $q = M(c/k)/M(c/K)$ is necessary for energy conservation in the Bremsstrahlung process. Thus, $q$ is an equivalent parameter for $ck = e$, with $d\varepsilon/e = dq/q$, and we can also eliminate $e$ completely in favor of $q$ in the integration. The latter has replaced here right away the corresponding summation over $q$, that is originally present.

We obtain for the resulting 2-particle "einselected" state a classically correlated system, a mixture of 2 pure states for each $\varepsilon$,

$$
\Psi = a\exp[i(p_{r_1}+p_{r_2})/2] + \int_{\varepsilon_0}^{\varepsilon_1} \rho_c e^{i\phi}\exp[\gamma(\varepsilon)-i\phi]\, d\varepsilon/\varepsilon^{1/2} + \int_{\varepsilon_0}^{\varepsilon_1} \rho_c \exp[\gamma(\varepsilon)-i\phi]\, d\varepsilon/\varepsilon^{1/2},
$$

(14)

By module squaring and averaging over the phases, we can thus write the autocorrelation function in space or time, as well as in "space and time"$

$$
<\Psi^\dagger \Psi> = 1 + \int_{\varepsilon_0}^{\varepsilon_1} [\rho_c^2 d\varepsilon/e + \int_{\varepsilon_0}^{\varepsilon_1} [\rho_c^2 \cos(q(r_2-r_1))d\varepsilon/e]
$$

$$
= 1 + \int_{\varepsilon_0}^{\varepsilon_1} [\rho_c^2 d\varepsilon/e + \int_{\varepsilon_0}^{\varepsilon_1} [\rho_c^2 \cos(\varepsilon)\cos(\varepsilon)\, d\varepsilon/e]
$$

$$
= 1 + \int_{\varepsilon_0}^{\varepsilon_1} [\rho_c^2 d\varepsilon/e + \int_{\varepsilon_0}^{\varepsilon_1} [\rho_c^2 \cos(\varepsilon)\cos(\varepsilon)\, d\varepsilon/e]
$$

(15)

(16)

(17)

Here $\varepsilon_0$ is the frequency, $\tau$ is the correlation time, and $\theta = t-\tau$ is the parameter describing the propagation of quantum 1/f fluctuations along the beam, or stream of matter, in the outgoing radial direction $r$. The fractional spectral density of conventional gravidynamical quantum 1/f fluctuations in concentration or current $j$ is given by the Wiener
Khitchine theorem as the third term divided by the constant term
\[ \langle (dj/j)^2 \rangle = \frac{2/N}{|\rho \varepsilon|^2 \varepsilon} \]
\[ = \frac{(2/N) \beta(f/\varepsilon_0)}{1 + \int \varepsilon \frac{d\varepsilon}{\varepsilon}}. \]
Here \( f = \omega/2\pi \) is the frequency, and \( N = 2 \) is the number of particles that define the notion of current \( j = v|\psi|^2 \).

IV. DISCUSSION

Eq. (18) gives the spectrum of the conventional gravodynamic quantum 1/f effect. This effect, with
\[ \beta = \beta_{\text{conv}} = \frac{(8G/5\pi^2 h c)^2 v^4 \sin^2 \theta}{(2/N) \beta(f/\varepsilon_0)}, \]
contains Planck's constant in the denominator and is at the angulation line of quantum mechanics and general relativity. This is the most fundamental form of quantum chaos, the quantum manifestation of classical, general relativistic, turbulence. It is a fundamental aspect of quantum mechanics, which can be used to define quantum mechanics as the actual form taken by the process of metrogenesis. Together with the coherent gravodynamic Q1/fE, it determines the total grav-dynamic quantum 1/f noise that is observable. All forms of quantum 1/f noise represent the main part, ontologically the most basic, of fundamental 1/f noise, verified experimentally [8] and important in engineering. In general, epistemologically or conceptually, fun-damental 1/f noise is the result of nonlinearity combined with a special type of homogeneity [9], [10].

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Noise and Dynamics in Diffusive Conductors

Edouard Pinsolle  
*Institut Quantique, Département de Physique*  
Université de Sherbrooke  
Sherbrooke, Qc Canada  
edouard.pinsolle@usherbrooke.ca

Christian Lupien  
*Institut Quantique, Département de Physique*  
Université de Sherbrooke  
Sherbrooke, Qc Canada  
christian.lupien@usherbrooke.ca

Samuel Houle  
*Institut Quantique, Département de Physique*  
Université de Sherbrooke  
Sherbrooke, Qc Canada  
Samuel.Houle@usherbrooke.ca

Bertrand Reulet  
*Institut Quantique, Département de Physique*  
Université de Sherbrooke  
Sherbrooke, Qc Canada  
bertrand.reulet@usherbrooke.ca

Abstract—In this article we present two experiments carried on diffusive metallic wires and aiming at understanding better current fluctuations and the informations we can extract from them. The first experiment studies the non-gaussianity of those fluctuations at low frequency. The second one studies the dynamic response of current fluctuations to an ac exitation. From the frequency dependence of this response function we obtain a direct determination of the inelastic relaxation times.

Diffusive contacts have attracted a lot of interest over the years [1]–[8]. This is probably due to their simplicity making them good candidates for reliable theoretical predictions of their transport properties. In addition, the ease of tuning the interactions at play by simply changing the length of the sample makes the diffusive wire a perfect platform for experimentalists. It allows the study of electron transport in presence of interactions or the test of new experimental probes in different regimes. In this paper we will demonstrate with two experiments the versatility of such a system. In a first one we will be interested in a short sample for which, at low temperature, interactions are negligible and the transport is elastic and diffusive. It has been predicted that current fluctuations in this elastic regime are non-gaussian and exhibit a third moment \(\delta I^3\). We will present one of the rare experiments performed on high order moment of current fluctuations. In a second experiment we will demonstrate a new experimental technique allowing a direct determination of relaxation times of electrons. We will distinguish two regimes. The diffusive regime for short wires where relaxation is dominated by the diffusion time \(\tau_D\) along the sample. And a macroscopic regime at higher temperature and longer wire length, where relaxation is described by electron-phonon interactions allowing the determination of the electron-phonon interaction time \(\tau_{e-\phi}\).

I. NON-GAUSSIAN CURRENT FLUCTUATIONS

At low frequency, the variance of current fluctuations in a diffusive wire has been calculated using many techniques [1]–[3], all providing the same answer for the spectral density of current fluctuations \(S_{\delta I^2}\) measured at temperature \(T\) with a voltage bias \(V\):

\[
S_{I^2} = \frac{1}{15} e^2 I + \frac{12}{5} k_B T \frac{dS_{I^2}}{dV}. \tag{2}
\]

This result differs from that of a tunnel junction \(S_{I^2} = e^2 I\) on two main factors: first it depends on temperature; second it has a much lower Fano factor at high voltage, \(F_3 = e^{-d} dS_{I^2}/dI \approx 1/15\) instead of \(F_3 = 1\) for the tunnel junction. Eq. (2) corresponds to a measurement performed with a noiseless voltage bias and an ammeter, i.e. an apparatus with an input impedance much lower than that of the sample. This situation can be achieved with a high impedance sample, but a typical metallic wire has a low impedance and one has to consider the effects of both the finite impedance of the environment, here a resistance \(R_A\) (the input impedance of the amplifier), and the current noise experienced by the sample, here generated by the amplifier used to detect current fluctuations and described by a noise spectral density \(S_A\). Those environmental effects are very subtle on the third moment of voltage fluctuations. They have been thoroughly studied both theoretically [9], [10] and experimentally [11], [12] and obey:

\[
S_{V^3} = -R_D^3 S_{I^2} + 3 R_D^2 (S_A + S_{I^2}) \frac{dS_{I^2}}{dV}. \tag{3}
\]
where $R_D = RR_A/(R + R_A)$ with $R$ being the sample resistance. As a consequence, a reliable way to characterize all the environmental terms is required to extract the intrinsic third moment of current fluctuations $S_{3I}$.

**Experimental setup.** The sample is a 1 µm long, 10 nm wide, 165 nm thick Aluminum (Al) wire of resistance $R_w = 30.5 \, \Omega$. Its contacts, also made of Al, are much larger (400 µm × 400 µm) and thicker (200 nm) to make sure they behave as good electron reservoirs [13]. An Al tunnel junction of resistance $R_J = 34 \, \Omega$ is used as a reference to calibrate the setup. Both samples have been made by e-beam lithography and the metal has been deposited by double angle evaporation [14]. The experimental setup is presented in Fig. 1. The samples are placed on the 7 mK stage of a dilution refrigerator. They are kept in their normal, non superconducting state with the help of a strong Neodymium permanent magnet. The two samples are connected to a cryogenic microwave switch which allows us to measure either of them without changing anything in the detection circuit. They are dc current biased through the dc port of a bias-tee and ac coupled to a cryogenic microwave amplifier in the range 40 MHz-1 GHz. The use of a cryogenic amplifier both optimizes the signal to noise ratio and minimizes the noise experienced by the sample which leads to environmental contributions. The signal is further amplified at room temperature in order to achieve a level high enough for digitization. Non-linearities in the detection are very detrimental since they lead to strong artifacts. Despite the use of ultra-linear amplifiers, non-linearities still give rise to a contribution which is an even function of $I$ in the sample. We simply remove this by considering $[S_{3I}(I) - S_{3I}(-I)]/2$. After amplification the signal is digitized by a 14 bit, 400 MS/s digitizer with a 1 GHz analog bandwidth. We measure real-time histograms of the signal from which moments are computed.

**Results: elastic transport.** In the inset of Fig. 2 we present the measurement of $S_{3I}$ for the tunnel junction (orange symbols) and the wire (purple). From the high current slope of $S_{3I}$ vs. $I$ for the tunnel junction we find the gain of the setup. Then, we deduce the Fano factor of the wire $F_2 = 0.35 \pm 0.02$, in good agreement with the theoretical value of $\frac{1}{2}$ in Eq. (1). This ensures that electron transport is elastic in the sample, in agreement with other measurements of similar wires [8], [15]. From $S_{3I}$ we also deduce the electron temperature for the wire and for the tunnel junction, as well as the noise temperature of the amplifier $T_a \approx 7.5 \, K$. Values of the temperature indicated in the various figures correspond to electronic temperatures deduced from the measurements of $S_{3I}$.

We then extracted $S_{3I}$ vs. $I$ for the tunnel junction and the wire at temperatures ranging from 130 K to 660 K. Following the procedure of [16], we use the measurements performed at all temperatures on the tunnel junction to extract the parameters that characterize the environment, i.e. the amplifier impedance $R_A = 44.8 \, \Omega$ and the effective environmental noise temperature $T_a^\ast = 0.54 \, K$. From the knowledge of the environmental parameters we can extract the intrinsic third moment of current fluctuations in the wire using Eq. (3). The corresponding results are plotted in Fig. 2. The theoretical predictions of Eq. (2) are plotted as solid lines with no fitting parameters. A clear agreement between experiment and theory is achieved at all temperatures for the current range explored. At higher temperature electron-electron interaction start to be important. We also performed this experiment at higher current where interactions have to be taken into account modifying the statistic of electron transport [17].

**II. Determination of relaxation times**

The environmental terms we have subtracted correspond to a feedback mechanism due to the non-linearity of the second moment of current fluctuation. In fact the intrinsic third cumulant itself is explained by such a feedback in the quasi-classical theory. At a given frequency the effect of the feedback is to modulate the noise, hence one could think of using a controlled excitation power $\delta P(f)$ to effectively reproduce, in a controlled manner, this modulation. This would permit to easily measure a response function $R(f) = \delta S_2/\delta P(f)$. At low frequency $R(0)$ correspond to the noise equivalent to the environmental terms of the third cumulant. At higher
frequency $R(f)$ should exhibit a cutoff frequency revealing any relaxation dynamic present in the sample.

The frequency dependence of $R(f)$ has been calculated for a metallic wire in different regimes [18]. For long enough samples the energy relaxation of the electron gas is dominated by electron-phonon interactions. This occurs when $L \gg L_{e-ph}$ where $L_{e-ph}$ is the electron-phonon scattering length given by $L_{e-ph}^2 = D \tau_{e-ph}$, with $D$ the diffusion coefficient. For shorter samples $L \ll L_{e-ph}$, electron-phonon processes are inefficient and the energy relaxation is dominated by diffusion of hot electrons into the contacts. In both regimes the frequency dependence of $|R(f)|^2$ is extremely well approximated by a Lorentzian decay:

$$|R(f)|^2 = \frac{R(0)^2}{1 + (2\pi f / \Gamma(T_e))^2}$$

(4)

where $\Gamma$ the energy relaxation rate depends on the relaxation process. The frequency dependence of $R(f)$ is a direct probe of $\Gamma$ without any assumption about the specific heat $C_e$ as in previous work [19]. In the presence of several relaxation processes, the fastest relaxation usually dominates. Since $\tau_{e-ph}$ is strongly temperature dependent and diverges at low temperature whereas $\tau_D$ is temperature independent, the energy relaxation is dominated by electron-phonon coupling at high temperature ($\tau_{e-ph} \ll \tau_D$) and diffusion at low temperature ($\tau_{e-ph} \gg \tau_D$). Our measurement allows continuous monitoring of $\Gamma$ as a function of temperature.

We have measured $\Gamma(T_e)$ for six samples made of different metals (Al, Ag) and different geometries. The wires have length $L$ ranging from 5 $\mu$m to 300 $\mu$m and thickness $d$ of 10 $\mu$m for the shortest and 20 $\mu$m for the others. The width has been adjusted to obtain a resistance of the order of 50 $\Omega$ for impedance matching purpose. The experimental setup is presented in Fig. 3. The sample, placed at the 10 mK stage of a dilution refrigerator, is dc and ac biased through the low frequency port of a diplexer by a time dependent voltage $V = V_0 + \delta V \cos(2\pi f t)$ with $\delta V < V_0$. The dc part $V_0$ is used to control the sample mean electron temperature through a constant Joule heating $P_J = GV_0^2$ and allowed us to work between $\sim$ 50 mK and $\sim$ 2 K. The superimposed ac power at frequency $f$, $\delta P(t) = 2GV_0\delta V \cos(2\pi f t)$ modulates the electron temperature of the sample. To detect this temperature, we measure the rms amplitude of the voltage fluctuations (Johnson noise) generated by the sample. Indeed, the noise spectral density of voltage fluctuations $S_V$ is related to the electron temperature by $S_V = 4k_B T_e G$. The voltage fluctuations are measured in the frequency band $\Delta F \simeq 1.5 - 5$ GHz (high frequency port of the diplexer) and amplified by a cryogenic amplifier placed at the 3 K stage of the dilution refrigerator. Their rms amplitude is detected by a power meter (diode symbol in Fig. 3) whose response time $\tau_{det} \sim 1$ ns limits the maximum frequency at which the noise modulation can be detected, $f \lesssim 1$ GHz. Experiments have been performed at a phonon temperature of 10 mK.

In Fig. 4, we present the normalized thermal impedance versus frequency for sample 2 for electron temperatures between 53 mK and 1.04 K. The symbols are the experimental data and the black dashed lines the fits according to Eq. (4). The frequency dependence of $|R(f)|^2$ is very well fitted by a Lorentzian, $\Gamma(T_e)$ being the only fitting parameter. We have performed this experiment for different samples and extracted $\Gamma(T_e)$ on 5 orders of magnitude.

We present in Fig. 5 the measured relaxation rates as a function of electron temperature for all the wires. At low temperature, we observe a Lorentzian behavior.

![Fig. 3. Experimental setup. Diode symbol represents a power detector. VNA=Vector Network Analyser](image)

![Fig. 4. Amplitude of the normalized thermal impedance as a function of frequency for sample 2. The symbols are the experimental data and the dashed lines are fits according to Eq. (4). The different curves correspond to different electron temperatures from $\pm$50 mK to $\pm$1 K.](image)

![Fig. 5. Energy relaxation rate as a function of electron temperature for all the samples. Dashed lines are fits according to Eq. (5).](image)
we observe a plateau, the relaxation rate does not depend on temperature. In this limit only diffusion cooling occurs, and \( \Gamma(T_e) \approx 10.01/\tau_D \). At high temperature the observed \( T^n_e \) dependency is characteristic of an electron-phonon cooling process [20]. In [21] the dynamic has only been calculated in the electron-phonon cooling or diffusion cooling regimes and not during the crossover. We thus assume that the frequency dependence of \( R(f) \) follows a Lorentzian even during the crossover between the two regimes with a relaxation rate given by the sum of the relaxation rates of the two processes:

\[
\Gamma(T_e) \approx \frac{10.01}{\tau_D} + A T^n_e. \tag{5}
\]

Dashed lines in Fig. 5 are fits according to Eq. (5). The plateau observed in \( \Gamma(T_e) \) at low temperature, see Fig. 5, provides a direct determination of the diffusion time \( \tau_D \) as a function of sample length. At high temperature the relaxation is dominated by electron-phonon interaction , the expected value for three dimensional phonon bath in the clean metallic limit [20], [22]. In previous experiments, \( \tau_e^{-1} \sim \tau_{ph}^{-1} \) has been reported to behave as \( T^n_e \) with \( n \) ranging from 2 to 4 depending on the nature of the disorder [19], [23]–[27]. Disordered gold wires have been observed to behave as \( T^{2.9}_e \) below 1K [26]. As far as we know, electron-phonon relaxation rates of Al and Ag have not been measured below 1K, a temperature range hardly explored [19], [26], [27]. From this experiment we are also able to extract the specific heat of electrons, for more information see [28].

III. CONCLUSION

We have measured the third moment of current fluctuations in a wire, thus demonstrating that even the simplest conductor exhibits non-Gaussian noise. Our data at low voltage are in very good quantitative agreement with the theory. In particular we have found a Fano factor \( F_3 = 1/15 \) characteristic of elastic transport in diffusive conductors. We have also demonstrated a sub-kelvin direct measurement of inelastic times in wires made of simple metals, which provides the determination of the electron-phonon scattering time, the diffusion time and the electron heat capacity of the sample. Our approach is however extremely versatile, and of great interest to study interactions and electron diffusion in modern materials.

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Fluctuation Scaling in Nano-Interconnects and its Application to Electromigration

Sofie Beyne\textsuperscript{1,2}
\textsuperscript{1}Department of Materials Engineering, MTM
\textsuperscript{2}KU Leuven and imec
Leuven, Belgium
sofie.beyne@imec.be

Tim Beyne\textsuperscript{3}
\textsuperscript{3}Department of Electrical Engineering, ESAT
KU Leuven
Leuven, Belgium
tim.beyne@esat.kuleuven.be

Abstract—The output current fluctuations in metallic interconnects exhibit fluctuation scaling. The temperature dependence of the scaling exponent is analyzed and found to increase up to a specific temperature, above which it remains constant. This temperature corresponds to the temperature at which a maximum in low-frequency noise (LFN) power spectral density (PSD) is observed. The model of Dutta et al. is applied to calculate the activation energy of the defect mechanisms contributing to the noise spectrum. This LFN activation energy is found to correspond to the electromigration activation energy, indicating that both phenomena depend on the same defect mechanisms. Studying the temperature dependence of the fluctuation scaling behavior of output current fluctuations can thus be used to study electromigration in advanced nano-electronic interconnects, in a fast and non-destructive way.

I. INTRODUCTION

We previously reported that the output current fluctuations of metallic nano-interconnects follow Taylor’s law \cite{1}, \cite{2}. This empirical law was originally obtained by studying the distribution of animal populations in a habitat, relating the variance of the population density to its mean by the following power function relationship:

\[ \sigma^2 = \alpha \mu^p, \]  

with $\alpha$ and $p$ positive constants. In physics, this phenomenon is known as Fluctuation Scaling (FS) \cite{3}.

The scaling exponent, $p$, of the current fluctuations in nano-interconnects was calculated, thereby studying temporal instead of spatial distributions. The scaling exponent was found to be approximately 2 on pristine interconnect samples at 200°C. Inducing voids in the samples, caused a sudden change in fluctuation scaling, indicated by the drop in $p$-exponent from 2 to $\approx 1$. Our previous study thus established the potential of using the scaling exponent of current fluctuations for defect characterization purposes in nano-interconnects.

In this paper, we study the temperature dependence of the scaling exponent and demonstrate how the results can be used for electromigration (EM) characterization.

Electromigration is the mass transport due to the momentum exchange between the electrons and metal ions of the conductor. It can lead to voiding and/or hillock formation, eventually resulting in failure of the entire component. Electromigration is indeed one of the main reliability issues in interconnects and becomes increasingly problematic as scaling of the line-width continues. It is of foremost importance for the semiconductor industry to test the EM reliability of new interconnect structures, materials, etc. The presently accepted EM test method is based on accelerated testing (high current densities and elevated temperatures speed up electromigration failure), but is nevertheless still too time-consuming and limited in providing sufficient fundamental understanding \cite{4}, \cite{5}.

As a solution, low-frequency noise measurements have been proposed to characterize the EM activation energy \cite{4}, \cite{6}, \cite{7}. We will briefly explain this methodology and demonstrate how this solution can be further simplified, by directly analyzing the output current fluctuations and their fluctuation scaling behavior. Other than being a fast characterization method and providing more fundamental understanding, the additional benefit of the methodology is that it is non-destructive.

II. METHODOLOGY

A. Sample description

The interconnects studied in this work are 100µm long and 22nm wide Cu lines without vias (aspect ratio 2). They have a 3nm TaN barrier and 1nm Co liner at the Cu/low-k dielectric interface. The lines are capped with 30nm SiCN and passivated with 300nm SiO$_2$ and 500nm SiN. The line resistance at room temperature is approximately 4.5kΩ.

B. Measurements

The electrical measurements are performed using a commercial Keysight E4727A Low-frequency noise analyzer and a B1500 semiconductor device analyzer.

1) Current fluctuations: To measure the output current fluctuations, a constant voltage is applied to the metal line, resulting in a current density of $\approx$ 1MA/cm$^2$. The measured output current $I_{\text{out}}$ is composed of the average current $E[I_{\text{out}}]$ and (small) fluctuations $\Delta I$. We are interested in studying these fluctuations because they are the result of electron scattering mechanisms in the conductor and as such elicit insights into the defects present in the metal line. The time-domain fluctuations were recorded taking 4096 samples with a sampling frequency of 190.7Hz. A bandstop filter was applied to the data to remove the 50Hz contamination. The temperature dependence was studied between 25 and 100°C.
2) Low-frequency noise measurements: The Low-frequency noise (LFN) Power Spectral Density (PSD) is calculated based on the recorded current fluctuations and is obtained from the spectrum analyzer.

Based on the temperature dependence of the low-frequency noise PSD (in the equations below denoted as $S(\omega)$), the activation energy ($E_A$) of defect mechanisms in metal films can be calculated by application of the model of Dutta et al. [8]. They show that the activation energy $E_A$ can be calculated as

$$E_A = -k_BT\ln(\omega \tau_0), \quad (2)$$

with $k_B$ the Boltzmann constant, $\omega$ the radial frequency, $T$ the temperature and $\tau_0$ an inverse attempt frequency, which for Cu is $\approx 10^{-13}$s. They then derive a distribution of activation energies $D(E_A)$ from $S(\omega)$, for which

$$D(E_A) \propto \frac{k_BT}{\omega} S(\omega, T). \quad (3)$$

The maximum in distribution function $D(E_A)$ indicates the activation energy.

This activation energy has previously been linked to diffusion mechanisms in aluminum [6], [9] and more recently we have shown that the activation energies obtained by LFN measurements correspond well with the values found for electromigration in nano-electronic interconnects consisting of copper and even alternative metals such as tungsten and ruthenium [4], [10], [11].

C. Fluctuation Scaling

As mentioned in the introduction, in this paper we will study the fluctuation scaling behavior of the output current fluctuations. To this aim, the scaling exponent $p$ will be calculated using the method of expanding bins [12]. This methodology works as follows: a wide-sense stationary discrete-time stochastic process, given by $(X_i)_{i=1}^N$, with length $N$ can be divided into subsequent bins of equal length. The absolute current fluctuations in each of the bins are summed and the variance and mean of this sum are estimated. This procedure is repeated for successively expanding bin sizes, until $N/2$ pairs of variance and mean are obtained. The sum of the values in bin $i$ is thus equal to

$$Z_i^{(m)} = \sum_{k=(i-1)m+1}^{im} X_k, \quad (4)$$

which can be shown to have variance

$$\text{Var}[Z_i^{(m)}] = \sigma^2 \sum_{k=1}^{m-1} (m - |k|) \gamma(k), \quad (5)$$

with $\mu$ and $\sigma^2$ the mean and variance of $X_i$ respectively and $\gamma$ its autocorrelation function. For processes exhibiting fluctuation scaling, one expects a variance-to-mean relationship as in eq. (1):

$$\text{Var}[Z_i^{(m)}] = a (m\mu)^p. \quad (6)$$

From the equalities (5) and (6), one can deduce that

$$\gamma(k) \sim \frac{(2-\beta)(1-\beta)}{2\mu^p} k^{-\beta}, \quad (7)$$

as $k \to \infty$ and with $\beta = 2-p$. Constructing a log-log plot of the variance-to-mean pairs allows estimating $p$ (hence $\beta$).

As $\beta$ is directly linked to the shape of the autocorrelation function, it allows one to identify whether the underlying stochastic process of electron scattering exhibits long-range-dependence (LRD), which is closely related to self-similarity and $1/f$ noise [13]. Specifically, a stationary stochastic process is said to exhibit long-range-dependence and therefore also $1/f$ noise, if the autocorrelation function $\gamma$ is of the form

$$\gamma(k) = k^{-\beta} L(k), \quad (8)$$

with $0 < \beta < 1$ and $L(k)$ a slowly varying function as $k \to \infty$ [13]. The lower $\beta$, the stronger the LRD.

The evolution of $p$ (or $\beta$) with temperature, will be studied in this paper because it directly provides information about the underlying stochastic processes; a sudden change in the fluctuation scaling behavior relates to a change in the stochastic process of the output current fluctuations.

III. RESULTS AND DISCUSSION

The $p$-exponent was calculated using the method of expanding bins, as explained above. An example of the application of the method is shown in Fig. 1. The $p$-exponent of the current fluctuations at 25°C is 1.7 and 1.4 at 65°C. This procedure is repeated to calculate the temperature dependence of $p$ and the results are given in Fig. 2. Note that $\beta = 2-p$, such that a larger $p$ corresponds to a lower $\beta$ and whilst $1 < p < 2$, $0 < \beta < 1$. A strong drop in $\beta$ is observed above 65°C, which is an indication of increased long-range dependence. To confirm this, the autocorrelation functions of the time domain data are studied. They were calculated based on the filtered time domain data, in the temperature range 25 to 100°C, as
shown in Fig. 3. The autocorrelation functions displayed in Fig. 3 indeed decrease with $k$ roughly as $k^{-\beta}$. Above 65°C, the autocorrelation function decreases more slowly, which conforms to the observation of an increased $p$-exponent.

The change in fluctuation scaling behavior up to 65°C, as seen in Fig. 3 and 2 reveals a gradual change in the stochastic process of the electron scattering.

This result can be directly compared with the temperature dependence of the LFN in these interconnects. Fig. 4 shows the temperature dependence of the LFN PSD (evaluated at 5Hz) and corresponding $D(E_A)$ for the Cu interconnect described in Section II-A. The PSD is maximal between 75 and 95°C, which, by application of the model of Dutta et al. (eq. (2)), corresponds to an $E_A$ of $0.80 \pm 0.02$eV. We previously showed that this value is in line with the electromigration activation energy [4], which in these samples was found to be $0.85 \pm 0.07$eV.

The temperature at which the scaling exponent $p$ reaches a ‘plateau’ and the temperature of the peak in LFN power spectral density, are indeed observed to almost coincide.

If we now use the temperature at which the $p$-exponent first becomes maximal ($\beta$ reaches a local minimum), in eq. (2), with $\omega = 2\pi f$ and $f$ evaluated between 1 and 5Hz (because this is the frequency range where maxima are typically visible in the PSD), an activation energy of $0.77 – 0.82$eV is found, which is in line with the value found using the LFN measurements and the standard electromigration tests.

Electromigration activation energies correspond to the activation energy of a diffusion mechanism. For Cu, surface diffusion is typically dominant, but in sub-30 nm interconnects grain boundary diffusion becomes also important due to the increased polycrystallinity of the metal [14], [15]. These diffusion mechanisms are vacancy-assisted and it is our understanding that the LFN spectra are impacted by the interaction of electrons with vacancies. The temperature dependence of $p$ indicates that the contribution of these vacancies to the current fluctuations, keep gaining importance up to 65°C and then remains constant at higher temperatures. This corresponds to the observation of a constant shape in autocorrelation functions above 65 – 75°C (Fig. 3). Based on the PSD data alone, this is not evident because the PSD peaks around a specific temperature and is then seen to decrease again.

Additionally, studying fluctuation scaling is much easier and faster than a full low-frequency noise analysis. For the fluctuation scaling analysis presented in this paper, one measurement point only requires 21 seconds, whereas low-frequency noise measurements can easily take up to several minutes, especially when a very accurate noise spectrum is desirable at low-frequencies (increased amount of averaging is needed).

One possible drawback of the fluctuation scaling method, is that it remains unknown how concurrent diffusion mechanisms...
affect the temperature dependence of the $p$-exponent.

IV. CONCLUSIONS

This paper demonstrates that the time-domain current fluctuations in nanoelectronic interconnects exhibit fluctuation scaling and that studying the temperature dependence of the scaling exponent can be used to calculate the activation energy of defects that are important during electromigration failure. Moreover, the activation of these defect mechanisms is found to correspond to increased long-range dependence in the autocorrelation functions of the output current fluctuations.

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Finite frequency noise: an original probe for topological superconductors

Jerome Rech
Centre de Physique Théorique
Aix-Marseille Université
Marseille, France
jerome.rech@cpt.univ-mrs.fr

Didier Bathellier
Centre de Physique Théorique
Aix-Marseille Université
Marseille, France
didier.bathellier@gmail.com

Laurent Raymond
Centre de Physique Théorique
Aix-Marseille Université
Marseille, France
laurent.raymond@univ-amu.fr

Thibaut Jonckheere
Centre de Physique Théorique
Aix-Marseille Université
Marseille, France
thibaut.jonckheere@cpt.univ-mrs.fr

Alex Zazunov
Institut für Theoretische Physik
Heinrich Heine Universität
Düsseldorf, Germany
zazunov@hu.de

Thierry Martin
Centre de Physique Théorique
Aix-Marseille Université
Marseille, France
thierry.martin@univ-mrs.fr

Abstract—Topological superconductor nanowires constitute a strong candidate for the observation of Majorana bound states, which are expected to lie at each of its ends. Here, we suggest that current-current correlations probed at finite frequency offer a promising and original tool for the further characterization of the presence of such states in condensed matter systems, complementary to properties studied thus far. Focusing on a voltage-biased junction between a normal metal and a topological superconductor nanowire, we use the nonequilibrium Keldysh formalism to compute the finite frequency emission and absorption noise. Our results suggest that the presence of a Majorana bound state leads to a characteristic behavior of the noise spectrum at low frequency. While more work is still required to ensure that this constitutes an unambiguous signature, we could already check that different features arise for a nontopological system with a resonant level, exhibiting a zero-energy Andreev bound state.

Index Terms—finite frequency noise, topological superconductors, Majorana fermions

I. INTRODUCTION

Majorana fermions [1], a concept which originally emerged in particle physics, now constitute a very active field of study in modern condensed matter physics. In this context, it is thought to arise from the collective behavior of a many-body electronic system, rather than being an elementary particle.

What brought these peculiar excitations to the forefront is the pioneering work of Kitaev [2]. Studying a tight-binding chain of electrons equipped with $p$-wave superconducting pairing between neighboring sites, he could show that for a whole region of the parameter space (the so-called topological phase) there exist spatially localized Majorana bound states (MBSs) emerging at the boundaries of this one-dimensional system.

The practical realization of such a toy model, now known as a “Kitaev chain”, has since been an ongoing experimental effort. One strong candidate for a real-life version of the Kitaev chain are topological superconductor (TS) nanowires. The latter consist of semiconducting nanowires with strong Rashba spin-orbit coupling, proximity induced s-wave superconductivity, and a Zeeman magnetic field [3]–[8]. This triggered many attempts at finding experimental evidence of the presence of MBSs in such realistic devices. In addition to an obvious fundamental interest for such an observation, Majorana fermions also represent a promising quantum information platform with a built-in topological protection, which constitute a very strong motivation for firm and robust realizations of MBSs.

However, despite the significant experimental progress, an unequivocal signature of these MBSs is still lacking. Promising experiments were conducted [9], but the features they reveal are ubiquitous and could be associated with other sources. This lead to many more theoretical proposals [10]–[15] for the detection of Majorana fermions, usually relying on the quantum transport properties of more elaborate devices. The search for such a “smoking gun”, a signature that would unequivocally signal the presence of Majorana fermions, is still quite active as it has not reached a definite answer so far.

Here, we propose to consider the finite-frequency noise (i.e. current-current correlations) of a voltage-biased normal metal-topological superconductor (NTS) junction. This transport property offers an original probe for the study of TS, as it provides valuable information, complementary with, say,
the differential conductance, opening the way for a different approach consisting in combining different resources in order to rule out alternative physical mechanisms.

II. Model

The setup we consider is a junction between a normal metallic lead and a semi-infinite topological superconductor wire. The latter corresponds to the continuous version of the Kitaev chain in the low-energy limit, and is described as an effectively spinless single-channel $p$-wave superconductor with a Hamiltonian of the form

$$H_j = \int_0^{\infty} dx \psi_j^\dagger(x)(-iv_F\partial_x\sigma_3 + \Delta_j\sigma_2)\psi_j(x).$$ (1)

The leads are labeled as $j = 0$ and 1, for the TS nanowire and the normal metal lead respectively. Here $\sigma_{1,2,3}$ are Pauli matrices in Nambu space, while $v_F$ stands for the Fermi velocity and $\Delta_0$ is the superconducting gap. The Hamiltonian for the normal lead in the absence of voltage is given by a similar form in the limit of vanishing gap parameter $\Delta_1$.

The Nambu spinor $\psi_j^\dagger(x) = [c_{j\uparrow}(x), c_{j\downarrow}(x)]$ introduced here combines right- and left-moving fermion operators $c_{R,L}(x)$.

The coupling between the two leads is then described by the following tunneling Hamiltonian

$$H_T = \lambda c_0^\dagger c_1 + \text{H.c.} = \Psi^\dagger_1 W_{10} \Psi_0,$$ (2)

where $c_j$ are boundary fermions given by $c_j = c_{Rj}(0) + c_{Lj}(0)$, combined under the Nambu notation $\Psi_j = (c_j^\dagger, c_j)$, and we introduced the tunneling matrix $W_{10} = \lambda \sigma_3$.

The current operator through the normal lead is readily obtained from this tunneling Hamiltonian as

$$I_1 = ic\Psi_1^\dagger \sigma_3 W_{10} \Psi_0 = ic\lambda \Psi_1^\dagger \Psi_0.$$ (3)

This then allows us to define the current-current correlations in real time, which reads

$$S_{11}(t,t') = \langle I_1(t)I_1(t') \rangle - \langle I_1(t) \rangle \langle I_1(t') \rangle.$$ (4)

In order to compute the physical properties of the junction, an essential tool is the Keldysh Green’s functions for the boundary fermions. For each lead, they are given by a set of 4 matrices in Nambu space defined as

$$G^{\eta_j\eta_2}_{\mu_1\mu_2}(t_1,t_2) = -i \left\langle T_K \Psi_{\eta_1}^\dagger(t_1^\eta_1) \Psi_{\eta_2}^\dagger(t_2^\eta_2) \right\rangle$$ (5)

where $\eta_j = \pm$ corresponds to the Keldysh time index specifying the position of time along the Keldysh contour, and $T_K$ denotes the Keldysh time-ordering prescription.

The average current can then be reexpressed in the form of a trace in Nambu space involving the Keldysh Green’s function

$$\langle I_1 \rangle = e\lambda \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \text{Tr}_N[G_{01}^+(\omega)]$$ (6)

and similarly, the real-time noise correlator reads

$$S_{11}(t,t') = e^2\lambda^2 \text{Tr}_N[G_{01}^+(t,t')G_{11}^+(t',t)$$

$$-G_{01}^+(t,t')G_{11}^+(t',t)].$$ (7)

Now the full Keldysh Green’s functions entering these expressions are readily obtained from the Dyson’s equation expressed in Keldysh-Nambu-lead space, involving the tunneling matrix $W$ along with the bare Green’s functions associated with the two leads [16]:

$$g^{R/A}_{00}(\omega) = \frac{\Delta^2 - (\omega \pm i0^+)^2}{\Delta^2 - (\omega \pm i0^+)^2 + \Delta\sigma_1}$$ (8)

$$g^{R/A}_{01}(\omega) = [g^{R/A}_{00}(\omega) - \sigma_{01}(\omega)] \tanh \left(\frac{\omega}{2\theta}\right)$$ (9)

$$g^{R/A}_{11}(\omega) = \mp i\sigma_0$$ (10)

$$g^{R/A}_{11}(-\omega) = -2i \tanh \left(\frac{\omega\sigma_0 - eV\sigma_3}{2\theta}\right),$$ (11)

where $eV$ is the voltage applied to the normal lead, and $\theta$ is the temperature of both leads.

III. Finite frequency noise

Since we apply a constant voltage, the NTS junction under study is in a stationary situation. It follows that the current-current correlations $S_{11}(t,t')$ introduced in Eq. (4) actually only depend on the time difference $t-t'$. This allows us to introduce two distinct correlators, and subsequently Fourier transform them, to define the emission and absorption noise as

$$S^+(\Omega) = \int_{-\infty}^{+\infty} d\tau S_{11}(0,\tau) e^{i\Omega \tau}$$ (12)

$$S^-(-\Omega) = \int_{-\infty}^{+\infty} d\tau S_{11}(\tau,0) e^{i\Omega \tau}.$$ (13)

These two quantities are not independent, as they satisfy $S^-(\Omega) = S^+(\Omega) - 2\Delta^2$, so that it is enough to consider the emission noise for all frequencies on the real axis to fully describe the whole range of physical parameters.

As it turns out, the structures arising in the emission noise tend to get smoothed out by increasing either temperature or
transparency, so that the most promising setups to observe characteristic signatures of the MBS are low temperature tunnel junctions.

Interestingly, this regime allows for a tractable analytic derivation, as the emission noise then reduces to

$$S^\pm (\Omega) = \frac{e^2}{\hbar} \int_{-\infty}^{\infty} \frac{dw}{2\pi} \text{TrN} \left[ g_{00}^+ (\omega) g_{11}^\pm (\omega) \right].$$

(14)

Working out the integral explicitly in the zero temperature limit, one is left with 4 contributions of the form

$$S^a_\pm (\Omega) = \frac{e^2}{\hbar} \lambda^2 \Delta \Theta (eV - \Omega) \left[ \frac{\sqrt{(eV - \Omega)^2 - \Delta^2}}{\Delta} \right]$$

(15)

$$S^b_\pm (\Omega) = \frac{e^2}{\hbar} \lambda^2 \Delta \Theta (eV - \Delta - \Omega) \left[ \frac{\sqrt{(eV - \Omega)^2 - \Delta^2}}{\Delta} \right]$$

(16)

$$S^c_\pm (\Omega) = \frac{e^2}{\hbar} \lambda^2 \Delta \Theta (-eV - \Omega)$$

(17)

$$S^d_\pm (\Omega) = \frac{e^2}{\hbar} \lambda^2 \Delta \Theta (eV - \Delta - \Omega) \left[ \frac{\sqrt{(eV + \Omega)^2 - \Delta^2}}{\Delta} \right]$$

(18)

where $\Theta(x)$ is the Heaviside distribution. As can be seen from Fig. 2, this result agrees very well with the emission noise obtained from a full numerical solution of the Dyson’s equations in the tunneling regime at low temperature.

The finite frequency noise is related to fluctuations of the current. It is therefore crucial to understand the processes that contribute to the current, by transferring electrons between leads along with the emission or absorption of a photon. They are pictured qualitatively in Fig. 3. As it turns out, based on their frequency and voltage dependence, along with the frequency range over which they contribute, one can find a one-to-one correspondence between these basic processes and the contributions arising in our analytic derivation of the finite frequency noise (which is why we used matching labels).

From this, one readily sees that the presence of a MBS is directly probed via the emission and absorption processes of Fig. 3(a) and (c). The process shown in Fig. 3(a) corresponds to electrons hopping from the normal metal to the Majorana state either by emitting or absorbing a photon (depending on the sign of the electron energy), therefore covering the whole range of frequency $\Omega \in [-\infty, +eV]$. Similarly, the process in Fig. 3(c) represents an electron transmitted from the Majorana state of the topological superconductor at zero energy to the empty states of the normal metal, accompanied by the absorption of a photon, leading to a finite result for frequencies $\Omega \in [-\infty, -eV]$. While the process from Fig. 3(c) is hard to resolve as it is dominated by other terms (typically $S^a_\pm (\Omega)$), the one from Fig. 3(a) is the leading contribution at low frequency. Furthermore, this contribution is frequency-independent, leading to a wide plateau in the finite frequency noise, a feature which arises from the sharp peak in the density of states (DOS) of the TS lead associated with the MBS.

IV. COMPARING WITH AN N-DOT-S SETUP

To illustrate the added value of finite frequency noise over other physical quantities, we now consider a junction involving a quantum dot between a normal metal and a conventional BCS superconductor. This device corresponds to a nontopological normal metal/superconductor junction, which bears Andreev bound states located at zero energy.

More importantly, this system can be tuned to yield a differential conductance which looks qualitatively similar to that of a low transparency NTS junction, with a conductance peak reminiscent of the signature expected for a MBS despite the absence of such excitations. This is achieved by considering symmetrical couplings $\Gamma_N = \Gamma_S$ from the leads to the dot, and setting the energy $\epsilon$ of the dot to zero.

We focus on this particular regime, and compute the finite frequency current correlations, using a Keldysh Green function formalism [17], [18]. Unlike the NTS case, the final result for the emission and absorption noise turns out to be non-perturbative in the coupling strength, preventing a simple analytical treatment in the regime of low transparency of the junction, even in the regime of large gap. We therefore rely on a full numerical approach which treats the tunneling between dot and leads at all orders. The resulting finite frequency noise is displayed in Fig. 4.

As in the NTS junction, the emission noise shows a clear plateau which drops sharply to zero at frequency $\Omega = eV$. This plateau arises from the presence of a narrow peak in the DOS, associated with weakly coupled discrete energy level of the dot. However, not only this plateau does not extend to the same energy scale for negative frequencies, it also shows a very different behavior at low frequency. Indeed, close to zero frequency, the emission noise suddenly dips down, reaching a value that corresponds to half that of the plateau, a known result in double-barrier symmetric junctions [19].
It follows that, while we cannot claim that the finite frequency noise signatures uniquely identify the presence of a MBS, our results clearly show that it is able to discriminate between the NTS and N-dot-S system when the differential conductance could not.

V. CONCLUSIONS

Focusing on a voltage biased NTS junction, we showed that the finite frequency noise at low transparency could be understood in terms of basic processes, and lead to distinctive features arising from the presence of a MBS. Considering a N-dot-S system, we argued that unlike the differential conductance, the finite frequency noise could be used to discriminate between a real MBS and an accidental Andreev bound state at zero energy.

Finally, this feature should be measurable in actual experiments at low temperature. In practice, in order to measure the finite-frequency noise, one needs a noise detector which, in principle, should be described within a quantum mechanical framework [20]–[22], just as the nanodevice it is connected to. Considering for the detection an LC resonant circuit inductively coupled to the junction, and following [20] and [23], one can predict the result of such a measurement and obtain an expression of the so-called measurable noise. In the regime of low detector temperature, $T_{LC} \ll \Omega$, this actually reduces to the emission noise, up to a constant prefactor [18], thus showing the same low-frequency behavior which we could associate with the presence of a MBS.

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Noise Measurements on MnSi thin films

Merlin Mitschek
Physikalisches Institut
Goethe-Universität Frankfurt
Frankfurt am Main, Germany
mitschek@physik.uni-frankfurt.de

David Schroeter
Institut für Physik der Kondensierten Materie
TU Braunschweig
Braunschweig, Germany
d.schroeter@tu-bs.de

Dirk Menzel
Inst. für Physik der Kond. Mat.
TU Braunschweig
Braunschweig, Germany
d.menzel@tu-bs.de

Stefan Süßlow
Inst. für Physik der Kond. Mat.
TU Braunschweig
Braunschweig, Germany
s.suesslow@tu-bs.de

Jens Müller
Physikalisches Institut
Goethe-Universität Frankfurt
Frankfurt am Main, Germany
j.muller@physik.uni-frankfurt.de

Abstract—Noise measurements are supposed to be a powerful tool to study the dynamical properties of a driven lattice of magnetic skyrmions. In analogy to the properties of driven superconducting vortices, broad-band and narrow band noise is expected to probe the microscopic pinning potential and low-frequency dynamics of the skyrmion lattice. We present first measurements of fluctuation spectroscopy on epitaxial grown MnSi thin film samples, which are expected to show a larger amplitude of fluctuations due to their strongly reduced volume as compared to bulk systems and enhanced absolute resistance values. We observe generic $1/f$-type noise, however with only a weak temperature dependence and no signatures of a magnetic skyrmion phase, which may be explained by electronic inhomogeneities or the absence of a well-defined skyrmionic phase in these samples subject to substrate-induced strain. Upon approaching the helical magnetically ordered phase in zero field, we observe an enhancement of the $1/f$-type fluctuations and two-level switching of a characteristic energy, possibly related to the switching of magnetic grains/clusters.

Index Terms—fluctuation spectroscopy, thin films, skyrmions, $1/f$-noise

Motivation

Magnetic skyrmions are topologically protected chiral spin textures with novel characteristics suitable for encoding data in memory applications. The formation of magnetic skyrmions is often facilitated by the presence of a strong Dzyaloshinskii-Moriya interaction, which prefers non-collinear spin orientation between adjacent magnetic moments. Since their experimental discovery in the last decade [1]–[3], magnetic skyrmions as a new type of magnetic order related to the field of topology have attracted great interest in condensed-matter physics. Manipulating magnetic skyrmions, individually or moving coherently in a lattice, is considered a promising scheme to resolve the limitations of state-of-the-art spintronics applications [1], [4]–[6].

Bulk single-crystals of the cubic helimagnetic B20 compound MnSi exhibit intrinsic skyrmions due to a crystal structure favoring spin-orbit coupling and lacking inversion symmetry [7]. In this system, the early-reported field-induced magnetic A-phase does represent an ordered magnetic skyrmion lattice [1]. Whereas the magnetic properties and phase diagram of bulk MnSi is investigated intensively and understood quite well, and where the signature of the skyrmion phase has been observed by various techniques, as e.g., neutron scattering [1], topological Hall effect [3] and many more, the skyrmionic behaviour in thin films of MnSi still remains rather unclear, see [8] and references therein.

As a potentially powerful probe to investigate the skyrmion dynamics, we employ fluctuation (noise) spectroscopy aiming to detect intrinsic $1/f$-type fluctuations. In analogy to what is observed for the driven superconducting vortex lattice [9]–[11], one expects to see pronounced signatures of the pinning/depinning transition of the skyrmion lattice [4] in the noise magnitude as a function of the current density. Furthermore, again similar to the findings for the superconducting vortex lattice, broad-band noise may be detected when the skyrmion lattice starts moving and narrow-band noise when the skyrmion velocity increases. This may allow to estimate pinning potentials, the skyrmion velocities and the characteristic energies of the dynamical processes.

In bulk MnSi, the large sample volume and low absolute impedances render the observation of $1/f$ noise rather difficult. Therefore, we have chosen to attempt such measurements on MnSi thin films trying to resolve changes in the low-frequency fluctuation properties upon entering different magnetically ordered phases. Thin film samples can be patterned by standard lithography techniques, therefore allowing for well-defined geometries. MnSi enters a helical ordered ferromagnetic state when cooling down the samples in zero magnetic field. The transition temperature of $T_C \sim 29 - 30 \text{ K}$ in bulk single crystals is shifted in thin-film samples to considerably higher temperatures, which is attributed to the effect of the substrate-induced strain of the thin film [13]. At finite fields between $B \sim 0.1 \text{T}$ and $0.2 \text{T}$, and between $T \sim 28 \text{K}$ and $29 \text{K}$, the skyrmion phase is seen in bulk samples, whereas it becomes considerably wider in the temperature-magnetic field phase diagram for thin films, see [12].
Sample and Experiment

The epitaxially-grown thin films of MnSi were prepared in ultra-high vacuum by molecular beam epitaxy with a base pressure of below $5 \times 10^{-11}$ mbar as described previously [8], [14]. The samples are single-phase MnSi and the surface roughness is well below 3 nm. A detailed description of sample preparation, surface analysis and a characterization of the (magneto)transport properties is presented in [8]. The samples were micro-structured by electron beam lithography [14] allowing to perform well-defined resistance measurements. The structures are measured allowing to perform well-defined resistance measurements. The structures are 

\[ 70 \mu m \]

Electrical contacts have been made by silver epoxy and were found to be in the ohmic regime for the current densities applied. Temperature dependent resistance and resistance noise measurements have been performed in a continuous helium-flow cryostat with variable temperature insert using a standard four-point AC technique and a lock-in amplifier. With a maximum applied current of \( I = 100 \mu A \) and the proportions described above current densities of up to \( j = 3.3 \times 10^{-8} \) A/m² have been achieved. The AC voltage is applied to a voltage divider circuit with a limiting resistor much larger than the sample resistance. After pre-amplification, the resulting voltage signal is processed by the lock-in and the voltage noise power spectral density \( S_V \) of the emerging fluctuations is calculated by a spectrum analyzer [15].

Results and Discussion

Fig. 1 shows the zero-field resistivity of a representative thin-film MnSi sample exhibiting an overall behaviour similar to results previous reported [8], [16]. In the whole measured temperature range the sample is in the metallic regime (see below for a discussion of the resistivity maximum at \( \sim 225 K \)) with an anomaly occurring at entering the helical state. Here the slope of the resistivity curve shows a pronounced kink at around \( T_C \sim 46 K \) upon entering the magnetic phase, the temperature of which is significantly shifted with respect to bulk MnSi, see for comparison the suggested phase diagram for thin-film samples in [12]. This is caused by the tensile strain/effective negative pressure induced in MnSi thin films by the lattice mismatch with the Si substrate [8]. With increasing temperature, the resistivity increases until \( \rho(T) \) shows a maximum at around 225 K, above which the resistivity decreases upon increasing the temperature up to 300 K. This behavior has been seen before and has been analyzed in terms of parallel conductivity through the Si substrate and a Schottky barrier at the interface between the contact pads and the Si wafer [8]. For the noise measurements discussed below, we discuss only the low-temperature behavior which remains unaffected by the parallel conduction path.

The inset of Fig. 1 shows a typical noise spectra, i.e. the voltage noise power spectral density \( S_V(f) \) at \( T = 10 K \) and \( B = 0 \). The spectra show typical \( 1/f \)-type behaviour, merging in a ‘white’ spectrum for vanishing current, and scaling \( S_V \propto I^2 \).

Furthermore, the noise magnitude shows the expected scaling \( S_V \propto I^2 \) according to Hooge’s empirical law [17]

\[
S_V(f) = \gamma \mu \cdot \frac{V^2}{\Delta f}
\]

where the Hooge parameter \( \gamma \mu \) is a material’s parameter, \( n \) the carrier concentration and \( \Omega \) the ‘noisy’ volume of the sample. Initially, the Hooge parameter was thought to be constant and of order \( 10^{-3} - 10^{-2} \) for various semiconducting materials. Later, however, experiments revealed that Hooges constant actually ranges from \( 10^9 \) to \( 10^{10} \) for different classes of materials, and is also temperature dependent. Despite all arguments against a physical meaning of Hooges equation, it remains a convenient way to compare the noise level of different systems [18]. With the dimensions of our thin film, the current density applied in the temperature-dependent measurements discussed below and an estimated charge carrier density of \( n = 1.7 \times 10^{22} \) cm\(^{-3} \) [8], we estimate a Hooge parameter of \( \gamma \mu \sim 3 \times 10^{-1} \).

In Fig. 2 we show (a) the normalized resistance PSD, \( S_R/\rho^2(T) \) evaluated at \( f = 1 \) Hz and (b) the frequency exponent \( \alpha(T) \) at different magnetic fields \( B = 0 \) and \( B = 0.2 T \). Zero-field data are shown up to 200 K. In a wide range of temperatures, both the noise magnitude and the frequency exponent are essentially flat and independent of the external magnetic field. Besides a slight increase of \( S_R/\rho^2(T) \) below about 20 K accompanied by a shift of spectral weight to higher frequencies observed for both the zero field and \( B = 0.2 T \) curves, no anomalous behavior in the magnetic phase is observed. We have not seen a dependence of the fluctuation properties on the driving current density. In particular, we have not observed any changes in the noise behavior upon varying the magnetic field below \( T_C \), i.e. we
have not seen a signature indicating a skyrmion phase. This likely is related to the fact that the very existence of a skyrmion lattice is difficult to prove in thin film samples. One argument is that there are electronic inhomogeneities on a spatial range of the order of MnSi islands within the thin film sample, i.e. a few tens of nanometers [8]. If this length scale sets an upper limit for the electronic mean free path, the latter would not be significantly larger than the diameter of a skyrmion, in which case the electrons may not at all see a skyrmion, as they might undergo scattering before traversing the skyrmion. The value of the Hooge parameter of order $10^{-1}$, which is of order to what has been observed for granular metallic thin films and metal-insulator composites [19], is compatible with such an interpretation. Another aspect is that the substrate-induced strain possibly destabilizes the skyrmion phase, which may be recovered by external pressure.

Interestingly, however, a signature is observed upon approaching the magnetically ordered phase. Fig. 2(a) clearly shows an enhanced noise magnitude below about 50 K and above 35 K, i.e. close to $T_C$, of the zero field data as compared to the measurements in $B = 0.2$ T. This is accompanied by a change of the frequency exponent $\alpha$ from smaller to greater than 1 for decreasing temperatures. Assuming a superposition of thermally-activated fluctuation processes causing the observed $1/f^\alpha$-noise one expects typical energies of $E = -k_B T \ln 2\pi f \tau_0$ for these fluctuations [20], which corresponds to $E \sim 110 - 130$ meV in the given temperature range using $\tau_0 = 10^{-14}$ s as a typical inverse phonon frequency. Furthermore, whereas the resistance noise PSD remains $1/f$-type at higher temperatures and inside the magnetic phase, Lorentzian-type spectra superimposed on the $1/f$ background have been detected in the temperature range between 50 K and 40 K, i.e. upon approaching the magnetic transition. Examples of such spectra are shown in the inset of Fig. 3 for selected temperatures $T = 41$ K and 47 K. Note that in Fig. 2 magnitude and frequency exponent of the "$1/f$ background" (on which a Lorentzian contribution is superimposed) is plotted. The enhanced $1/f$-noise level observed close to $T_C$, which may be a signature of enhanced magnetic fluctuations, becomes suppressed by a small magnetic field of 0.2 T.

![Fig. 2. (a) Normalized resistance noise PSD $S_{R^2}/R^2(f = 1$ Hz) against temperature $T$ for different magnetic fields $B = 0$ T and $B = 0.2$ T. (b) Frequency exponent $\alpha$ against temperature $T$ at for different magnetic fields, namely $B = 0$ T and $B = 0.2$ T. The horizontal line marks $\alpha = 1$, see text for details.]

![Fig. 3. Arrhenius plot of the corner frequency $f_c$ of the Lorentzian contributions against $1/T$. A linear fit yields the activation energy $E_A = 70$ meV of the underlying two-level process. Inset: Selected spectra at $T = 44$ K and 47 K in a plot $S_{R^2}/R^2 \times f$ vs. $f$ with corresponding Lorentzian fits.]

Very often, $1/f$-noise originates in the superposition of many two-level fluctuation processes with a certain distribution of time constants (or activation energies). In the present case, a Lorentzian spectrum with a characteristic corner frequency $f_c$ is enhanced in our 'noise window'. The inset of Fig. 3 clearly shows for two selected temperatures that $f_c$ shifts to higher frequencies with increasing temperatures. The main panel of Fig. 3 shows the corner frequency $f_c$ of the observed Lorentzian spectra determined by fitting a single-two level fluctuation process superimposed on a $1/f$-like background against $1/T$ between $T = 50$ K and 40 K in an Arrhenius plot. The characteristic energy of this thermally-activated process can be determined from

$$f_c = f_0 \exp \left(-E_A/k_B T\right)$$

with $k_B$ as the Boltzmann constant and an attempt frequency $f_0$. A linear fit yields a value for the activation energy of around $E_A \sim 70$ meV. This energy may correspond to a certain region within the thin film sample that switches between the paramagnetic and magnetically ordered state. However, the corner frequency does not significantly shift with the applied magnetic field. As shown in Fig. 4 below, the process rather vanishes for larger magnetic fields. It remains almost unaffected within the magnetic phase, i.e. for fields up to $B = 0.5$ T and is not observed in the fully field-polarized state above 0.9 T. In the intermediate field region, the process cannot be reproduced, i.e. it occurs for certain field values.
large volume/small sample impedances could be overcome by micro-structuring bulk MnSi.

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Conclusion and Outlook

In summary, we have performed first noise noise measurements on epitaxial grown MnSi thin films. In the context of skyrmion dynamics, our work was motivated by the expected effects of driven skyrmion lattices on the low-frequency fluctuation properties. In analogy to driven superconducting vortex lattices, the characteristic pinning/pinning energies, skyrmion velocities and other dynamical parameters can be accessed by analyzing broad-band and narrow-band noise. We have chosen thin-film samples which exhibit an enhanced 1/f-noise level due to the small sample volume and larger absolute impedances. We have observed generic 1/f-type spectra, but no clear signatures of changes inside the magnetic phase. A possible reason is the existence of electronic inhomogeneities, which is compatible with a Hooge parameter of order 10^{−1} and the observation of distinct two-level fluctuations. Another aspect is that, in general, the existence of a skyrmion phase in epitaxial grown thin films is less clear than in bulk single crystals, likely due to the suppression of the skyrmionic phase by substrate-induced strain. However, an enhanced 1/f-noise magnitude and redistribution of spectral weight as well as signatures of two-level fluctuations are observed upon approaching the helical magnetic phase in zero magnetic fields.

Our results show that fluctuation spectroscopy of MnSi is feasible. Systematic investigations of the noise properties may help to better characterize the transport characteristics of thin-film samples and even to access dynamical properties of the different magnetically ordered phases. However, bulk MnSi, where the existence of a skyrmion phase can be more easily verified [21] and which do not suffer from strain effects imposed by the substrate, may be a better starting ground for accessing skyrmion dynamics.
Low-frequency Noise and Random Telegraph Noise in Nanoscale Devices: Modeling and Impact on Circuit Operation

Christofooros Theodorou
IMEP-LAHIC
Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS,
Grenoble INP
Grenoble, France
christofooros.theodorou@grenoble-inp.fr

Gerard Ghibaudo
IMEP-LAHIC
Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS,
Grenoble INP
Grenoble, France
gerard.ghibaudo@grenoble-inp.fr

Abstract—In this work, we present our latest modeling approaches regarding low-frequency noise (LFN) and random telegraph noise (RTN) in advanced MOSFETs, with a special focus on the fully depleted (FD) SOI CMOS technology. Concerning the channel mobility fluctuations, the Hooge parameter is shown to be inversion charge dependent, thus not constant with bias. Furthermore, we present a method that accounts for the impact of quantum mechanical effects on the RTN trap kinetics and a complete 1/f carrier number with correlated mobility fluctuations noise model for FDSOI MOSFETs. Finally, various methods of model implementation are shown, allowing for accurate defect-aware circuit noise and reliability studies. Oscillators and SRAM circuits are taken as examples.

Keywords—Low frequency noise, Random Telegraph Noise, Modeling, Verilog-A, FDSOI, MOSFET

I. INTRODUCTION

As the intensity of Low frequency noise (LFN) and Random Telegraph Noise (RTN) fluctuations increases with the reciprocal device area [1]-[2], they can therefore jeopardize the functionality of both analog [3] and digital [4] circuits. They could even appear as an ultimate variability source [5] due to carrier dynamic trapping in undoped channel devices. For these reasons, the proper modeling and simulation of LFN and RTN phenomena in nano-scale devices is a key requirement for the technology evaluation and evolution.

In Ultra-Thin Body and Box (UTBB) Fully Depleted Silicon-On-Insulator (FDSOI) MOSFETs [6] in particular, LFN and RTN can be further influenced by coupling effects. Due to this coupling, it is difficult to predict precisely the contribution of each interface on the measured noise. Moreover, application of a positive or negative bias voltage on the buried oxide can possibly lead to the appearance of either Lorentzian-type noise [7], or significant increase of the flicker noise level [8], [9]. Thus, analytical study of the noise sources and their dependence on the bias conditions is crucial for both device characterization and noise modeling.

In this work, we present some important aspects concerning the LFN/RTN modeling in advanced devices, as well as the development of circuit noise simulation methods. In Section II, some new noise modeling approaches are presented regarding the Hooge channel mobility fluctuations, the impact of quantum mechanical effects on the trap kinetics and the flicker noise of FDSOI MOSFETs. In Section III, we demonstrate a series of different model implementation methods that can provide realistic defect-aware circuit noise simulations of high accuracy.

II. REVISION OF NOISE MODEL APPROACHES

A. Dependence of Hooge parameter on inversion charge

According to the Hooge mobility fluctuations (HMF) noise model [10], the drain current noise is the result of carrier mobility fluctuations stemming from variations in the scattering probability due to phonon number fluctuations. This results in a flicker noise with amplitude inversely proportional to the total number of carriers in the device. The normalized drain current noise in linear operation then reads [11]:

$$\frac{S_{id}}{T_{id}} = \frac{q\alpha_H}{W LD_f}$$

(1)

where $Q_i$ is the inversion charge and $\alpha_H$ is the Hooge parameter ($10^{-7}$-$10^{-4}$). In (1) it is implied that $\alpha_H$ is independent of voltage bias or inversion charge. However, as the Hooge mobility fluctuations depend only on the phonon scattering rate [12], $\alpha_H$ should be modulated by its contribution among other scattering mechanisms limiting the carrier mobility. Therefore, in the case of a MOSFET, the Hooge parameter should be expressed as:

$$\alpha_H = \alpha_{H0} \left( \frac{1}{\mu_{ph}} + \frac{1}{\mu_{CS}} + \frac{1}{\mu_{SR}} \right)^2$$

(2)

where $\alpha_{H0}$ refers to the intrinsic Hooge parameter and $\mu_{ph}$, $\mu_{CS}$, and $\mu_{SR}$ are respectively the phonon, Coulomb and surface roughness scattering limited mobility in the inversion layer [13]. If we also account for the universal mobility law [33], against effective electric field as plotted in Fig. 1(a), the dependence of $\alpha_H$ can be evaluated theoretically versus the inversion charge from weak to strong inversion. As shown in Fig. 1(b), $\alpha_H$ is far from being independent of the inversion charge, and is maximized when

![Fig. 1. Theoretical variations of $\mu_{eff}$ and Hooge parameter $\alpha_H$ (b) with MOSFET inversion charge $Q_i$ for various interface charge $Q_i$ levels modulating the Coulomb scattering rate ($\alpha_{H0}=10^{-4}$).](image-url)
the PH contribution prevails with respect to CS and SR rates.

Fig. 2 shows the impact of the Hooge parameter dependence with inversion charge (2) on the associated normalized drain current noise. In this situation, $S_{Id}/I_d$ is no longer simply inversely proportional to the inversion charge as it were the case for HMF model with constant mobility.

\[ S_{Id}/I_d = q \frac{Cox}{kT} \frac{\sigma f_{Q1}}{Q_1} \]  

where $\sigma$ is the thermal conductivity, $f_{Q1}$ is the trap frequency, and $Q_1$ is the trap charge.

\[ \bar{\tau}_c = \frac{1}{\sigma n_e v_{th}} (a) \quad \bar{\tau}_e = \frac{1}{\sigma n_i v_{th}} (b) \]  

where $n_e$ is the thermal velocity, $\sigma$ is the trap cross section, $n_i$ is the surface carrier concentration, and $v_{th}$ is the thermal velocity.

Regarding the trap kinetics, in general, the RTN capture and emission times are governed by the Shockley-Read-Hall statistics [14] and read:

\[ \bar{\tau}_c = \frac{1}{\sigma n_e v_{th}} (a) \quad \bar{\tau}_e = \frac{1}{\sigma n_i v_{th}} (b) \]  

However, when the trap is not located right at the oxide-channel interface, but at a depth $x_t$ in the oxide, the apparent trap energy $E_t$ depends on the band bending in the gate dielectric as:

\[ E_t = E_{tr} - q \frac{x_t}{Cox} (V_g - V_f) - \psi_s \]  

where $\psi_s$ is the surface potential and $V_f$ is the gate voltage.

Another way to express this difference is through $n_i$ in 3(b), if we replace it with:

\[ n_i = n_{eq} e^{\frac{\psi_s}{kT}} = n_{eq} e^{\frac{\psi_s - \Delta \psi_s}{kT}} \]  

where $\psi_s$ is the surface potential for which $E_t$ coincides with $E_s$, and $\Delta \psi_s = x_t Q_0/d_{ox}$ corresponds to the potential drop across the oxide, from the front interface to the trap depth.

It should also be noted that the capture and emission times of (3) are evaluated within the classical statistics i.e. using carrier volumetric concentration at the surface. They have to be updated when quantum mechanical effects become important in the MOSFET inversion layer, since $n_i$ is cancelled out at the surface. Indeed, the capture probability is proportional to the escape frequency, $\bar{\tau}_c$ ($\approx 2 \times 10^{13} \text{Hz}$), of the electrons in the quantized sub-band and to the barrier tunneling transparency to reach the trap in the oxide. If in addition we take into consideration (5), the capture and emission time can be expressed in a way that accounts for the trap depth $x_t$ within the single sub-band approximation as:

\[ \bar{\tau}_c = q \frac{Cox}{kT} \frac{\sigma f_{Q1}}{Q_1} (a) \quad \bar{\tau}_e = \frac{q e^{\frac{x_t}{E}}}{\sigma f_{Q1} C_{ox}} (b) \]  

where, $\epsilon_{ox}$ is the oxide permittivity, $Q_0$ the inversion charge when the Fermi level $E_F$ crosses the trap energy $E_t$ and $Q_3$ is the depletion charge. It should be emphasized that this formulation (6) of the capture and emission times can also be of great interest for compact modelling applied to circuit simulation (see section III).

C. Flicker noise (1/f) modeling in FDSOI MOSFETs

A very useful quantity for noise model implementation in circuit simulations is the input-referred gate voltage noise $S_{Vg}=S_{Id}/g_{m}$, because it can be inserted as a voltage noise source at the transistor gate. Following this conversion, the Carrier Number Fluctuations (CNF) with Correlated Mobility Fluctuations (CMF) model [15] is expressed as [16]:

\[ S_{Vg} = S_{Vfb} \left( 1 + \frac{g_{m1}}{g_{m2}} \right)^2 \]  

where $S_{Vfb}$ is the flat band voltage power spectral density given by (8) and $\Omega = \omega_{sc} d_{ox} C_{ox}$ is the CMF factor with $\omega_{sc}$ being the remote Coulomb scattering coefficient.

\[ S_{Vfb} = \frac{q^2 kT N_{t1}}{W IC_{ox} L_f} \]  

Now if we consider that in SOI devices, there are two interfaces present - the channel/gate oxide and the channel/buried oxide, (8) shall be rewritten as [17]:

\[ S_{Vg} = S_{Vfb1} \left( 1 + \Omega_1 \frac{g_{m1}}{g_{m2}} \right)^2 + S_{Vfb2} \left( 1 + \Omega_2 \frac{g_{m1}}{g_{m2}} \right)^2 \]  

where index 1 refers to the front and 2 to the back interface. Combining (8) and (9) and accounting for the fact that in front-gate mode, $\Omega_2$ can be considered negligible (shown in [17]), we obtain:

\[ S_{Vg1(SOI)} = S_{Vfb1} \left( 1 + \Omega_1 \frac{g_{m1}}{g_{m2}} \right)^2 + \frac{C_{21}}{C_{21} + C_{22}} \left( \frac{1}{N_{t1}} \right)^2 \]  

with $C_{21}= g_{m2}^2 g_{mn}$ being the coupling factor. Equation (10) reveals that in the simplest case where $\Omega_2=0$ and $g_{m2}^2 g_{mn}= C_{mn}/C_{ox}$ (subthreshold region), the total noise level is equal to the front interface noise multiplied by $(1+N_{t2}/N_{t1})$. Thus, in the case of same quality oxide interfaces, the total 1/f noise would be two times higher in amplitude than the typical bulk MOSFET 1/f noise. If we further consider the FDSOI case where the channel is depleted when no significant back-bias is applied and add the access resistance noise term, we can create a generic model approach as below:

\[ S_{Vg1} = S_{Vfb1} \left( 1 + \Omega_1 \frac{g_{m1}}{g_{m2}} \right)^2 + \frac{N_{t1}^2}{N_{t2}^2} \left( \frac{1}{C_{ox} L_f} \right)^2 \]  

where $\Omega_1=\omega_{sc} d_{ox} C_{ox}$ and $\omega_{sc}$ being the remote Coulomb scattering coefficient.
where the index “i” corresponds to the operating gate interface, i.e. 1 for front-gate (FG) mode and 2 for back-gate (BG) mode, and index “j” to the opposite side interface. This equation reveals that the contribution of the opposite interface to the total 1/f level depends on both the trap density ratio and its oxide to channel capacitance ratio.

It should be noted that the last term in (11) is obtained by considering that the drain current sensitivity with respect to the access resistance \( R_a \) variation is given by:

\[
\frac{\partial I_d}{\partial R_a} = g_a + \frac{g_m}{2}
\]  

(12)

III. FROM NOISE MODELING TO CIRCUIT SIMULATIONS

For our noise model implementations, we used the Verilog-A [18] behavioral description language, since it provides the capacity of both frequency and time domain simulation approaches, while allowing for full description of the device behavior and its interface connections.

A. Frequency vs time domain modeling

Provided that there is a compact flicker noise model expression, one can easily include a noise source in the Verilog-A code. The CNFCMF model (8) for example can be implemented by adding two voltage noise sources at the transistor gates, as follows:

\[
V(g) \sim flicker\_noise(Sv_{g1}, 1, \text{CNFCMF}1); \\
V(b) \sim flicker\_noise(Sv_{g2}, 1, \text{CNFCMF}2);
\]

where \( Sv_{g1} \) and \( Sv_{g2} \) are the power spectral density (PSD) values at 1 Hz and can be given through (9) in the Verilog-A code, accounting for both front/back transconductances \( g_{m1} \) and \( g_{m2} \). The number 1 represents the value of the exponent \( \gamma \) and “CNFCMF1-2” are the names of the noise sources. These voltage sources will automatically induce a drain current PSD equal to \( Sv_{g1}g_m^\gamma + Sv_{g2}g_m^\gamma \).

The above method is very efficient for circuit simulations in the frequency domain accounting for 1/f noise. A good example of such case is the phase noise, because in frequencies close to the oscillation frequency it is directly proportional to the LFN amplitude [19]. In order to demonstrate the importance of accurate LFN modelling in FDSOI circuits, we took the example of a 3-stage ring oscillator circuit. Fig. 3 shows three examples: one case where only the front interface noise is considered (\( N_{t2} = 0 \)), one where \( N_{t2} = N_{t1} \) and finally a case with equally defective front and back oxides (\( N_{t2} = N_{t1} \)). From the figure becomes clear that if the \( N_{t2} \) contribution is not taken into account, both the phase noise level and the 1/f corner frequency are underestimated by 2-3 times, which may lead to false design decisions.

Regarding the time domain, even only with 1/f noise modules, realistic fluctuation-aware transient results can be obtained by using the “Transient Noise” option in Spectre (Cadence) or ELDO (Mentor Graphics) simulators with proper time constraints (speed and duration) as we have shown in [20] and [5]. Fig. 4 shows such an example, where we simulated the measured Supply Read Retention Voltage dynamic variability of a 6T SRAM cell, using the Periodic Transient Noise approach [5].

However, 1/f noise is rarely the dominant LFN source in advanced nano-scale area MOSFETs like FDSOI [17], FinFETs [21] or Nanowire FETs [22] where gate or RTN related Lorentzian noise often prevail. Thus, the non-1/f frequency dependencies cannot be taken into account by applying the “flicker noise” method, making it unsuitable for reliable sub-\( \mu \)m circuit noise simulations. A compromising way would be to use the “noise table” function in Verilog-A and insert Lorentzian PSD table values for each voltage bias. Nonetheless, apart from this method’s complexity, the corresponding “Transient Noise” results cannot reproduce the RTN abrupt current shifts which can cause digital circuit errors [4]. Fortunately, Verilog-A can be also used to create time-dependent modules, since it gives access to the running time value of a transient simulation, as well as control over parameters such as the permitted time-step etc. This proves very useful for defect-aware transient simulations, from RTN and LFN to BTI (time-dependent degradation).

An example of such defect-aware module can be found in [23]: the trap occupancy is checked in every time-step and/or bias modification, resulting in realistic transient and PSD results, as shown in Fig. 5. Using the same module, we showed that the RTN-induced static-noise margin (SNM) dynamic reduction reaches 20% (Fig. 6), which corresponds

![Fig. 3. Simulated 3-stage oscillator (W=10\( \mu \)m, L=30nm) phase noise versus frequency for three different \( N_{t2}/N_{t1} \) ratios, using the model of (9). The flicker/thermal corner frequency points are noted with a circle.](image1)

![Fig. 4. Noise-induced dynamic variability of 6T SRAM cell SRV (after [5]): periodic transient noise simulations with 1/f Verilog-A noise module (a) and I-V measurements with Agilent B1530A (b).](image2)

![Fig. 5. Simulated RTN signal examples for \( N_{t2} = 1, 2 \) and 10 traps (left) and corresponding normalized FFT spectra (right) (after [23]).](image3)
to 1/3 of the mismatch-induced SNM reduction. This means that the total (static+dynamic) variability in nano-scale circuits can be increased by 30%.

B. Implementing defect-aware models in existing PDKs

In one of our recent works [23], we integrated the aforementioned Verilog-A defect-aware module in an open FDSOI compact drain current model, achieving one-step bias-dependent transient simulations. However, in order to make the use of our module completely generic and current model independent, it needs to be implemented using already existing PDK device instances. To this end, the simplest way would be to create a sub-circuit instance that contains the PDK transistor, along with a RTN voltage source in series with the gate, as shown in Fig. 7(a), so that $V_{G} = V_{G} + \Delta V_{RTN}$. Because the PDK device models usually don’t provide access to the inversion charge values, obtaining the drain current, $I_{d}$, values during a transient is needed for the calculation of $\tau_{c}$ and $\tau_{r}$ for each trap. If the mobility degradation effects are neglected for simplicity, one can express the inversion charge as:

$$Q_{i}(t) = \frac{L}{W} \frac{I_{d}(t)}{\mu V_{d}} \quad (a) \quad Q_{it} = \frac{L}{W} \frac{I_{d(t)}}{\mu V_{d}} \quad (b) \quad (13)$$

However, this implementation method has a serious issue: $I_{d}(t)$ is the device current that contains the defect activity induced through $\Delta V_{t}$, which causes a $\Delta I_{d}$ shift. Thus, the capture time calculation during a transient simulation is sensitive to the trap occupancy itself, creating continuity errors. The solution we propose is to use an ideal defect-less “dummy” transistor inside the sub-circuit, which will always provide the $I_{d}(t)$ values without accounting for the trap activity. This method is illustrated schematically in Fig. 7(b).

Finally, regarding the energetic position of the traps, since we cannot express a charge-voltage relation as in [23], the trap energy level can be declared through a characteristic $I_{dt}$, for which $E_{t}$ coincides with $E_{f}$ and calculate $Q_{it}$ through 13(b), to use it in 6(b) for the calculation of $\tau_{c}$.

Fig. 6. Right versus left node voltage plot to extract Read Static Noise Margin with and without the impact of defect activity (after [23]).

Fig. 7. Module implementation methods in existing PDKs: without (a) and with (b) noise-less dummy transistor.

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In-situ noise system measurement for SiGe HBT characterization at 150 GHz

S. Bouvot¹, J.C. Azevedo Gonçalves¹, T. Quêmerais², D. Gloria³
¹STMicroelectronics, 38920 Crolles, France
²STMicroelectronics, 38000 Grenoble, France
simon.bouvot@st.com

Abstract—This paper presents an extraction of the four noise parameters of a BiCMOS SiGe heterojunction bipolar transistor (HBT) at 150 GHz using an in-situ characterization system. Due to the non-availability of commercial G-band impedance tuner, an integrated active impedance tuner designed in the BiCMOS 55 nm technology was used to characterize a HBT from the same technology. Furthermore, the active part of the tuner increases the measurement accuracy by the validation of the minimum detectable signal (MDS) condition. Lane’s algorithm was employed to extract transistor noise performances from 146 to 156 GHz.

Keywords—in-situ, impedance tuner, HBT, SiGe

I. INTRODUCTION

Advanced silicon technologies today have cutoff frequencies above 300 GHz [1]. Such performances make it possible to design millimeter-wave (mmW) circuits. For applications to be robust with reliable component models, the noise characterization of HBTS is of utmost importance. The main objective of its characterization is the extraction of the noise parameters [2]: it includes the minimum noise factor $F_{\text{min}}$, the optimum matching of HBTS remains sensitive with an external tuner.

\begin{equation}
F_{\text{DUT}}(Y_s) = F_{\text{min}} + \frac{R_n}{G_{\text{av}}} |Y_s - Y_{\text{opt}}|^2
\end{equation}

Today, commercial off-wafer impedance tuners are limited to 110 GHz. Furthermore, the probes used to connect these tuners to the silicon DUT reveal important losses. That significantly limits the magnitude of the maximum reflection coefficient $Y_{\text{opt}}$ provided by the tuner to the DUT. Therefore, due to the required optimal source admittance $Y_{\text{opt}}$ needed to reach minimum noise factor, that are far from 50 $\Omega$, the optimum matching of HBTS remains sensitive with an external tuner.

Fig. 1. Block diagram of a typical bench used for noise characterization of a DUT integrated on-wafer

This constraint motivated the development of in-situ impedance tuners integrated close to the DUT. In W-band (75-110 GHz) [3] and D-band (110-170 GHz) [4][5], tuners are designed in STMicroelectronics BiCMOS technologies. Recently, an active impedance tuner [6] was developed to increase measurement accuracy. The latter, used in this study, will be directly connected at the input of a DUT in order to extract its noise parameters while validating the receiver sensitivity condition.

The sensitivity condition is of utmost importance to ensure a correct measurement and extraction of noise parameters, which corresponds to the minimum detectable signal (MDS) imposed by the noise receiver. Thus, the MDS must be guaranteed over the entire studied frequency band. As a matter of fact, for the measurement to be valid, the receiver noise figure (NF$_{\text{RCV}}$) must be less than the sum of the noise figure (NF$_{\text{tot}}$) and the power gain (G$_{\text{tot}}$) of the characterized structure (see Fig. 1), with a 3-dB margin (2).

\begin{equation}
NF_{\text{tot}} (dB) + G_{\text{tot}} (dB) \geq NF_{\text{RCV}} (dB) + 3
\end{equation}

II. METHODOLOGY

To extract the four noise parameters of the HBT under test, the multi-impedance methodology is employed by the resolution of the Lane’s algorithm by knowing at least four tuner presented admittances $Y_s$ and their corresponding noise factors $F_{\text{DUT}}$. The several steps leading to the four noise parameters extraction are proposed in the procedure shown in Fig. 2.

![Fig. 2. Procedure established for noise parameters extraction using multi-impedance method](image-url)
First, the standalone 64 positions active tuner (LT structure, see Fig. 2) is characterized in small signal in order to extract its output reflection coefficient $\Gamma_{LT}$ after having removed the contribution of the output pad of the test structure. This de-embedding step was performed by first measuring the S-parameters of a pad-OPEN structure comprising only a test pad. Then, this pad is reformed into a matrix chain in order to be able to remove it from the raw measurement. It enables to know exactly the $S_22$ parameter of the active tuner which will be presented at the input of the DUT. The available gain of the active tuner $Gav_{LT}$ is calculated by knowing S-parameters with input reflection coefficient (see Fig. 2). Its corresponding noise contribution $NFLT$ is then extracted with the characterization bench depicted in Fig. 3.

The LTD structure, composed of the active tuner (LT) directly connected to the DUT, is also characterized for each position of the tuner to extract its available gain $Gav_{LTD}$ and noise factor $NF_{LTD}$. The extracted available gains and noise factors of LT and LTD structures enable to check the MDS condition for each structure and to calculate the DUT contribution in available gain $Gav_{DUT}$ and noise $NFDUT$.

Then, several filtering steps are applied to the extracted DUT noise factor and corresponding $\Gamma_{LT}$ in order to select the positions of the tuner enabling to execute accurately the Lane’s algorithm. A final check is implemented in order to validate that the extracted noise parameters are physical.

### A. D-band Active Impedance Tuner Integration

To perform source-pull characterization, an in-situ active impedance tuner based on previous work [6] was designed to cover area located in the inductive part of the Smith chart, close to the optimal source admittance ($Y_{opt}$) of the DUT corresponding to its minimum noise factor. This on-wafer integration removes losses induced by the use of high frequency probe to contact the off-wafer impedance tuner to the silicon DUT. The active impedance tuner is composed of a passive tuner preceded by a low noise amplifier (LNA) which main objective is to compensate the losses induced by the passive tuner in order to increase $P_{OUT}$ and ensure the sensitivity condition validation.

Fig. 4 Microphotograph of the $0.8 \, \text{mm} \times 1.2 \, \text{mm}$ active impedance tuner

The output active tuner reflection coefficient ($S_{22}$) for the 64 states is proposed in Fig. 5 at 146, 150 and 156 GHz. The obtained output $|\Gamma_{LT}|$ is higher than 0.5 in the 146-156 GHz band. The available gain of the active tuner $Gav_{LT}$ is then calculated from S-parameters measurements of the latter for the 64 positions, de-embedded of its output pad.

### B. D-band Active Tuner Characterization

First, an off-wafer LRRM (Line-Reflect-Reflect-Match) calibration is done using the Cascade® ISS calibration kit. Then, S-parameters characterization is performed using a 0.01-24 GHz vectorial network analyzer (VNA) and frequency extender modules enabling to operate in the G-band (140-220 GHz).

The output active tuner reflection coefficient ($S_{22}$) for the 64 states is proposed in Fig. 5 at 146, 150 and 156 GHz. The obtained output $|\Gamma_{LT}|$ is higher than 0.5 in the 146-156 GHz band. The available gain of the active tuner $Gav_{LT}$ is then calculated from S-parameters measurements of the latter for the 64 positions, de-embedded of its output pad.

Fig. 5. $S_{22}$ parameter of the 64 positions of the active tuner (LT) de-embedded of the output pad at 146, 150 and 156 GHz

The bench depicted in Fig. 3 is used to extract noise performances of the active tuner. High frequency noise characterization of this structure was performed using Y-factor measurement (3), which corresponds to the ratio of noise powers measured through the NFM when the noise source is at ON state ($P_{hot}$) and OFF state ($P_{cold}$). A 12-dB ENR solid-state noise source from ELVA-1® and a noise receiver developed at IEMN laboratory were used, both on D-band frequency range. Noise power measurements were performed at 30 MHz through a HP 8970B® noise figure meter (NFM) located downstream the noise receiver.

$$F_{LTtot}(Y_S) = \frac{ENR - (Y - 1)\left(\frac{T_{cold}}{T_{hot}} - 1\right)}{(Y - 1)} \tag{3}$$

With $Y = \frac{P_{hot}}{P_{cold}} \gg 1$

The tuner noise factor $F_{LT tot}$ is obtained after calibration.
Noise de-embedding of Cascade Infinity® input and output probes and output GSG pad of the active tuner is then carried out thanks to FRIIS formula. The available gain of the probe is deducted from S-parameters of the probe and the output reflection coefficient of the active tuner.

Sensitivity can be checked by applying (2) to the active tuner structure. The sum of its available gain and the corresponding noise figure shown in Fig. 6 is compared to the noise figure of the receiver including output RF probe (red curve) at each frequency. We observe that several tuner states do not guarantee the minimum detectable signal condition. However, these non-convenient states will be rejected afterward, before executing the Lane’s algorithm without any loss of convergence of the extraction procedure.

The active tuner being characterized, the following steps are to achieve the same characterizations (noise and S-parameters) on the in-situ test structure (LTD) composed of the active tuner and the DUT in order to extract its performances.

C. Transistor Noise Figure Extraction

The transistor under test is a 5 μm × 0.2 μm BiCMOS 55 nm high-speed HBT in CBEBC configuration and was biased at a 1.2 V collector-emitter voltage and at current density of $J_C=17.8$ mA/μm. By knowing LT and LTD performances in noise and available gain, DUT contributions can be extracted. Fig. 7 shows that the sum of the contributions in noise and available gain of the active tuner and DUT leads to a minimum value of 13 dB over the 146-156 GHz frequency range for all tuner states. This validity of the MDS condition allows the noise receiver to be sufficiently sensitive to accurately detect the noise power range at its input.

By knowing noise factor and available gain of both LT and LTD structures, the noise factor $F_{\text{DUT}}$ and available gain $G_{\text{av DUT}}$ of the transistor can be extracted (see Fig. 8 and Fig. 9) with (4) and (5) respectively.

$$F_{\text{DUT}} = (F_{\text{LTD}} - F_{\text{LT}}) G_{\text{av LT}} + 1$$  \hfill (4)

$$G_{\text{av DUT}} = \frac{G_{\text{av LTD}}}{G_{\text{av LT}}}$$  \hfill (5)

D. Filtering Datas from Measurements

Several filtering steps were established in a Matlab® program in order to keep a selection of triplet $(F_{\text{DUT}}$, real part of $Y_S$ and imaginary part of $Y_S$) sufficiently spaced from each other to be able to accurately execute the Lane’s algorithm. The selection of kept tuner states is based on a minimum gap between two noise factors values, then between two values of real parts of $Y_S$ and finally between two values of imaginary parts of $Y_S$. 

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**Fig. 6.** Comparison of the receiver noise figure (NF$_{\text{RCV}}$) with the sum of the noise figure (NFLTD) and the available gain (G$_{\text{av LTD}}$) of the 64 tuner positions.

**Fig. 7.** Comparison of the receiver noise figure (NF$_{\text{RCV}}$) with the sum of the noise figure (NFLTD) and the available gain (G$_{\text{av LTD}}$) of the test structure for the 64 tuner positions.

**Fig. 8.** Extracted available gain of the DUT according to the 64 output impedances of the active tuner.

**Fig. 9.** Extracted noise figure of the DUT according to the 64 output impedances of the active tuner.
E. Noise Parameters Extraction

The four noise parameters were extracted using multi-impedance method associated to Lane’s algorithm [2] after filtering data from measurements. To ensure the physical character of the extracted noise parameters, a check (6) has been performed. Thus, the 4 noise parameters of the DUT extracted are preserved if \( F_{\text{min}} \) is greater than 1 and when \( R_n \) and \( G_{\text{opt}} \) are positive. In addition, a last filtering is applied thanks to the Lange’s criterion defined in (6). The latter must be between 1 and 2 [7].

\[
1 \leq \frac{4R_nG_{\text{opt}}}{F_{\text{min}}} \leq 2
\]

III. Results

Extracted noise parameters of the 5 μm × 0.2 μm BiCMOS 55 nm high-speed HBT biased at \( J_C=17.8 \) mA/μm² (\( V_{dd}=1.2 \)V) are shown in Fig. 10 from 146 to 156 GHz. The result of extracting the four noise parameters at 146 GHz is to be discussed since the previously defined validity criterion is not assured.

IV. Conclusion

The four noise parameters of a HBT were extracted in the 146-156 GHz frequency range using an active 64 positions impedance tuner designed in the STMicroelectronics BiCMOS 55 nm technology. To ensure the minimum detectable signal condition requirement given by the off-wafer noise receiver, a LNA has been designed and associated with a passive tuner. The Y-factor method was employed for noise characterization of the tuner and dedicated test structure. Then, Lane’s algorithm used for noise parameters extraction after a relevant selection of tuner positions.

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Integrated W-Band Measurement System Combining IMD, S-Parameters and Noise Figure Suitable for Coax, Waveguide and On-Wafer Test

Joel P. Dunsmore  
Keysight Technologies  
Santa Rosa, CA, USA  
joel_dunsmore@keysight.com

Suren Singh  
Keysight Technologies  
Santa Rosa, CA, USA  
Suren_Singh@keysight.com

Abstract—New commercial applications for mm-wave systems have driven the need for systems that can accurately and efficiently measure attributes of mm-wave amplifiers, including S-parameters, Two-Tone Intermodulation Distortion (IMD), and Noise Figure. Implicit in the system is automated control to seamlessly switch between measurement applications such as S-parameter (including gain compression measurements), IMD, and noise figure.

Keywords—mm-wave, millimeter wave, noise figure, w-band

I. INTRODUCTION

The expansion of communications systems into mm-wave frequencies and the growing role of automotive radar has greatly increased the commercial use of radio spectrum in the W-band. Previously, W-band measurements were largely the province of industrial and academic research labs, and as such the convenience and cost of mm-wave test systems were secondary to the need to simply make the desired measurement. With the advent of advanced systems operating in the mm-wave band becoming commercialized, there is a need for a system that can combine multiple measurements into a single-system with a single connection. This is particularly true for the case of on-wafer testing, where the process of “touching down” on a die-site always carries the potential to damage the device bonding pad, so the number of touch-downs needs to minimum; ideally one.

While W-band noise measurements systems have been described in the past [1] we introduce a compact system, based on cold-source noise methods, that allows a single-connection multiple-measurement of all these attributes, and is suitable for on-wafer testing. This system utilizes the cold-noise method applied to mm-wave frequencies (up to 110 GHz) and novel in this system is removing the need for utilizing mm noise sources for the noise figure calibration, greatly reducing the system uncertainty. The system, based on a 4-port Vector Network Analyser, with a built-in 50 GHz noise receiver, allows automated switching between measurement applications such as S-parameter (including gain compression measurements), IMD, and noise figure.

Calibration at mm-frequencies is performed using power-meter methods, traceable to the accuracy of the power meter used. Based on cold-source methods, no mm-wave noises are required or used in the calibration or measurement of the system. Additionally, extending this system for transistor noise parameter measurements is possible [2].

II. SYSTEM CONFIGURATION

Figure 1 shows a block diagram of the measurement system configured for all measurements. The VNA used is from Keysight Technologies, model N5291A, with the option 029 (internal 54 GHz noise receiver), and includes three mm-extension heads, and a mm-head controller unit. Also shown is a W-band noise down-converter module which includes an integrated W-band switch to route the output of the DUT either to the integrated low-noise down-converter or to the external mm-head. Port 1 of the VNA system is connected to the input of the DUT and is utilized to measure the gain, compression, and for measuring the noise figure using the cold source method. Port 2 (which includes a 54 GHz dedicated noise receiver) is connected to the output of the W-band down converter. This receiver has an internal noise figure (as measured at the test port) in the range of 9-12 dB. An important part of the system design is ensuring the DUT noise as presented at the output of the W-band converter is at or above the Port 2 noise receiver.

Port 3 of the VNA system is used as a second source for IMD measurements and is combined with the Port 1 source to generate the two-tone signals at the input of the DUT. Port 4 of the system is used as the output port for measuring S-parameters (with full correction), as an IMD Spectrum Analyzer receiver, and for gain-compression measurements. Further, this version of VNA includes the capability of performing as a spectrum analyzer, and port 4 can be used to look for spurious oscillations over in the W-band at the output of the amplifier. Similarly, Port 2 of the VNA can be re-configured to also be a spectrum analyzer, up to 70 GHz, and can identify any system spurious created in the mm-wave down-converter. This can be useful to identify if spikes in the measured noise figure are from the DUT or from internally generated spurs.

The details of the W-band down-converter are shown in Figure 2. The input of the converter is directly connected to a waveguide switch, which either returns the signal to the port 4 test head of the VNA, or passes it through to a W-band downconverter, after an appropriate LNA. Also included is an electronic control of the waveguide switch, which allows the VNA to automatically switch between normal (S-parameter) mode and noise mode. The down-converter system includes a built-in 60 GHz local oscillator, derived from the 10 MHz output of the VNA. The output of the mixer is followed by
a low-pass filter to remove LO feedthrough as well as the
higher image out of the mixer.

Figure 2: W-band Down-Converter

The test system can be mounted on a bench top (as shown in
Figure 3). As shown, the system is configured using 1-mm
connectors between the DUT. The W-Band noise converter
has a W-Band waveguide input, so in the case of a 1-mm coax
DUT, a W-Band to 1-mm adapter is used. For the on-wafer
case, the W-band converter may be mounted to the wafer
prober. Ports 1 and 3 of the VNA feed to a combiner at the
input of the DUT. It has a small size (similar to older style
mm-heads), that makes it reasonable to mount it on directly
to an on-wafer test system.

Figure 3: Photo of the Test System

III. SYSTEM CALIBRATION

The calibration of the test system utilizes a modified version
of the built-in calibration feature of the VNA. In the first step,
the VNA system is calibrated as a Scalar Converter Noise
Figure measurement, where the noise figure and the gain of
the W-band converter are measured and stored. The calibration consists of four steps:

1) Noise Receiver IF Calibration: The VNA Port 2 noise
receiver is calibrated to measure noise power at the IF
(output) frequency of the W-band converter. The typical method is to do a noise-parameter calibration using a hot/cold
noise source (to characterize the gain-bandwidth product
of the VNA Port 2 noise receiver) and a vector noise calibration
where several standards of a mechanical cal kit, or more
typically, an electronic calibration kit (Ecal) is used to
perform noise pulling on the noise receiver (thus finding the
noise parameters of the VNA noise receiver), as well as used
to calibrate the Port 2 reflectometer (which is used to measure
the S22 of the W-band converter to correct for any noise parameter effects of the VNA noise receiver).

2) Port 1 Power Calibration and S11 Calibration: A W-
band power meter and a W-band cal kit are used to perform a
match-corrected power calibration at the W-band frequencies of interest [3]. After this calibration, the reference receiver of
Port 1 is calibrated to measure true incident-power applied to
the DUT during the noise figure measurement.

3) Port 2 Power Calibration and S22 Calibration: A 1-
mm coaxial power meter which operates at the over W-Band
and IF (output) frequencies is use to calibrate the Port 2 VNA
receiver, as well as a vector calibration of Port 2 in the VNA
mode. In this way a match-corrected power calibration is obtained at Port 2 for the IF output frequency of the W-band
converter.

4) Port 1 to Port 4 2-port S-parameter calibration: The
calibration for S-parameters is performed between Ports 1 and
4, using typical S-parameter calibration methods. Further, a
match-corrected power calibration may be performed between
these ports for purposes of power measurements such as IMD
measurements and gain compression. The power calibration at port 1 is also used in measuring the W-
band converter gain.

IV. MEASUREMENT METHOD

The measurement method for W-band noise figure consists of
measuring the noise power from the DUT, as well as the
gain from the DUT, and computing the noise figure from
these two values. From the definition of noise figure [3]

\[ N_f = N_{noise} = \left( \frac{\text{Signal}_{out}/\text{Noise}_{out}}{\text{Signal}_{in}/\text{Noise}_{in}} \right) \left( \frac{S/N}{S/N} \right) \left( \frac{N_0}{N_0} \right) \]

(1)

From which we can compute noise figure as

\[ N_{noise} = \frac{1}{\text{Gain}} \left( \frac{N_{out}}{N_0} \right) = \frac{N_{out}}{G \cdot N_0} = \frac{N_{noise, avail}}{G \cdot N_0} \]

(2)

From this we can find the noise factor using more commonly
used factors such as S21 and incident noise such that

\[ G = \frac{1 - |\Gamma |^2}{1 - \Gamma \cdot S_\text{in} \cdot (1 - |\Gamma |^2)} \]

where \( \Gamma = \frac{S_\text{in} + S_\text{out} - S_{\text{in}} \cdot S_{\text{out}} \cdot \Gamma}{1 - S_{\text{in}} \cdot \Gamma} \)

And

\[ N_{noise, avail} = N_f = kTB \]

(4)

From which we can compute noise factor as

\[ N_f = \frac{N_{noise, avail}}{1 - |\Gamma |} \]

(5)

And for the most common case of noise figure, defined as the
noise figure for a matched input, where \( Z_{in} = 50 \, \text{ohms} \), and \( \Gamma = 0 \), the noise figure becomes simply

\[ N_f = \frac{N_{noise, avail}}{1 - |\Gamma |} \]

(6)
In the W-band noise figure system, the noise figure of the DUT is not measured directly; instead the noise figure of the entire chain, or system, comprised of the DUT plus the W-band converter is measured. In addition, a one-time measurement of the W-band converter by itself is required. From this we can compute the noise figure of the DUT by itself, utilizing the Friis equation. In this system we are looking for the noise figure of only the DUT, $F$, which we compute as

$$ F_{\text{DUT}} = F_{\text{sys}} + \frac{F_{\text{in}} - 1}{G_{\text{DUT}}} $$

Where $F_{\text{in}}$ is the noise figure of the W-band converter. To perform the measurement on the DUT, a first measurement of the W-band converter is performed and saved as a reference trace. The measurement is made from characterizing the output noise of the converter and the gain of the converter (called SC21 to indicate it is converter gain), which is also saved as a reference trace. This one-time measurement can be considered a kind of second-tier calibration. After this, the noise figure of the W-band DUT is computed from equation, by computing the gain of the system divided by the reference gain of the W-band converter, and the noise figure of the W-band converter, and applying equation (7) as

$$ F_{\text{DUT}} = F_{\text{in}} + \frac{F_{\text{in}} - 1}{G_{\text{DUT}}} $$

in the built-in equation editor function of the VNA. This was done for a wide-band mm-wave module, as shown in Figure 4, lower window.

Here we see one trace is overall system noise figure, $F_{\text{sys}}$, labeled as SYNSNF, one trace for the SC21sys, labeled SC21/M, and a trace for DUT NF, labeled DUTF using equation (8).

### V. MULTIPLE MEASUREMENT SCENARIO

#### A. S-Parameter Measurements

One key aspect of this solution is the ability to make multiple measurements with a single connection. The noise figure measurement can include the gain, but the gain is simple ratio of the input system SC21 and the W-band converter SC21, and so this ratio does not include mismatch effects between the DUT and W-band converter. For noise figure, this doesn’t matter because the mismatch from the DUT output to the W-band converter input is common to the noise power and the SC21 measurement. The SC21 measurement method does correct for input mismatch between port 1 of the VNA and the DUT or system being measured. However, it is typical to want to measure the 2-port S-parameters of the DUT as normally defined, and this can be accomplished using the switched path to port 4 of the system. A normal 2-port calibration is used and the key S-parameters can be measured as illustrated in Figure 4, upper left. Note the S21 trace is a quite a bit smoother than in the NF gain, due to 2-port calibration.

#### B. Gain Compression Measurements

The P1dB or gain compression point is generally not a key specification, but it is desired to know what the value is to ensure the LNA is used well away from its compression point. The VNA system, utilizing a match-corrected power sweep, can do single-frequency power sweeps, but can also provide the 1 dB compression point as a function of swept frequency, utilizing a kind-of 2-dimensional sweep. For this amplifier the worst-case point is near 90 GHz with an input P1dB of -32 dBm. For noise measurements we should back off at least 10 to 15 dB from this value.

#### C. IMD Measurements

It is common for the intermodulation (IMD) performance of an LNA to be specified, and it can be directly measured in this case using the second source for the VNA to create a second tone, which is combined before the input of the LNA, as illustrated in Figure 1. Here the two sources are set to be equal at the output of the DUT and the IM products are measured utilizing the built-in spectrum analyzer function and the marker values to generate the IMD value in dBc. This is illustrated in Figure 4, upper right. Here the power in each input tone is set to about -35 dBm, just below the 1 dB compression point. We see from the markers the IMD level is about -20 dBc. The input signals are equalized at the output to less than 0.35 dB offset.

#### D. Single-Connection Multiple-Measurement

Due to the integrated nature of the system, as well as automated control of the W-band converter, all of these measurements can be combined into a single instrument state that can be triggered on a per-device basis. As such an on-wafer test system can simply move the probes to a new LNA and trigger a system sweep, to generate a complete data acquisition cycle.

### VI. MEASUREMENT CONSIDERATIONS

Making noise figure measurements in the W-band region brings some special challenges. Foremost is creating a system than can measure relatively low noise powers in the W-band region. For this purpose, a W-band converter system was designed. The first element is an input switch that allows the W-band converter to be bypassed for measurements such as S-parameters, IMD and gain compression, by routing one leg of the switch from the input mm-head to a second mm-head. The other leg of the switch is routed to the W-band...
down converter. Because mm-wave converters often have relatively high conversion loss, an input amplifier (W-band LNA) is inserted between the switch and W-band down converter. The LNA is preceded by an isolator as the input match to the LNA is not very good (on the order of -5 dBc) and this would cause some additional uncertainty due to mismatch with the DUT output. The W-band system provides a built-in hi-pass function so no filtering is needed after the LNA.

A typical noise converter system uses a swept Local Oscillator to a narrow-band noise receiver. But cost and complexity rules out a swept LO in this system, and we can make use of the relatively wide-band noise receiver built into the VNA. So in this case we use a 69 GHz local oscillator to derive the W-band mixer. The LO is derived from a phase locked 11.5 GHz oscillator which is multiplied by six-times to get to 69 GHz. The 11.5 GHz is locked to the internal 10 MHz reference. The 11.5 GHz signal must also be filtered to ensure a clean signal to the 6x multiplier.

A. Noise Considerations

The gain of the input LNA must be carefully managed, as mm-wave LNAs do not have very large P1dB (approximately -2 dBm output) and the system must have at least 15 dB back-off from P1dB. The LNA in this case has a gain on the order to 20-28 dB, and an input noise figure of about 4 dB. The input isolation has about 1 dB of loss and the input switch has about 5 dB of loss. Since the only active device is the LNA, it is the only device that adds excess noise above the loss factor. The narrow-band filter from the LO to the mixer input is necessary to prevent LO broadband noise from converting to the IF band in the mixer. The bandwidth of the filter is 3 GHz, meaning it rejects noise offset more than about 1.5 GHz from the LO, thus it does not mix into the IF output frequencies.

B. Power Considerations

The output power of the first stage LNA is quite limited, to about -2 dBm P1 dB. At maximum gain, this means the input power to the LNA is about -30 dBm at P1 dB, and through the switch (at the input of the W-band converter) the P1 dB is about -35 dBm. For reasonable noise figure measurements, the compression should be held to less than 0.1 dB, which means about a 10 dB back-off from P1 dB, so the maximum input to the W-band converter is ~45 dBm. For an example test DUT with 20 dB gain, the input power must be set to no higher than ~65 dBm, which is low enough to start having noise-effects on the SC21 measurements (noisy measurement of the input power due to the low level).

For amplifiers that have gain on the order of 30 dB, we run into additional problems of compression of the W-band converter and compression of the VNA low-noise receiver (which has a maximum of about -30 dB excess noise input power): the broadband noise at the output of the test DUT can itself cause compression of the W-band converter. Consider a full-band W-band LNA, with 30 dB gain and a 5 dB noise figure. The excess noise density at the output is 35 dBm/Hz above KTB. Over the 40 GHz BW of the amplifier the integrated noise power is on the order of -33 dBm, above our maximum input of ~45 dBm to avoid any compression. In fact this level of broadband noise will place us near the P1dB point of the W-band converter LNA. In such a case an attenuator should be used between amplifier and the W-band converter (about 10 dB for this example). Most on-wafer devices will not have such high gain but packaged devices may, as the device in the previous figures shows. We use a rotary variable attenuator on the input to allow flexibility for high gain devices (visible after the DUT in the waveguide path before the W-band converter, in Figure 3). To remove the effects of the attenuator, the DUT is removed and the attenuator is set to a desired value. The S2P file is measured of the resulting setting and saved, then de-embedded from port 2, removing the attenuator effects on S-parameter and IMD channels. For the noise figure channel, the gain and noise power are reduced equally, and so the attenuator does not affect the noise figure to the first order. The reference trace for the converter SC21 and converter noise figure are updated with the attenuator in-place before the DUT is added.

A final detail about calibration: the current method for calibrating the noise figure requires a path from port 1 to the VNA port 2 (bypassing the W-band converter) which will pass both the W-band frequencies and the IF output frequencies of the W-band converter. In this case the IMD combiner must be removed for the calibration step, and then must be measured in the W-band so its loss can be de-embedded from the noise figure measurements. In practice, adding the combiner can degrade the raw source match and cause some additional ripple in the noise figure measurement due to noise parameter effects. Adding a tuner before the DUT port 1 gives the potential to generate noise parameters.

VII. UNCERTAINTY CONSIDERATIONS

Principal sources of uncertainty are the noise source ENR uncertainty used to characterize the VNA noise receiver in the IF range. This is on the order of 0.1 to 0.2 dB from low to high frequencies. The power meter uncertainty contributes to the error in characterizing the W-band converter loss, which has a second order effect on the noise figure, but also contributes to error in the system noise figure due to error in the system gain measurement, which directly effects the system noise figure. This error is on the order of 0.1 dB for the IF frequency range and 0.2 dB for the W-band frequency range. The final error is associated with the source match of the system, which if different from 50 ohms, can cause pulling of the noise receiver due to noise parameter effects. To fully understand this effect the noise parameters of the DUT must be known.

VIII. REFERENCES

Analysis on Noise Requirements of RF Front-End Circuits for Spin Qubit Readout

Yatao Peng
École Polytechnique Fédérale de Lausanne (EPFL)
Neuchâtel, Switzerland
yatao.peng@epfl.ch

Andrea Ruffino
École Polytechnique Fédérale de Lausanne (EPFL)
Neuchâtel, Switzerland
andrea.ruffino@epfl.ch

Edoardo Charbon
École Polytechnique Fédérale de Lausanne (EPFL)
Neuchâtel, Switzerland
edoardo.charbon@epfl.ch

Abstract — Currently, radio frequency (RF) measurement setups for semiconductor qubits are implemented with high performance discrete devices operating at cryogenic temperatures, and the output signals are detected by room temperature instruments. This is impractical for a real quantum processor, and integrated circuits (ICs) should be designed for spin qubit readout in a practical quantum computer, to ensure system reliability and compactness. To guide the system-level IC design, we analyze in this work noise specifications of RF front-end circuits for qubit readout, mainly focusing on noise figure (NF) of the low-noise-amplifier (LNA) and phase noise (PN) of the voltage-controlled-oscillator (VCO). RF reflectometry and RF dispersive gate sensing (DGS) are two main spin qubit readout techniques, both of which should detect weak reflected signals as low as -135 dBm and distinguish <1% amplitude or phase variation, which results in high sensitivity and low noise requirements for the readout circuits, to achieve the high measurement fidelity. For the analysis, we model the RF reflectometry and RF-DGS as demodulators respectively of the on-off keying (OOK) signal and phase modulated (PM) signal from qubits. From this, we provide a first estimation of the required NF and PN for a given fidelity and readout bandwidth, as well as an insight into the relationship between phase resolution of RF-DGS and performance degradation due to PN of the VCO. We then review some state-of-the-art CMOS LNA and VCO topologies, which can be utilized in the spin qubit readout ICs to satisfy the specifications.

Keywords — Cryo-CMOS, qubit, spin qubit, quantum computer, noise, phase noise, LNA, VCO.

I. INTRODUCTION

Quantum computing is a fast growing field addressing the constant request for increased computational power [1]. Quantum computers require classical electronics to read out and control the state of quantum bits, to perform quantum error corrected computation. In particular, solid-state quantum processors are typically realized by semiconductor or superconductor quantum devices operated at cryogenic temperatures below 0.5 K [2]. The required radio-frequency control electronics is currently implemented by discrete commercial devices wired to the quantum devices on one end and to room-temperature instruments on the other end [3].

This is however only feasible for the limited number of qubits available today and cannot be scaled to the million qubits required to realize a practical quantum computer [4]. Therefore, to address compactness and scalability, CMOS electronic circuits have been proposed to be designed and operated directly at cryogenic temperatures around 4.2 K and to realize custom integrated circuits for qubit control and readout. Such an approach can also lead to the possible integration with the quantum devices directly.

In the case of semiconductor qubits, the quantum devices are typically realized by transistor-like structures with high electrostatic gate control, so to be able to create a quantum dot capable of trapping single electrons in the transistor channel. Such quantum devices are typically read out by radio-frequency techniques employing readout front-ends very similar to RF receivers for wireless communication systems [5].

There are several functional blocks in RF readout architectures for semiconductor qubits. The LC impedance matching network (LC-IMN), the low noise amplifier (LNA), the directional coupler (used to decouple the incident RF signal from the reflected signal carrying the readout information), the LNA, the mixer and the voltage-controlled oscillator (VCO). In this paper, we will introduce the two main techniques for semiconductor qubit readout and analyze the noise specifications for the readout circuits. CMOS circuit topologies for front-ends are also reviewed briefly.

II. RF READOUT TECHNIQUES AND NOISE SPECIFICATIONS

For the readout of semiconductor quantum dots, two main techniques have been developed in the context of radio-frequency probing. The first one is radio-frequency reflectometry [6][7], the second one is dispersive gate sensing [8]. In radio-frequency reflectometry, an intermediate charge sensor such as a single-electron transistor (SET) or a quantum point contact (QPC), as shown in Fig. 1 (a), is used as an electrometer to sense the state of the qubit and convert it into an electrical quantity, which is then read out. For an RF-QPC, the resistance of the sensor will change according to state of the quantum device. The impedance matching network used to convert the high impedance of the QPC into 50 Ω is tuned to have optimal match at the readout frequency, but changes in the resistance of the QPC will lead to a change in the portion...
of the radio-frequency carrier power reflected from the RF-QPC. Therefore, such a readout technique is called radio frequency reflectometry and one can relate the state of the quantum device to a change in the amplitude of the reflected signal.

More recently, direct connection to the gate of the quantum device has been proposed in order to reduce the complexity of the readout scheme as shown in Fig.1 (b) [8], providing direct in situ access to the quantum device without the need of an external electrometer. In such a technique, the semiconductor quantum device is addressed at its gate with an RF carrier signal and the phase of the reflected signal is read out. According to the state of the quantum device, the capacitance at the gate will be different, due to the presence in the |1⟩ state of an additional quantum capacitance ΔC. This contributes to a phase shift ΔΦ in the response of the resonant matching network, which carries the quantum information about the qubit state. For this reason, such a technique is called dispersive gate sensing.

The cryogenic readout circuit is shown in Fig. 2: the qubit readout circuit mainly consist of LC-IMN working at mK, the directional coupler (DC), low noise amplifier (LNA), mixer and voltage controlled oscillator (VCO) working at 4.2 K. Differently from the current measurement setup of semiconductor qubit for theoretical demonstration and verification [3], the demodulator and the VCO are moved to 4.2 K to simplify the electrical connection between cryogenic and room temperature circuits. The VCO generates incident RF signal (attenuated not to affect the spin state of the quantum dot) and guided down to the QPC or SET through the DC and LC-IMN. Then, the signal is reflected by the quantum devices. The reflected signal travels through the direct path of DC and is amplified by LNA. Finally, the high-frequency signals are mixed with local oscillating (LO) signals and down-convert the signals in a low-IF (or zero-IF) detection scheme, to be then digitized by an ADC.

The RF reflectometry readout of SET is shown Fig. 3 (a), and the LC resonator is used to match the output impedance of the SET to 50 Ω at the operating frequency f0. As the spin status of the electron trapped in the quantum dot changes, the conductance of the SET or QPC (g_a) changes accordingly (by ~10%). The changes of the g_a then modulate the RF carrier power reflected from the matching network at resonance [3]. Here, we model it as a variable resistor.

Similarly, the gate dispersive readout is illustrated in Fig. 3(b), where the change of the loaded capacitance is modeled as a varactor, where capacitance changes according to the spin state of the electron. Here, we must clarify that for the RF reflectometry readout of SET or QPC, the detected signal is amplitude variation, while the gate dispersive readout only the phase change of the reflected signals is detected.

The reflection of a microwave signal depends on change of the load impedance, according to:

\[
A_r = A_{in} \cdot \Gamma = \frac{Z_{out} - Z_0}{Z_{out} + Z_0},
\]

where \( Z_0 \) is the character impedance of the transmission line, \( \Gamma \) is the reflection coefficient and \( Z_{out} \) is the output impedance of the quantum device with LC-IMN. i.e.:

\[
Z_{out} = \frac{R_q}{1 + j\omega R_q (C_p + C_M)} + j\omega L_M.
\]

For the change of the spin status for an electron, the variation for the amplitude and the phase of the reflected signals can be deduced as,

\[
\Delta A = \frac{|A_{in}| \Delta R_s}{2 Q R_s},
\]

\[
\Delta \Phi \approx - \frac{\pi Q \Delta C_s}{C_p},
\]

where \( Q \) is the quality factor of the matching network, \( R_s = 50 \Omega \) is the real part of \( Z_s \), and \( \Delta R_s \) is the change of the real part of the impedance of \( Z_{out} \).

Assuming \( P_{in} \) is incident power to the quantum device, to prevent incident signal from coupling to quantum dot, \( P_{in} \)
should be set <-99 dBm for both readout techniques [5]. For the amplitude based radio-frequency reflectometry, given the variation of the resistance is around 10% and $Q = 15$, based on (3), the reflected signal power can be calculated as -124 dBm. The readout fidelity $F_R$ is determined by the SNR of the front-end circuit [9] according to:

$$F_R = \frac{1}{2} \text{erfc} \left( \frac{1}{2\sqrt{2}} \sqrt{SNR} \right)$$

The noise temperature specification of the LNA for $Q = 15$ can be calculated as 6.3 K for readout frequency of 1 GHz.

For the RF gate dispersive readout, the reflected signal at input port of the LNA is determined by the performance of LC-IMN. Assuming the reflected scatter parameter $S_{11} = -10$ dB, the sensed power at the input port of the LNA can be estimated as -110 dBm. Since the phase change of reflected signal is smaller than 1° [8], the demodulation of the reflected signal should be modeled as high order phase shift keying (PSK) signals, such as 512-PSK and 1024-PSK, depending on the $Q$-factor of the matching network. Based on the results in [10], the required SNR for 512- and 1024-PSK under BER=0.1% are approximately 35 dB and 40 dB, with some margin respectively. Then we can calculated the noise temperature requirements of the LNA are around 22 K and 4.2 K respectively.

To demodulate, the reflected signals are mixed with the LO signal provide by the VCO. The calculated PN requirement of the VCO in the thermal noise should be better than $-147$ dBc/Hz at 10 MHz offset from the carrier and $-115$ dBc/Hz in the sub flicker noise corner region [5].

III. CIRCUIT IMPLEMENTATION ON CMOS TECHNOLOGIES

The CMOS process are proposed to design the qubit readout controller system on chip (SOC) [5]. Deep submicron CMOS devices work well and show acceptable noise performance at 4.2 K [11] [12], and low power consumption of the devices also alleviates the cool down requirements to reach cryogenic temperature.

Three typical topologies are applied to fulfil CMOS LNAs at room temperature, i.e. common gate amplifier with Feedback (CGAF) [13] [14], inductively degenerated common source (IDCSA) [15], and the noise cancellation amplifier (NCA) [16]. The CGAFs are broadband and the input impedance match can be easily achieved by the intrinsic input impedance of the active devices, while these structures suffer from inferior noise performance (>2 dB) since theoretical optimum achievable noise ($1+\sqrt{2} \approx 2$) is heavily deviated from device minimum noise figure $F_{\text{min}} < 0.5$ dB at sub-10 GHz. The NCA utilizes an auxiliary amplifier path to create out-of-phase noise signal at the output node. In principle, the noise of the main amplifier will be nullified, while the auxiliary amplifier can still contribute considerable noise. Given the power consumption is nearly doubled due to dual-path, these structures are not attractive for qubit readout application, especially when the qubit readout frequency is set to higher than 6 GHz to make matching inductors small enough to be integrated on chip [17].

The IDCSAs are widely used in narrowband LNAs design, and $F_{\text{min}}$ can be achieved by optimum noise impedance ($Z_{\text{opt}}$) matching. However optimum noise impedance can be only matched at single frequency and, as CMOS processes advance, the effect of gate-drain capacitor ($C_{gd}$) becomes influential, which leads to the input impedance of transistor $Z_{in} \neq Z_{opt}$. Thus the input impedance matching and the noise matching cannot be achieved simultaneously. As shown in Fig.4 (a), a capacitive load is introduced to adjust $Z_{in}$ of the amplifier while keep $Z_{opt}$ unchanged [18]. This structure was used to design LNAs in 90 nm CMOS process with NF smaller than 0.5 dB at 1 GHz, which satisfies NF specifications for qubit readout, while the broadband design at higher frequency is still open to be investigated.

Since the flicker noise of the transistors goes higher at cryogenic temperature and the bandwidth of the phase-locked-loop is limited, when designing VCOs at 4.2 K for qubit readout both the thermal noise region $(1/f^2)$ and the flicker noise $(1/f^3)$ region should be treated seriously. Recently, harmonic oscillator techniques were proposed to optimize phase noise of the VCO [19], where the second harmonic is used to lower the flicker noise corner and the third harmonic is applied to reduce impulse sensitivity function (ISF) and thus PN, due to thermal noise. A typical VCO structure with second harmonic optimization is illustrated in Fig.4 (b) [20], where LC tank resonates at both operating frequency and second harmonic. The PN of the VCO is $-139.7$ dBc/Hz at $\Delta f = 3$ MHz from 3 GHz, and the flicker noise corner is
around 200 kHz. Fig. 4 (c) depicts a class F VCO implemented by transformer where the tank shows high impedance at fundamental frequency and 3rd harmonic [21], and the VCO endows extremely low PN of -142.5 dBc/Hz, with $\Delta f = 3$ MHz at 3.7 GHz, and $1/f^3$ corner is around 300 kHz. Therefore, a low PN and flicker noise corner VCO can be achieved for qubit readout when both 2nd and 3rd harmonics are considered for the tank design.

IV. Conclusion

In this paper, we analyzed the operation of semiconductor qubit readout techniques. From this, we deduced the noise performance requirement of RF front-end circuits for qubit readout. The noise figure requirement of the dispersive readout is higher than the RF reflectometry due to the high resolution of the phase variation. The phase noise requirements are also extremely strict comparing to the existing RF wireless communication systems. We also reviewed some typical circuit topologies for LNA and VCO on advanced CMOS process, to show that new circuit techniques should be proposed to satisfy the specifications of the qubit readout application.

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Contact effects, Stability and Noise Investigation in Organic Thin-Film Transistors

G. Giusi¹, G. Scandurra¹, S. Calvi², G. Fortunato², M. Rapisarda², L. Mariucci² and C. Ciofi²

1. Department of Engineering, University of Messina, Messina I-98166, Italy
2. Institute for Microelectronics and Microsystems, National Research Council, Rome 00133, Italy

Abstract—In this work we report on the results of Direct-Current (DC) and Low-Frequency Noise (LFN) measurements in p-type staggered top-gate Organic Thin-Film-Transistors (OTFTs). The analysis involves the effects of Source/Drain contacts and the stability characteristics of OTFTs induced by Gate and Drain bias stress. Noise data are interpreted in the context of a multi-trap correlated-mobility-fluctuations (CMFs) model, showing that noise is dominated by acceptor-like traps. The influence of noise sources at contacts is found to be negligible. However contacts affect the measured noise by a non negligible differential resistance. The product between the scattering parameter and the effective mobility \(\eta_{\text{eff}} = 2 \times 10^{3} \text{ cm}^2\text{V}^{-1}\text{s}^{-1}\), which measures the strength of CMFs, is similar to what reported for a-Si:H and much higher with respect to c-Si MOSFETs revealing a strong correlation between CMFs and the state of disorder of the active layer. Instability is observed in presence of Drain bias stress and for sufficient short channel length (<10μm). The measured shift in LFNMs appears correlated with the shift of the measured channel current. In the context of the CMF model the noise shift can be interpreted as due to the increased scattering between the charged channel carriers and the charged traps at the interface.

Keywords— organic, OTFTs, low frequency noise, LFNMs, stability, contacts, LFN measurements, bias stress.

I. INTRODUCTION

As for c-Si MOSFETs, Low Frequency Noise (LFN) measurements have been largely used as characterization tool in order to investigate the material properties and conduction mechanisms in Organic Thin-Film-Transistors (OTFTs). However, noise modeling in OTFTs suffers for the fact that conduction mechanisms are still not collected in a general accepted framework. For this reason, OTFT noise models are borrowed from the c-Si MOSFET models. In particular, it has been found a higher level of 1/f noise in OTFTs with respect to c-Si MOSFETs [1-8]. This result has been interpreted as a higher defect density in OTFTs with respect to c-Si MOSFETs, due to the disordered structure of OTFTs. In most cases the measured data fits within the Hooge mobility-fluctuation (MF) model [1-3] while, in other cases, the number-fluctuation (NF) [4-6] has been used to interpret the measured noise. It should be pointed out that, in OTFTs, the LFN analysis is complicated by the non negligible influence of the contacts. It has been reported, in fact, that contacts are responsible for additional fluctuations which, in some cases and particularly for short channel devices, completely dominate the overall LFN at strong current intensity [1, 2, 9, 11]. In this work we report on the results of DC and LFN measurements in p-type staggered top-gate OTFTs. Measurements are performed by the integrated system reported in [11]. The investigated OTFTs are based on a small molecule organic semiconductor (SmartKem p-FLEX, 20nm thick) and the amorphous fluoropolymer gate dielectric CytopTM (550nm thick, relative dielectric constant ~ 2.1).

II. BASIC NOISE MEASUREMENTS

Fig. 1 shows the power spectral density (PSD) of current fluctuations \(S_{\text{ID}}\) as function of the frequency \(f\) and of the drain current in linear regime. A clear 1/f, \(\gamma=1\), is observed from subthreshold to the strong accumulation region.
frequencies is due to the LNA background noise (LNA-BN).

III. INFLUENCE OF CONTACTS ON LFNMS

In the linear region the PSD of the measured current fluctuations \( S_{\text{ID}} \) can be considered as due to the sum of a contribution coming from the intrinsic device, and a contribution coming from the contacts

\[
S_{\text{ID}} = S_{\text{ID,CH}}(R_{\text{CH}}/R_t)^2 + S_{\text{ID,RC}}(R_c/R_t)^2
\]

(1)

where \( S_{\text{ID,CH}} \) and \( S_{\text{ID,RC}} \) are the current PSDs associated to the intrinsic channel and to the contacts respectively, \( R_{\text{CH}} \) and \( R_c \) are the differential resistances of the channel and contacts respectively, and \( R = R_{\text{CH}} + R_c \) is the total differential device resistance. Since we are interested in the intrinsic noise component \( S_{\text{ID,CH}} \), we have to evaluate the impact of the differential contact resistance \( R_c \) and of the contact noise \( S_{\text{ID,RC}} \). Since \( R_c \ll 1/R_t \ll L \) while \( R_t \) is expected to be independent on \( L \), the contact resistance can be extracted by linear extrapolation of \( R_t \) measured in devices with different \( L \) at the same Gate voltage overdrive [14]. Fig. 2(top) shows the total resistance and the extrapolated contact resistance as function of the Gate overdrive \( (V_{\text{GS}} - V_t) \). The contact resistance reduces with the Gate overdrive from \( ~10^7 \) to \( ~10^5 \) in the subthreshold region and to \( ~10^3 \) in the linear extrapolation of \( R_t \) measured in different devices, for instance in the subthreshold region the shorter channel devices is significantly affected by \( R_c \), especially in the long channel region where \( R_c \) is almost negligible. In order to estimate the impact of the contact resistance \( S_{\text{ID,RC}} \), Eq. (1) can be written as

\[
S_{\text{ID}} = S_{\text{ID,CH}} + S_{\text{ID,RC}}
\]

\[
S_{\text{ID}} = S_{\text{ID,CH}} + S_{\text{ID,RC}} = S_{\text{ID,CH}} + S_{\text{ID,RC}} = S_{\text{ID,CH}} + S_{\text{ID,RC}}
\]

(2)

where \( S_{\text{ID,CH}} \) and \( S_{\text{ID,RC}} \) are the PSDs \((V^2/Hz)\) related to \( R_t \), \( R_{\text{CH}} \) and \( R_c \) fluctuation respectively. As in the case of pMOSFETs, it is reasonable to assume that \( S_{\text{ID,CH}} \approx L \). The physical origin of the contact noise \( S_{\text{ID,RC}} \) is an argument still not clear. However, it is expected that, as in the case of c-Si MOSFETs, the contact noise \( S_{\text{ID,RC}} \) is almost independent on \( L \). Based on these assumptions, \( S_{\text{ID,RC}} \) can be extrapolated by the linear fit of the curve \( S_{\text{ID}} \) vs. \( L \) [9]. Fig. 2 (bottom) shows \( S_{\text{ID}} \) in all investigated devices as function of the Gate overdrive. The noise \( S_{\text{ID,RC}} \) scales well with \( L \), meaning that \( S_{\text{ID,RC}} \) is negligible \((<S_{\text{ID,CH}})\) in the whole bias range. In other works significant effects of \( S_{\text{ID,RC}} \) have been reported in both staggered [10] and coplanar [9] OTFTs, possibly resulting in a not-negligible defect density in the contact region due to different materials and/or process. From the previous analysis it is apparent that, in all the explored bias range from the subthreshold to the strong accumulation regime, the contact noise \( S_{\text{ID,RC}} \) can be neglected while it is necessary to take into account the differential contact resistance \( R_c \), especially in the shorter channel devices.

IV. NOISE ANALYSIS

Besides classical MFs and NFs, noise in MOSFETs has been also interpreted in a larger context by means of a correlated number fluctuation-mobility fluctuation (CMF) theory [15]. CMFs are different from Hooge MFs as they are due to the statistical fluctuation of the scattering cross section induced by the fluctuation of oxide charge which, in turn, is due to trapping/detrapping of charged carriers. For this reason the MFs are correlated with NFs. In the case of a pMOSFETs the model is

\[
S_{\text{ID,CH}} \frac{S_{\text{ID,CH}}}{\mu_{\text{eff}}} = S_{\text{ID}} \left[ 1 + \alpha \mu_{\text{eff}} C_{\text{ox}} \left( \frac{L}{\mu_{\text{eff}} V_{\text{FB}}} \right)^2 \right] \]

where \( S_{\text{ID,CH}} \) is the PSD \((V^2/Hz)\) related to the equivalent intrinsic Gate voltage fluctuations, \( S_{\text{ID}} \) is the flat-band Gate voltage PSD, \( \mu_{\text{eff}} \) is the effective mobility in the active layer, \( C_{\text{ox}} \) is the oxide capacitance, \( V_{\text{FB}} \) is the absolute temperature, \( q \) is the elementary charge, \( C_{\text{ox}} \) is the oxide capacitance, \( N_0(E_F) \) is the trap density corresponding to the oxide energy level aligned with the Fermi level \( (E_F) \) in the active layer and \( \lambda \) is the tunneling parameter. The scattering parameter \( \alpha \) takes into account for oxide-induced mobility fluctuations. In the CMF model (Eq. 3) \( S_{\text{ID,CH}} \) is a quadratic function of the ratio \( \mu_{\text{eff}}/\lambda \). In the case of donor traps \((\text{sign} - \text{in Eq. 3})\) \( S_{\text{ID,CH}} \) increases with \( -\mu_{\text{eff}}/\lambda > 0 \) for pFETs, while in the case of acceptor traps \((\text{sign} - \text{in Eq. 3})\) \( S_{\text{ID,CH}} \) has a minimum, which is also a zero, with respect to \( -\mu_{\text{eff}}/\lambda \). However, a zero in the measured noise is not possible from a physical point of view and it is necessary to admit a distribution of more dominant trap species, each of which is characterized by a couple of parameters \( N_i \) and \( \sigma_i \). In fact, if more trap species contribute
Current vs. Gate Voltage curves and NOISE measurements are recorded after each bias stress. In particular, the NOISE measurement is performed in a point in the strong accumulation region with a Gate voltage $V_G = -15V$ and a Drain voltage $V_D = -1V$ (the Source is always grounded). Different stress conditions are investigated varying $V_{G\text{,stress}}$ and $V_{D\text{,stress}}$. After the stress phase, the device is left to relax with $V_G = V_D = 0$ for 7000s, while DC and NOISE are monitored as in the stress phase every 1000s. Fig. 4 shows the $I_D-V_G$ curve of the fresh device along with the $I_D-V_G$ after each stress pulse with $V_{G\text{,stress}}=-30V$, $V_{D\text{,stress}}=-10V$. It is clearly evident the instability of the investigated device due to the shift in the threshold voltage, as well as the degradation of the subthreshold slope and of the transconductance $g_m$ (apparent increase). Since the (absolute) value of the Drain current reduces (increase of the threshold voltage), from a macroscopic point of view the instability could be attributed to positive charge trapping or positive charged defects creation at the interface between the semiconductor and the dielectric, or in the dielectric. The subthreshold slope degradation may suggest that traps/defects are located at the interface rather than in the dielectric bulk. Reverse $I_D-V_G$ curves (not shown) have lower threshold shift, suggesting that traps/defects are preferably located close to the source region. Fig. 5 (top) shows the time evolution of the monitored current during stress ($V_{G\text{,stress}}=-30V$) for different $V_{D\text{,stress}}$ and during relax. It can be observed that i) the threshold voltage shift increases with $V_{D\text{,stress}}$, ii) the stress appears almost partially reversible since the monitored current comes back close to the fresh value. This last result may suggest that no (or few) new defects are created during the stress phase, and that the observed instability is caused by trapping (during stress) and release (during relax) of mobile carriers into trap sites located at the interface (source side). The instability increases with $V_{D\text{,stress}}$ for large $V_{G\text{,stress}}$ (as reported in Fig. 5) and also with $V_{D\text{,stress}}$ for large $V_{G\text{,stress}}$ (not shown). In particular no instability has been observed for large $V_{G\text{,stress}}$ and $V_{D\text{,stress}}=0$. This last result is in agreement with other reports where no Gate bias instability is observed for Cytot based OTFTs [17, 18]. Device simulations [19] show that the observed instability phenomena can be explained in terms of injection of high energetic carriers at the source side into the dielectric. In fact, carriers can gain enough energy due to the large depletion and consequent high longitudinal field at the Schottky contact between the source and the semiconductor. Fig. 5 (bottom) shows the normalized noise $S_{I_D/CH}I_D^0$ monitored during the stress and relax phases. It can be observed that the noise increases during stress, and recovers during the relax phase, according to the current in Fig. 5 (top). As discussed in Section IV, the noise in the investigated devices can be interpreted in the context of the correlated-mobility-fluctuation model. Since the monitor bias is in the strong accumulation region,

$$S_{I_D/CH} I_D^0 \approx S_{I_{YS}} \left(\frac{C_{ox} \varepsilon}{L_D R_{on}}\right) \left(\frac{I_D}{I_D^0}\right)^2$$

(4)

that is the normalized noise $S_{I_D/CH} I_D^0$ is independent on the bias and is proportional to $N_e(\mu_{app})^2$. Let us notice that the increase of $S_{I_D/CH} I_D^0$ cannot be explained with the apparent increase of $g_m$, since $S_{I_D/CH} I_D^0/g_m^2$ is not independent on the stress but reduces with it (not shown). Indeed, the apparent increase of $g_m$ has been explained as an electrostatic effect due to the presence of defect regions close to the contacts [20].
The effective mobility $\mu_{eff}$ can only reduce with stress, due to the increased electrostatic scattering between charged carriers and the traps/defects at the interface. The almost complete relax of $S_{I,CH/ID}$ comes back to the initial value due to trap discharging and to the reduction of $\alpha$. DC and NOISE measurements have been performed also in devices with a longer channel length ($>5\mu m$). No significant instability have been observed in DC and NOISE measurements in devices with L≥10mm for the same range of $V_{G,stress}$ and $V_{D,stress}$. This result is consistent with the hypothesis of trapping of high energetic carriers as contact effects and longitudinal fields are reduced in longer channel lengths.

VI. CONCLUSIONS

LFN measurements show that the origin of current fluctuations in the investigated OTFTs can be interpreted in a context of a multi-trap correlated-mobility-fluctuations model, and that noise is dominated by acceptor-like traps. Contacts affect the measured noise only by a non negligible differential resistance. The higher scattering parameter found with respect to c-Si devices, can be used and interpreted as due to the higher state of disorder. Moreover, instability induced by Drain bias stress is found to be almost reversible and the related noise shift is found to be correlated with the shift of DC characteristics. This behavior can be interpreted as due to the higher scattering parameter caused by the increased scattering between the charged channel carriers and the charged traps at the interface.

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Low-frequency Noise Characterization of Si Nanonet Field Effect Transistors

Thibault Cazimajou  
IMEP-LaHC  
Univ Grenoble Alpes,  
Univ. Savoie Mont Blanc,  
CNRS, Grenoble INP  
Grenoble, France  
thibault.cazimajou@grenoble-inp.fr

Christoforos Theodorou  
IMEP-LaHC  
Univ Grenoble Alpes,  
Univ. Savoie Mont Blanc,  
CNRS, Grenoble INP  
Grenoble, France  
christoforos.theodorou@grenoble-inp.fr

Maxime Legallais  
IMEP-LaHC, LMGP, LTM  
Univ Grenoble Alpes,  
Univ. Savoie Mont Blanc,  
CNRS, Grenoble INP  
Grenoble, France  
maxime.legallais@cea.fr

Thi Thu Thuy Nguyen  
LMGP  
Univ Grenoble Alpes,  
Univ. Savoie Mont Blanc,  
CNRS, Grenoble INP  
Grenoble, France  
thuthuynguyen2004@gmail.com

Mireille Mouis  
IMEP-LaHC  
Univ Grenoble Alpes,  
Univ. Savoie Mont Blanc,  
CNRS, Grenoble INP  
Grenoble, France  
mouis@minatec.grenoble-inp.fr

Celine Ternon  
LMGP, LTM  
Univ Grenoble Alpes,  
CNRS, Grenoble INP  
Grenoble, France  
celine.ternon@grenoble-inp.fr

Bassem Salem  
LTM  
Univ Grenoble Alpes,  
CNRS, CEA  
Grenoble, France  
bassem.salem@cea.fr

Gerard Ghibaudo  
IMEP-LaHC  
Univ Grenoble Alpes,  
Univ. Savoie Mont Blanc,  
CNRS, Grenoble INP  
Grenoble, France  
gerard.ghibaudo@grenoble-inp.fr

Abstract—This paper deals with the low frequency noise characterization of silicon nanonet field effect transistors. Electrical noise parameters such as surface trap density and remote Coulomb scattering coefficient were extracted using the carrier number and correlated mobility fluctuation model. Their variation with density of nanowires and source-drain distance is presented, as well as a method to obtain information about the order of magnitude of the electrical active area variation.

Random network; silicon nanowire; electronic transport; Low frequency noise; percolation

I. INTRODUCTION

Nanowire (NW) based field effect transistors (FET) seem to be promising building blocks for biosensing applications, because of the NW sensitivity to surface charge [1]. However, their large scale integration can be complex. In the Front-End-of-Line, it normally requires processes for nanowires patterning, as well as a toughly etching process through the thick Back-End-of-Line stack, in order to open an access to the active nanowire-based channel. Moreover, a minimum opening size is required to allow fluid penetration, thus increasing footprint and cost [2]. Other integration concepts can be adapted from the techniques that have been developed for CNT-based transistors [3], allowing for above-IC integration. However, being based on essentially aligned nanostructures, these techniques would be limited to the fabrication of short devices, with a source-drain distance smaller than the mean nanostructure length. Here instead, the active material is a network of randomly interconnected nanowires, also called nanonet (NN). Silicon Nanonets can be fabricated using low-cost bottom-up processes, they can be transferred on any type of substrate and they can be processed using standard microelectronics technology [4]. The thermal budget required for such processing is compatible with CMOS integration (maximum temperature below 400°C) [5]. This opens the route to the 3D integration of sensor arrays above their CMOS readout. The first NN-based FET has been obtained recently [5] and has been studied through static measurements [6]. In this paper, we present an analysis of the low frequency noise (LFN) measurements performed for different densities of NWs and source-drain distance L_{source-drain}. Furthermore, we demonstrate a method to extract the apparent density of oxide interface traps. Finally, based on its dependence with NW density and source-drain distance, we present a model approach for the physical active area variation.

II. DEVICES UNDER STUDY

The NNs used in this study were self-assembled from Si-NWs grown by Vapor Liquid Solid chemical vapor deposition [6]. NW length and diameter were respectively 7 ± 3 μm and 40 ± 7 nm. NNs can be fabricated with different NW densities [8]. For this paper, 4 different densities (from d = 0.28 NWs·μm⁻² to 0.66 NWs·μm⁻²) NWs were then transferred on a Si / Si₃N₄ wafer passivated with alumina and contacted by Ni / Au metal pads. Field-effect control was obtained by using the Si / (200nm) Si₃N₄ substrate as a backgate. Several device geometries were available with Source-Drain (S-D) distance L_{source-drain} ranging from 5 μm to 1000 μm and a constant pad width W_{source-drain} = 200 μm. For this study, only devices with L_{source-drain} = 30 μm and 50 μm were tested. In this case, because L_{source-drain} is longer than the average NW length, it means that the channel is made of several conduction paths where each path is constituted by a successive alternation of NWs and NW-NW junctions.

III. EXPERIMENTAL PROTOCOL AND RAW RESULTS

The drain current noise power spectral density (PSD) was measured in the low frequency range (4 Hz-10 kHz) in a metal
Fig. 1. Transfer characteristics \( I_d - V_g \) for the sample A (\( L_{\text{mask}} = 30 \mu m \) et \( d = 0.28 \text{ NWs.} \mu m^{-2} \)).

Fig. 2. LFN-PSD analysis for the sample A (\( d = 0.28 \text{ NWs.} \mu m^{-2}, L_{\text{mask}} = 30 \mu m \)), for different \( I_d \) at \( V_d = -9 \text{ V} \). Black line indicates 1/f dependency.

shielding box, as a function of back gate voltage (\( V_g \)), at room temperature. Source contact was grounded and drain contact was set at a given bias voltage (\( V_d = -9 \text{ V} \)). A Programmable Point-Probe Noise Measurement System was employed. Fig. 1 shows typical \( I_d - V_g \) characteristic for sample A (with a NW density, \( d = 0.28 \text{ NWs.} \mu m^{-2} \) and \( L_{\text{mask}} = 30 \mu m \)). Fig. 2 shows normalized drain current noise power spectral density (LFN-PSD) \( S_{Id/Id^2} \) for sample A, for different values of drain current. The LFN-PSD is generally 1/f like, except for some Lorentzian contributions. The total LFN behavior can be attributed to a large ensemble of Lorentzian noise sources based on individual NWs and junctions between NWs.

IV. ANALYSIS AND DISCUSSION

Within the carrier number and correlated mobility fluctuation model (CNF-CMF), the drain current noise is given by:

\[
\frac{S_{Id}}{I_d^2} = \left(1 + \Omega \left(\frac{I_d}{g_m}\right)^2\right) \left(\frac{g_{\text{eff}}}{I_d}\right) S_{\text{fb}} \tag{1}
\]

with flat band voltage PSD \( S_{\text{fb}} \) and CMF factor \( \Omega = \alpha \mu_{\text{eff}} C_{\text{ox}} \) [9] and \( \alpha \) the remote Coulomb scattering coefficient. The flat band voltage PSD \( S_{\text{fb}} \) is related to the oxide trap density \( N_{st} \) with the equation:

\[
S_{\text{fb}} = \frac{q^2 k T N_{st}}{W L C_{\text{ox}} f} \tag{2}
\]

with electron charge \( q \), Boltzmann constant \( k \), temperature \( T \), channel width \( W \), channel length \( L \), oxide capacitance \( C_{\text{ox}} \), frequency \( f \). In this case, the input referred gate voltage noise \( S_{V_g} \) now reads:

\[
S_{V_g} = S_{Id} \frac{S_{\text{fb}}}{g_{\text{st}}^2} = \left(1 + \Omega \left(\frac{I_d}{g_m}\right)^2\right) S_{\text{fb}} \tag{3}
\]

Fig. 3 shows \( S_{V_g} \) measured at \( f = 10 \text{ Hz} \) as a function of drain current for sample A. A plateau is visible for small value of \( I_d \), corresponding to \( S_{V_g} \) equal to \( S_{\text{fb}} \) (approximately equal to \( 2 \times 10^{-6} \text{ V}^2 \cdot \text{Hz}^{-1} \) for \( f = 10 \text{ Hz} \)). The increase of \( S_{V_g} \) in higher values of \( I_d \) can be attributed to the CMF component in (3) or to the access resistance. For sample A, \( S_{\text{fb}} = 2 \times 10^{-6} \text{ V}^2 \cdot \text{Hz}^{-1} \) for \( f = 10 \text{ Hz} \), and considering \( L_{\text{mask}} = L \) and \( W_{\text{mask}} = W \), an apparent trap density can be extracted of about \( 10^{15} \text{ cm}^{-2} \cdot \text{eV}^{-1} \).

Fig. 3. \( S_{V_g} \) extracted at \( f=10\text{Hz} \) as a function of drain current for sample A. A plateau is visible at low current, corresponding to \( S_{V_g} = 2 \times 10^{-6} \text{ V}^2 \cdot \text{Hz}^{-1} \), and a trap density equal to \( 10^{15} \text{ cm}^{-2} \cdot \text{eV}^{-1} \).

Fig. 4. \( S_{V_g} \) extracted from plateau of \( S_{V_g} \) and corresponding apparent trap density as a function of the density. Squares are for experimental points and stars for mean values. Dotted line is a guidance for the eyes.
Fig. 4 shows the extracted values of $S_{\text{vfb}}$ from the weak inversion plateau of $S_{\text{vg}}$ and the corresponding apparent trap density, as a function of NW density for devices with $L_{\text{mask}} = 30 \, \mu m$. The apparent trap density decreases when the NW density increases. However, even though a dispersion of trap density value is expected from one device to another, a mean variation with the density is not reasonable. This apparent variation can be attributed to a variation of the electrical active area, especially of the physical width. Since $1/S_{\text{vfb}}$ reads,

$$
\frac{1}{S_{\text{vfb}}} = \frac{NWLC_{\text{ox}}^2}{q^2kTN_{\text{st}}}
$$

considering the typical description of conductance in percolating problem [10] and the small variation of channel length when the NW density increases (the average length of shortest conduction paths are found only 10% longer than $L_{\text{mask}}$ in [9]), the channel width $W_{\text{channel}}$ can be described by the equation:

$$
W_{\text{channel}} = \text{cst} \times (d - d_c)\beta
$$

with $d_c$, the percolation threshold and $\beta$ an exponent equal to 4/3 for 2D networks. In this case, considering (4) with $W_{\text{channel}}$ as $W$, $1/S_{\text{vfb}}$ should vary as:

$$
\frac{1}{S_{\text{vfb}}} = \text{cst}' \times (d - d_c)\beta
$$

Fig. 5 shows the successful fit of the inverse of mean $S_{\text{vfb}}$ with the model (6). Assuming $W_{\text{channel}}$ as $W$ in (2), another trap density can be extracted. Fig. 6 shows this trap density as a function of the NW density. This $N_{\text{st}}$ seems to be constant with the NW density increase. Because of the unknown value of the constant term in (5), the study of the mean value of $N_{\text{st}}$ is meaningless. However, the difference between the apparent trap density value (approximately $5 \times 10^{15} \, \text{cm}^{-2}\text{eV}^{-1}$) extracted from LFN data and the trap density extracted from the subthreshold slope of the I-V data ($7 \times 10^{12} \, \text{cm}^{-2}\text{eV}^{-1}$) can provide an insight regarding the difference between the physical area ($W_{\text{mask}}L_{\text{mask}}$) and the effective (channel) area. The ratio between the two areas seems to be around 2 orders of magnitude.

Fig. 7 shows fits with CNF-CMF model of the normalized LFN-PSD of sample A as a function of $I_d$. One of the fits is realized without remote Coulomb scattering factor. These 2 fits show the importance of the remote Coulomb scattering in these devices ($\Omega = 0.14 \, \text{V}^{-1}$). Extracted $S_{\text{vfb}}$ ($S_{\text{vfb}} = 1.6 \times 10^{-6} \, \text{V}^2\text{Hz}^{-1}$) is consistent with the one extracted by considering only the plateau of $S_{\text{vg}}$. Fig 8 shows $\alpha_{\text{mu}}(= \Omega/C_{\text{ox}})$ as a function of NW density for $L_{\text{mask}} = 30 \, \mu m$ and $L_{\text{mask}} = 50 \, \mu m$. The mean value of $\alpha_{\text{mu}}$ is in the order of $5 \times 10^{-6} \, \text{cm}^{-2}\text{C}^{-1}$ for $L_{\text{mask}} = 30 \, \mu m$. This value is consistent with the ones observed in silicon [11].
Fig. 8. $G\mu_{\text{eff}}$ as a function of NW density for different Source-drain distances. For $L_{\text{mask}}=30\mu\text{m}$, black squares are experimental points and black stars mean values. For $L_{\text{mask}}=50\mu\text{m}$, red circles are experimental points and red stars mean values.

For a small number of devices (Fig. 9), (1) does not fit the normalized LFN-PSD, because of an increase of $S_{\text{id}}/I_d^2$ in strong inversion regime. This increase can be related to the access resistance noise, which can be added to the model as:

$$\frac{S_{\text{id}}}{I_d^2} = \left(1 + \Omega \left(\frac{1}{g_m}\right)^2 \frac{g_m^2}{L_c^2}\right) S_{\text{Vfb}} + S_{\text{Rds}} \left(\frac{g_m}{2}\right)^2$$

with $S_{\text{Rds}}$ the PSD of the access resistance [12]. With this equation, a good fit of $S_{\text{id}}/I_d^2$ is obtained, as shown in Fig. 9. Note that in general, the variation is that of Fig. 7 which cannot be well fitted with the assumption of a serial resistance noise but where account for CMF model is required.

![Fig. 9: $I_d$ normalized LFN-PSD at f=10Hz (symbols) with CNF-CMF (stars) fitting curves, for a device with $d=0.28$ NWs.$\mu\text{m}^2$ and $L_{\text{mask}}=30\mu\text{m}$. Red stars correspond to a fit with remote Coulomb scattering but without access resistance. Blue stars correspond to a fit with remote Coulomb scattering and access resistance.](image)

V. CONCLUSION

We measured the LFN characteristics of passivated nanonet-based FET devices with different source-drain distances and NW densities. Flat band voltage PSD and remote Coulomb scattering factors were extracted using the CNF-CMF model. Apparent density of traps is extracted using $S_{\text{Vfb}}$ and the gate area. Comparison of this trap density with the one extracted from static measurement provides the ratio between the gate area and the real electrical active area where percolating transport occurs.

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A cost-competitive low Noise Junction-FET (JFET) for high-precision Analog Application

Ai Qiang, Suba Subramaniam, Michaelina Ong, C.R. Manoj
X-FAB Sarawak Sdn. Bhd., 1 Silicon Drive, 93350 Kuching, MALAYSIA
X-FAB Branch Office Plymouth, Tamerton Road, Plymouth, PL6 7BQ, UK
ai.qiang@xfab.com

Abstract—In this paper, we present an optimized cost-effective JFET integrated in 0.13μm CMOS technology with excellent Sub-threshold Swing (SS), transconductance, Gate leakage, tunable pinch-off voltage. The optimised JFET shows excellent DIBL short channel effect, which does not only exist in CMOS but also in JFET devices. Based on the output noise performance, the JFET shows more than one decade improvement in the output noise level when compared against a buried channel PMOS from the 0.35μm LOCOS CMOS technology with excellent low noise performance. The integrated JFET can greatly benefit analog applications by offering the low noise performance without compromising on amplifier performance.

Keywords — JFET, MOS, Noise, DIBL

I. INTRODUCTION

Recent years, many consumer communication amplifier products adopted common MOSFET for integrated circuit design as CMOS devices have high input impedance and lower Gate leakage. The low noise pre-amplifier performance significantly depends on the noise of basic CMOS components in the integrated circuits due to internal noise impact on the very first signal stage.

A lot of efforts have been made to improve 1/f noise performance of the primitive CMOS devices. It is well known that buried channel (BC) devices show significantly lower 1/f noise than that of surface channel (SC) devices due to deeper current transport and reduced carrier number fluctuation in Gox/Si interface [1,2]. In addition, it is reported both LOCOS and STI edge effect can influence 1/f noise level [3,4]. However, the direct comparison between STI and LOCOS is not feasible due to different wafer processes between the two technologies. As STI plasma dry etching process can generate plasma damage and surface roughness on the sidewall of the bulk silicon [5], it could degrade noise performance much more for devices with STI process when compared to traditional LOCOS isolated device.

It is reported that JFETs have superior 1/f noise performance than conventional MOSFET[6], but the STI used in the JFET can potentially degrade noise performance due to STI interface impact. Replacing MOSFET with non-STI based JFET in an amplifier has resulted in improved signal-to-noise ratio (SNR) [7]. However, the single Top-Gate JFET device operation was not fully characterized in [7]. And few publications reported STI and silicon interface impact on JFET devices noise in the same CMOS technology.

In this paper, we demonstrated a cost effective JFET integrated in 0.13μm CMOS technology with excellent device characteristic, and it can be an alternative solution for low noise analog amplifier design. By comparing a very low noise buried channel PMOS in 0.35μm LOCOS CMOS technology, the JFET shows more than one decade improvement in the output noise level by using unified noise characterization technique [8].

II. DEVICE STRUCTURE AND EXPERIMENT

The designed P-channel JFET structure is illustrated in Figure 1. CMOS Source/Drain implantation is used for the JFET S/D region and N-WELL pillar acts as Bottom Gate (G’) connection. Standard P-WELL is used for p-channel formation and two n-type dopants control p-channel height of type A JFET, which is integrated with STI process, as shown in Figure 1 (a). Type B JFET has simplified p-channel region but without STI design between Gate and Source/Drain, as shown in Figure 1 (b). An I-LINE PHOTO and Phosphorus/ Boron implantation process is dedicated for the channel formation of the two devices. Meanwhile, two N-JFET devices layout are designed with similar construct as experiment. All the JFET devices are processed in a 0.13μm CMOS MPW using Design of Experiment.

III. ANALYSIS AND DISCUSSION

The Sub-threshold Swing of the JFET device is defined as the change in Gate voltage that produces a one decade increase in the output current. It is observed that the JFET Sub-threshold Swing has significant improvement under both Top and Bottom Gate (GTGB) operation when comparing to under Top Gate (GT) operation only. A measured N-JFET shows steep and improved Id-Vg slope in Figure 2. The GT operation forces voltage at GT terminal and sweep it from 0V to -2.3V, and GB terminal is connected to ground. GTGB operation connects both Top Gate and Bottom Gate with same voltage sweep. Based on our calculation, the Sub-threshold Swing is improved from 160mV/decade to 62mV/decade by
using both Gates operation instead of a single Gate operation. Additionally, the transconductance is maximized. Meanwhile, the Gate leakage (to underneath WELL and P-sub) is suppressed by using two Gates operation method. Furthermore, the result indicates that the JFETs missing Bottom Gate design, which can gain process cost saving with less implantation steps, have weaker channel control ability and the additional leakage concern due to the channel pinched off with Top Gate connecting to underneath WELL.

\[ DIBL = \frac{|V_{p\_sat} - V_{p\_linear}|}{(V_{DH} - V_{DL})} \times 1000 \]

Where \( V_{p\_sat} \) is the pinch-off voltage measured at high Drain voltage (VDD), and \( V_{p\_linear} \) is the pinch-off voltage measured at a low Drain voltage, typically at 0.1 V.

It is also notable in Figure 3, the Gate leakage current (Ig) increases with increasing Gate voltage \( |V_g| \) at higher VDD (Vdh), but it reduces when the JFET channel is close to the pinch-off region (Vg is near to -5V). However, the Gate current is stable at lower VDD (Vdl). It indicates that the Gate to Drain leakage not only reflects widened channel depletion width impact in lateral channel direction which is due to increased reverse voltage bias between Gate to Drain junction, but also worsen the DIBL effect.

The Vp stability against VDD variation can be improved by adjusting S/D junction depletion dopant or optimizing the channel length. In addition, the type B device Gate to S/D spacing can be fine-tuned to meet terminal to terminal breakdown voltage requirement without considering STI design. Figure 4 and 5 shows no DIBL effect for both N and P-JFET optimized devices (5/1.6μm) under two Gates operation as there is no obvious pinch-off voltage dependency on high and low VDD. Figure 6 shows that the Vp is tunable by adjusting channel dopant to meet design requirement.

In short-channel MOS devices, when the channel become shorter, the Drain is close enough to the Gate channel and the Threshold voltage reduces when Drain voltage increases, namely the DIBL (Drain Induced Barrier Lowering) effect. The DIBL effect affects analog Small-Signal parameters (gm/Id, rD), and it is shown to increase output noise Power Spectral Density (PSD) [1][2]. It is also known that DIBL effect can affect not only MOS but also JFET devices [9] in the form of Threshold voltage stability. Figure 3 shows measured I-V curves of an unoptimized JFET device in one of process splits with obvious DIBL of 385mv/V.
for the buried channel device construct. The optimized Type B N-JFET noise is around a factor of 100 lower than the 0.35μm low noise NMOS, as shown in Figure 8. This is due to general surface effect eliminated in the N-JFET as the channel current is far away from silicon interface.

![Figure 4](image1.png)  
**Fig. 4.** Measured Drain current at high & low VDD for an optimized N-JFET for device size 5/1.6μm

![Figure 5](image2.png)  
**Fig. 5.** Measured Drain current at high & low VDD for an optimized P-JFET for device size 5/1.6μm

![Figure 6](image3.png)  
**Fig. 6.** The JFET Pinch-off voltage dependence on implanted channel dose

Furthermore, to have a completed comparison of STI impact on JFETs in the same technology, type A JFETs noise is also measured. Figure 9 shows that for both N and P-JFET, the output noise is degraded with STI design (type A) when compared to without the STI design (type B). It is because of the accumulated current density near the interface of STI and ACTIVE region, as shown in the TCAD simulation of Figure 10. In the JFET devices without STI design, the N-JFET (B) output noise improvement is more prominent than P-JFET (B). It could be that the P-JFET device channel formed by counter doped implantation has more lattice damages, which can be improved by adjusting the related implanted dopant depth or fine tuning the annealing conditions [10].

In addition, in the higher frequency region (near 10kHz), thermal noise is visible for Type B JFET devices as shown in Figure 9. It indicates that there could be a process margin to improve the channel resistance as the JFETs operates as voltage controlled resistors. Lower Rds JFETs can result in lower
thermal noise. This is because thermal noise is influenced by channel resistance and the spectral density of the thermal noise is given by

\[ S_v(f) = 4kTR \]

Where, \( k = 1.38 \times 10^{-23} \text{J/K} \) is the Boltzmann constant, \( T \) is absolute temperature of the resistor \( R \), and \( S_v(f) \) is expressed in \( \text{V}^2/\text{Hz} \).

IV. CONCLUSION

In this paper, we have demonstrated a cost-effective JFET integrated in 0.13\( \mu \)m CMOS technology with improved device DC performance and the noise measurement results show low 1/f noise performance when compared to buried channel PMOS. The results discussed in this paper shows that JFET can be an alternative solution for low noise amplifier design.

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Low-Frequency Noise Behavior of nMOSFETs with Different Al₂O₃ Capping Layer Thickness and TiN Gate

Danghui Wang
School of Materials Science and Engineering
Xi’an Shiyou University
Xi’an, China
wdhyxp@163.com

Eddy Simoen, Bogdan Govoreanu, Stefan Kubicek, Julien Jussot, B.T. Chan, Nard Dumoulin-Stuyck, Iuliana Radu, Dan Mocuta
Imec
Leuven, Belgium
Eddy.Simoen@imec.be

Cor Claeys,
ESAT
KU Leuven
Leuven, Belgium
cor.claeys@kuleuven.be

Abstract—In this paper, the impact of the thickness of the high-κ Al₂O₃ capping layer on the low-frequency noise performance of nMOSFETs with 8 nm SiO₂ and TiN gate metal has been investigated. It is shown that the predominant 1/f noise is governed by the number fluctuations mechanism. The presence of an Al₂O₃ cap increases the noise Power Spectral Density and, hence, the oxide trap density.

Keywords—Low-frequency noise, Al₂O₃ capping layer, Oxide trap density, TiN gate

I. INTRODUCTION

The threshold voltage (Vth) and the effective workfunction of an MOSFET with high-κ/metal gate can be tuned using an appropriate capping layer. Normally, Al₂O₃ is one of the popular capping materials for tuning the Vth of pMOSFETs [1],[2]. At the same time, it has been shown that deposition of a high-κ layer on an SiO₂ interfacial layer increases the effective trap density Nt [2],[3] and degrades the gate stack reliability [4]. These oxide defects can trap and detrapping electrons and holes, which cause number fluctuations of carriers during the transport process in the channel between Source and Drain, giving rise to increased 1/f or flicker noise.

There is currently a strong interest in the implementation of quantum computing on a Complementary Metal-Oxide-Semiconductor (CMOS) platform. Several integration schemes exist, whereby one popular way to define a quantum dot is by electrostatically creating a potential well, a quantum bit as information carrier is stored. Such devices are extremely sensitive to the presence of traps in the vicinity of the qubit, so that defect characterization is of high importance for a proper understanding, modeling and optimization of the qubit performance.

In this paper, the impact of the thickness of the high-κ Al₂O₃ capping layer on top of an 8 nm SiO₂ gate dielectric of bulk nMOSFETs with TiN metal gate has been investigated using the input characteristics and low-frequency noise. It is shown that the predominant 1/f noise behaves according to the number fluctuations model. The oxide trap density near the SiO₂ interface has been extracted and compared for the different Al₂O₃ capping layer thicknesses using the power spectral density (PSD) of the input-referred voltage noise, showing an increase compared with the cap-free references.

II. EXPERIMENTAL DETAILS

The n-channel devices have been defined by e-beam lithography, with different geometries W×L: 0.1 μm×1 μm; 1 μm×1 μm and 1 μm×10 μm. Three types of gate dielectric stacks, represented schematically in Fig. 1, have been processed, consisting of 8 nm thermal SiO₂; 8 nm SiO₂ + 5 nm Al₂O₃ deposited by Atomic Layer Deposition (ALD) and 8 nm SiO₂ + two times 5 nm Al₂O₃.

![Fig. 1. Schematic of the three types of gate stack studied.](image)

According to the input drain current – gate voltage (I_d-V_g) transfer characteristics in linear operation (V_d=0.05 V), the presence of an Al₂O₃ cap results in a higher threshold voltage Vth. The Vth increases from 0.4 V to 1.4 V (5 nm Al₂O₃) and to 1.55 V (10 nm Al₂O₃) for the 1 nm×10 μm nMOSFETs in Fig. 2.

For noise measurements, the devices were biased in the linear operation region at a drain voltage V_d=0.05 V and the drain current was stepped from weak to strong inversion, from a drain current of about 10 nA up to 10 μA. The input-referred voltage noise Power Spectral Density (PSD) (S_{V_d}) is derived from the measured drain current noise PSD (S_I) by dividing with the transconductance squared (g_m²). As shown in Fig. 3, the obtained noise spectra are predominantly of the 1/f noise type (flicker noise) with γ close to 1.

III. RESULTS AND DISCUSSION

According to Fig. 4, the normalized current noise PSD (S_I/L²) at a frequency f=10 Hz exhibits a plateau in weak inversion and a roll off with L² in strong inversion, for all devices investigated. This strongly suggests that the 1/f noise is due to number fluctuations [8]-[11] and can be
interpreted in terms of trapping/detrapping by defects in the SiO₂ at a distance between about 1 and 2 nm from the interface with the silicon substrate. This is further confirmed by the comparison of the normalized noise with $g_m^2/I_d^2$, in Fig. 5.

Fig. 2. Input characteristics corresponding with the reference nMOSFETs (upper), the nMOSFETs with 5 nm Al₂O₃ (middle) and the devices with 10 nm Al₂O₃ (lower).

Fig. 3. Low-frequency noise spectra for a 1 µm x 1 µm reference nMOSFET with 8 nm SiO₂, measured at $V_{DS}=0.05$ V and different $I_d$ (10 nA, 100 nA and 1 µA).

Fig. 4. Normalized current noise PSD as a function of the drain current in linear operation for 1 µm x 10 µm nMOSFETs, corresponding with the three gate stacks studied.

Fig. 5. Normalized current noise PSD at 10 Hz as a function of the drain current in linear operation for 1 µm x 10 µm nMOSFETs, corresponding with the three gate stacks studied. It is compared with $g_m^2/I_d^2$ in each case.
For MOSFETs, the number fluctuations 1/f noise PSD can be used to derive the oxide trap density at the interface of the gate stacks. This follows from the correlated mobility fluctuations model, given by [8]-[11]:

\[ S_{\text{vfb}} = S_{\text{vfb}} \left( 1 + \alpha C_{\text{EOT}} \epsilon_{\text{ox}} \right) \]  

(1a)

with the flat-band voltage noise PSD given by:

\[ S_{\text{vfb}} = \frac{q^2 N_{\text{tot}}}{W l k_{\text{B}} T C_{\text{EOT}} A f} \]  

(1b)

In Eq. (1a), \( \alpha \) is the Coulomb scattering coefficient, \( \mu_{\text{e}} \) the effective low-field mobility, while in Eq. (1b) \( q \) is the elementary charge, \( k_{\text{B}} T \) is the thermal energy, \( W \) and \( L \) the effective width and length of the gate, \( f \) is the frequency and \( C_{\text{EOT}} \) is the capacitance density corresponding with the Equivalent Oxide Thickness (EOT). The parameter \( \alpha \) is the attenuation factor of the electron wave-function in the gate oxide. If assuming elastic tunneling, one can demonstrate a unique relationship between the trap depth \( z \) and the frequency \( f \), given by [9]-[11]:

\[ z = \alpha^{-1} \ln \left( 1 / (2 \pi f \tau_{\text{ox}}) \right) \]  

(2)

with \( \tau_{\text{ox}} \) the Shockley-Read-Hall recombination lifetime at the Si/SiO\(_2\) interface, for which a typical value of 10\(^{-8}\) s is assumed. Thus, a 1/f noise spectrum can be converted into an oxide trap density profile with depth with respect to the Si/SiO\(_2\) interface according to Eqs (1)-(2).

Figure 6 compares the \( N_{\text{tot}} \) values for the three studied gate stacks at 10 Hz, indicating that the deposition of an Al\(_2\)O\(_3\) cap layer results in an increase of \( N_{\text{tot}} \). It is also clear that the trap density profile in the SiO\(_2\) is rather uniform, which is in line with the frequency exponent close to 1.

![Fig. 6. Oxide trap density versus trap depth derived from the flat-band voltage noise PSD for the three gate stacks studied (1 \( \mu \)m x10 \( \mu \)m nMOSFETs).](image)

From the foregoing, it can be concluded that the deposition of an Al\(_2\)O\(_3\) layer on top of 8 nm SiO\(_2\) leads to a degradation of the oxide quality, due to the increase of the oxide trap density in the vicinity of the Si/SiO\(_2\) interface. This is in line with previous observations for Al\(_2\)O\(_3\) cap pMOSFETs [1],[2],[4]. There, it was shown that the deposition of such a Vth shifter yields a roughly one decade increase in \( N_{\text{tot}} \), for the case of an SiO\(_2\) interfacial oxide with 2 nm H\(_2\)O on top. However, there the traps were aligned with the valence band in silicon. In nMOSFETs, we are probing oxide states close to the silicon conduction band.

One can speculate on the origin of such a degradation of the oxide. It has been for example put forward that the TiN metal gate can cause a gettering of oxygen, creating oxygen vacancy traps [12]-[14]. However, from the result of the reference transistors with a TiN gate on top of SiO\(_2\) it can be concluded that this is not the main reason for the oxide trap creation. Indeed, the values found for the trap density in Fig. 6 are typical for good thermal oxide transistors [8]-[11],[15]. Apparently, it is the deposition of the Al\(_2\)O\(_3\) layer that leads to a higher \( N_{\text{tot}} \) in the gate stacks with a cap layer. This could possibly result from the in-diffusion of Al during ALD, which has been shown to correspond with a higher trap density [1],[2].

IV. CONCLUSIONS

The low-frequency noise of nMOSFETs with 8 nm SiO\(_2\) and different Al\(_2\)O\(_3\) cap thickness has been investigated. It is shown that the higher 1/f noise PSD found for the capped devices can be ascribed to an increase in the oxide trap density, possibly related with the in-diffusion of Al.

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Noise in Interacting Biological Systems

1st Tomislav Stankovski
Faculty of Medicine
Ss Cyril and Methodius University
Skopje, North Macedonia
t.stankovski@ukim.edu.mk

2nd Peter V. E. McClintock
Department of Physics
Lancaster University
Lancaster, United Kingdom
p.v.e.mcclintock@lancaster.ac.uk

3rd Aneta Stefanovska
Department of Physics
Lancaster University
Lancaster, United Kingdom
aneta@lancaster.ac.uk

Abstract—Biological systems are never isolated, usually oscillatory, and invariably subject to noise and fluctuations that may be of either internal or external origin. We present a methodological framework for studying the deterministic interactions of biological oscillatory systems, while at the same time decomposing and evaluating the noise strength. Based on dynamical Bayesian inference, the method models coupled phase oscillators in the presence of dynamical noise. We demonstrate first the potential and the precision of the method on a predefined numerical system. Then we illustrate its usefulness in detecting how the noise strengths from three human physiological systems – the heart, the lungs and the brain – are affected by general anaesthesia. The results demonstrate the potential of the method for detecting and quantifying noise from biological dynamical systems, quite generally.

Index Terms—dynamical Bayesian inference, noise, interactions, coupling functions, biological systems

I. INTRODUCTION

Biological systems are never isolated, usually oscillatory in nature, and invariably subject to many different perturbing influences [1]–[4], often resulting in fluctuations and variability around their dynamical states. Some of these influences are of known deterministic origin; others are of currently unknown, seemingly random, origin and may be treated as stochastic noise [5]. The noise can act at many different levels, ranging from the most basic molecular, sub-cellular processes up to the dynamics of tissues, organs, organisms, populations and their interactions.

Often, oscillatory biological systems mutually interact, with the noisy perturbations in addition. Here, we focus on such systems – studying their deterministic interactions while, at the same time, decomposing and quantifying the noise.

In particular we will study oscillatory interactions through their phase dynamics [6], subject to white noise, described by bivariate stochastic differential equations. We will use the method of dynamical Bayesian inference to model the coupled phase oscillators in the presence of noise [7], based on real biological data. The modeling will yield the deterministic interactions and the coupling functions [8] between the systems, as well as the stochastic noise strength. The deterministic interactions described by the coupling functions have recently been studied intensively [8]–[14]. Here we will instead focus primarily on the stochastic part of the inference, i.e. on the inferred noise strength, and will discuss how it changes in different states of the biological system.

After first presenting the inference method, we will demonstrate its usefulness in a numerical example of a predefined model of coupled phase oscillators. The stochastic model will be inferred from the numerically generated time-series. The results of the inference will provide an insight into the properties and precision of the inference on the deterministic dynamics, as well as on the noise strength.

The approach will then be used to illustrate its usefulness in detecting how the noise strengths in human subjects are affected by general anaesthesia. We will study three states: awake resting, anaesthesia with propofol, and anaesthesia with sevoflurane anaesthetic [15], [16]. We will study three human physiological systems – the heart, the lungs and the brain. The recordings include the electrocardiogram (ECG), the respiration signal and the electroencephalogram (EEG), all measured simultaneously in individual subjects. We will be interested in three groups of noise strengths: (i) for the heart; (ii) for respiration; and (iii) for different brainwaves (including delta, theta, alpha and gamma brainwaves), noting how they are affected by general anaesthesia with the two different anaesthetics.

II. DYNAMICAL BAYESIAN INFERENCE

In order to tackle the problem in hand – to infer the noise from the data of interacting biological systems – we will use a method based on dynamical Bayesian inference [7], [17]. The method is able to infer separately the deterministic part of the model, and the stochastic part of the dynamics. Considering such a model defined by stochastic differential equations (SDEs), we will infer the noise as a residual part of the dynamics that is complementary to the deterministic part of the model.

So we consider oscillating biological systems that mutually interact. Their self and interacting dynamics will constitute the deterministic part of the model. Moreover, due to their oscillatory nature we will concentrate on the phase dynamics reduction approach [6], leaving a single time-series for the instantaneous phase of each system. Having these as input, we will then apply the method of dynamical Bayesian inference to find the coupling functions between the systems and the
noise perturbing them. Our focus here will be on the strength of the noise.

We therefore consider a model pair of coupled phase oscillators [6] described by the stochastic differential equation:

$$\dot{\phi}_i(t) = \omega_i(t) + q_i(\phi_i, \phi_j, t) + \xi_i(t),$$

with $i \neq j$ for $i, j = \{1, 2\}$ and where $\omega_i(t)$ is the parameter for the natural frequency. The deterministic part given by the base functions $r_i(\phi_i, \phi_j, t)$ describes the self and the interacting dynamics. The external stochastic dynamics $\xi_i(t)$ is considered to be Gaussian white noise $\langle \xi_i(t) \xi_j(\tau) \rangle = \delta(t - \tau)D_{i,j}$. Due to the periodic nature of the deterministic dynamics, the base functions can be decomposed into infinite Fourier series:

$$q_i(\phi_i, \phi_j, t) = \sum_{k=-\infty}^{\infty} \sum_{r,s} \tilde{c}(t)_{r,s,k} e^{i 2 \pi \phi_i(t)} e^{i 2 \pi \phi_s(t)}.$$ 

In practice, however, the dynamics is well-described by a finite number of Fourier terms, so that one can rewrite the phase dynamics as:

$$\dot{\phi}_i(t) = \sum_{k=-K}^{K} \tilde{c}^{(i)}_{k}(t) \Phi_{i,k}(\phi_i, \phi_j, t) + \xi_i(t),$$

where $\tilde{c}^{(i)}_{k}$ are the $K$ most important Fourier components. The Fourier components $\Phi_{i,k}$ act as base functions for the dynamical Bayesian inference, through which the parameters $\tilde{c}^{(i)}_{k}$ are evaluated. In the analysis we used a second-order Fourier expansion ($K = 2$) because the signals come from narrow-band intervals. Two phase time series and the order of expansion $K$ act as inputs for the phase model which is inferred for each interaction (e.g. δ-α), from each subject.

Dynamical Bayesian inference [7], [9] enables us to evaluate the model parameters $\tilde{c}_k$, which gives the time-evolving coupling functions and coupling strength in the presence of noise. From Bayes' theorem one can derive the minus log-likelihood function, which is of quadratic form. Assuming that the parameters are represented as a multivariate normal distribution (with mean $\bar{c}$, and covariance matrix $\Sigma = \Sigma^{-1}$), and given such a distribution for the prior knowledge using the likelihood function, one can calculate recursively [7], [17] the posterior distribution of the parameters $\tilde{c}_k$ using only the following four equations:

$$D = \frac{h}{L} \left( \phi_n - c_k \Phi_k(\phi_{*n}) \right)^T \left( \phi_n - c_k \Phi_k(\phi_{*n}) \right),$$

$$r_w = \left( \Xi \right)_{k,w} = \frac{1}{L} \sum_{n=1}^{N} \frac{\partial \Phi_k(\phi_{*n})}{\partial \phi_n} \left( D^{-1} \right) \phi_n +$$

$$\frac{1}{2} \frac{\partial \Phi_k(\phi_{*n})}{\partial \phi_n} \left( D^{-1} \right) \phi_n,$$

$$\Xi_{k,w} = \left( \Xi \right)_{k,w} = \frac{1}{L} \sum_{n=1}^{N} \frac{\partial \Phi_k(\phi_{*n})}{\partial \phi_n} \left( D^{-1} \right) \phi_n,$$

$$\bar{c}_k = \frac{1}{L} \sum_{n=1}^{N} \phi_n.$$

where summation over $n = 1, \ldots, N$ is assumed, and summation over the repeated indices $k$ and $w$ is implicit. We used informative priors and a special procedure for the propagation of information between consecutive data windows [7], [18], which allowed inference parameters that varied with time (for implementation, software toolbox and usage see [19]).

Given its ability to infer time-varying and noisy dynamics, the Bayesian method is especially well-fitted for applications to biological and physiological signals, like for example the EEG, ECG and respiration signals.

Once we have the inferred parameters $\bar{c}$, we can calculate the deterministic coupling quantities, including the coupling strength and the form of the coupling function. Complementary to these are the noise strengths $D_{i,j}$ inferred for each system with phase $\phi_i(t)$. The inference also gives the correlated noises $D_{i,j}$, between two systems given with phases $\phi_i(t)$ and $\phi_j(t)$.

III. NUMERICAL EXAMPLE

Before considering applications to biological data, we first demonstrate the Bayesian inference method on a simple numerical example. We consider a pair of interacting phase oscillators subject to white noise – a numerical example of a type of dynamics we will analyse later from biological systems. The systems are given by the following stochastic differential equations:

$$\dot{\phi}_1 = \omega_1 + a_1 \sin(\phi_2) + a_2 \sin(\phi_2 - \phi_1) + \xi_1(t),$$

$$\dot{\phi}_2 = a_2 + 2 \sin(\phi_1) + a_4 \sin(\phi_1 - \phi_2) + \xi_2(t).$$

Each phase oscillator is described by its frequency parameter $\omega_1 = 2$, $\omega_2 = 4.53$, and the parameters for their interaction dynamics $a_1 = 0.4$, $a_2 = 0.6$, $a_3 = 0.8$ and $a_4 = 0.5$. The noises are set to be white Gaussian and mutually uncorrelated with the following noise strengths $D_1 = 0.04$ and $D_2 = 0.02$. The two oscillators are not synchronized. Fig. 1 shows samples from the resultant time series to which dynamical Bayesian inference is to be applied.

In this example we know beforehand the phase model and the deterministic terms on the rhs of the coupled system (3) that are the actual base functions to be used for inference of the six parameters $(\omega_1, \omega_2, a_1, a_2, a_3, a_4)$. The inference results from a single block of data (40 seconds long) from the first system are presented in Table III. The agreement between the actual (intrinsic) parameters and their inferred values is good, and the method evidently works to high precision. In

![Fig. 1. The instantaneous phases $\phi_1(t)$ and $\phi_2(t)$ simulated by the mathematical model (3). The effect of noise on the phase is more clearly visible in the enlarged inset on the right. The dark line and the inset show $\phi_1(t)$, and the light line shows $\phi_2(t)$.](image-url)
Parameters $\omega_1 \ a_1 \ a_3 \ D_1 \ D_{12}$

<table>
<thead>
<tr>
<th></th>
<th>$\omega_1$</th>
<th>$a_1$</th>
<th>$a_3$</th>
<th>$D_1$</th>
<th>$D_{12}$</th>
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<td>0.8</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>Inferred means</td>
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<td>0.4040</td>
<td>0.8017</td>
<td>0.0400</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

TABLE I
RESULTS FROM INFERENCE OF THE $\phi_1(t)$ DYNAMICS OF MODEL (3).

addition, the intensity and the correlations of the noise are inferred very precisely.

IV. BIOLOGICAL EXAMPLE – NOISE IN THE PHASE DYNAMICS OF CARDIAC, RESPIRATION AND NEURAL OSCILLATIONS AS AFFECTED BY ANÆSTHESIA

In order to demonstrate the potential of this approach in tackling the noise and fluctuations in biological systems, we now consider an example of dynamical Bayesian inference applied to the phase dynamics of interacting biological oscillators. In particular, we study the phase dynamics of oscillations from the cardiac, respiration and neural activity, in each case measured simultaneously from the same subject. Moreover, this was done both in the awake state and under general anaesthesia induced by either propofol or sevoflurane i.e. three states were compared: the awake (A); anæsthetized with propofol (P); and anæsthetized with sevoflurane (S). The inference gives two outputs, the deterministic dynamics and the noise strength. The deterministic part comprises the self-dynamics of each of the cardiac-respiration-neural oscillations plus those related to the interactions described by the inter-oscillator coupling functions. The focus here is on the latter output and, in particular, on the noise strength of each oscillation.

The measurements included the electrocardiogram (ECG), the respiration signal measured with an elastic belt proportional to the thorax expansion, and an electroencephalogram (EEG) signal. There were 25 awake and 29 anaesthetized healthy subjects, aged 18 to 60 years, who were about to undergo elective surgery. Of the 29 anaesthetized subjects, 14 were given propofol and 15 sevoflurane. The oscillation intervals were estimated by standard digital filtering procedures, including a FIR filter followed by a zero-phase digital filtering procedure to ensure that no time or phase lags were introduced by the filtering. The boundaries of the intervals extracted were: $\delta = 0.6 – 2$Hz for the cardiac oscillations from the ECG signal, $\theta = 0.145 – 0.6$Hz from the respiration signal, while for the brainwaves from the EEG signal they were $\alpha = 7.5 – 14$Hz, $\beta = 14 – 22$Hz, and $\gamma = 22 – 100$Hz. Special care was taken in dealing with frequency spillage between intervals, heart artifacts, and powerline artifacts [20]. The phases of the filtered signals were estimated by use of the Hilbert transform and the protophase-to-phase transformation [21]. To determine whether the deterministic coupling relationships were genuine, or just happened by chance, we used surrogates data testing [22], [23]. From the large number of investigated relationships, only those exhibiting a statistically significant difference compared to their corresponding surrogates were retained and, similarly, the noise from the $\beta$ oscillation was not presented as the couplings to this oscillation were insignificant and there were no significant noise difference. To present visually the differences between the distributions we used standard boxplots that refer to the descriptive statistics (median, quartiles, maximum and minimum). Extended technical details can be found in [15], [16].

In Fig. 2 we present the noise strength results from the heart and the lung activity, representing one of the most important parts of the cardiovascular system. The cardiac noise strength Fig. 2 (a) did not change significantly between the three states, awake and anaesthetized with propofol and sevoflurane. There was more variation (wider box plot) in the anaesthetized states, but this change was not significant. The respiration noise strength Fig. 2 (b), on the other hand, was significantly different i.e. it decreased in the two anaesthetized states in comparison to the awake state. There was also significant difference between the two anaesthetics, indicating that sevoflurane reduced the noise level more than the propofol induced anesthesia.

![Fig. 2](image-url)
The effect of general anæsthesia on the noise strength of the neural cognitive brainwaves is presented in Fig. 3. There was no much difference on the noise level in the delta oscillations Fig. 3 (a). In the theta oscillation band (b) the anæsthesed state exhibited different noise strengths both for propofol and sevoflurane, the noise intensity being higher than in the awake state. In the case of the higher frequency oscillations, alpha in (c) and gamma (d), anæsthesia induced a significant decrease in noise strength compared to that in the awake state. In none of the neural oscillations (all panels in Fig. 3) was there any significant difference in noise strength dependant on anaesthetic used, propofol or sevoflurane.

V. Conclusion

This study illustrates the potential of dynamical Bayesian inference for describing the noise and fluctuations from biological oscillatory systems. Our demonstration on the numerical example showed that the method is able to infer both the deterministic and the stochastic dynamics with high precision. The application to a biological system demonstrates that anæsthesia changes, not only the deterministic couplings, but also some of the random fluctuations acting on the oscillations. The decrease in the noise level in \( \alpha \), \( \gamma \) and respiratory oscillations might be because the processes associated with the onset of anæsthesia induce order, coupling and coherence of the oscillations [24], [25].

It is worth pointing out that, without direct observation or understanding of the processes that generated the noise, we infer it just as a residual. It remains conceivable that some parts of the noise are attributable to deterministic non-autonomous influences [26]–[29], e.g. from some of the other (finite number of) processes in the human body. This inherent limitation of the study is, of course, shared, with investigations of noise in other contexts. The nature and origin of biological noise raises questions of some depth and subtlety, and especially for an inverse approach they are ones that still remain largely open and unanswered in general terms.

Finally, we comment that the methodological framework described here in relation to oscillatory biological systems also carries wider implications for applications to dynamical systems, quite generally.

References

Parameter change of propagating waves in dominant arms and non-dominant arms using multichannel surface elecrogram

Marzieh Aliaabadi Farahani  
Graduate School of Informatics and Engineering  
The University of Electro-Communications  
Tokyo, Japan  
a1740002@edu.cc.uoe.ac.jp

Miki Haruna  
Graduate School of Informatics and Engineering  
The University of Electro-Communications  
Tokyo, Japan

Kota Akehi  
Graduate School of Informatics and Engineering  
The University of Electro-Communications  
Tokyo, Japan

Kazuyuki Mito  
Graduate School of Informatics and Engineering  
The University of Electro-Communications  
Tokyo, Japan

Tota Mizuno  
Graduate School of Informatics and Engineering  
The University of Electro-Communications  
Tokyo, Japan

Naoki Itakura  
Graduate School of Informatics and Engineering  
The University of Electro-Communications  
Tokyo, Japan

Abstract—In this study, muscle potentials were obtained by applying various loads to the dominant arm and the non-dominant arm, and analysis was performed using the Multi Channel method (m-ch) to investigate the muscle contraction mechanism. In the conventional methods, only the size of the integrated electromyogram was compared, but according to the m-ch method, the type of motor units active in the dominant arm and the non-dominant arm, and the tendency of motor units to participate in the contractile activity of each arm was clarified.

Keywords—dominant arm, non-dominant arm, surface EMG, motor unit, multi channel method

I. INTRODUCTION

Skeletal muscle is a type of striated muscle, attached to the skeleton and they are used to facilitating movement, by applying force to bones and joints via contraction. Skeletal muscle is made up of thousands of muscle fibers that run the length of the muscle. An action potential occurs when the membrane potential of a specific axon location rapidly rises and falls. Action potential of muscle fibers is related to the neuromuscular junction which is a chemical synapse formed by the contact between a motor neuron and a muscle fiber. Within a muscle, the muscle fibers are functionally organized as motor units (MU). A motor unit consists of a single motor neuron and all of the muscle fibers it innervates. There are 2 different types of motor units: 1) Fast motor units 2) Slow motor units. While the Slow motor units do not produce as much force as fast motor units, they are able to develop the force in a relatively long period and they are very resistant to fatigue compare to Fast motor units. The propagation velocity of the action potential is called muscle fiber conduction velocity and surface EMG is used for measurement.

In the previous work, Kosuge et al. proposed a method (m-ch method) of extracting all propagation waves from multi-channel surface EMG and quantitatively determining the velocity and number of them to investigate the features. A calculation method was designed to measure the conducting waves’ propagation velocity.

![Fig. 1. The structure of muscle](image)

Fig. 1. The structure of muscle

As an application for efficient muscle strengthening training, consideration is given to the type of motor unit involved in contraction from the parameter change of the dominant and non-dominant arms obtained by using the m-ch method. There are few studies on dominant and non-dominant arms using Integrated Electromyogram and they had been considered just on amplitude. There is no consideration on the type of MU, such as the slow and fast motor units.

In this study, we analyzed the difference of muscle fibers composition and influence on cortex and spinal cord from the type of MU in dominant and non-dominant arms. It is expected that the results of this research will be a new indicator for strengthening muscle during rehabilitation and training.

II. EXPERIMENT

In the experiment, the biceps brachii muscle of left and right hands was used as the test muscle. The subjects were 5 healthy adult males with an average age of 22. The subjects were asked to keep the elbow joint angle at 90 degrees in the sitting position to measure 100% of their maximum voluntary contraction (MVC); maximum muscular strength. After that the subjects were asked to keep the same posture for 2 different experiments.

1) 30 seconds with 10% of the maximum exertion muscle load of each arm
2) 10 seconds with 40% of the maximum exertion muscle load of each arm. 

And data were acquired. For cutoff frequency, the high-pass filter (Low Cut Filter) was set to 5Hz and the low-pass filter (High Cut Filter) was set to 1kHz. The amplification factor was 80dB and the sampling frequency was 5kHz. All the data was input to a computer to be analyzed. Data was collected for 10 minutes. Next, collected data was analyzed with the high-pass filter (Low Cut Filter) at 5Hz, and the finite impulse response (FIR) filter was added just in case of being sure to define right filters. The experimental system is shown in Fig. 2 and the results and analysis are presented in this paper.

III. METHOD

In this research, Multi Channel (m-ch) method had been used for analysis. The interval where the zero crossing occurs twice is defined as the analysis unit, and the channel with the analysis unit is defined as the conduction source. The adjacent channels around the conduction source that are within 10 ms before and after the starting point of it (the conduction source) are defined as the conduction destination. The propagation determination is performed over multiple channels, and then the propagation velocity is calculated. In the propagation judgment, if the shape of waveforms between channels is similar, it is determined to be a propagation wave, and the similarity ratio and wavelength ratio are 0.9 Hz or more, and the amplitude ratio is 0.7 dB or more.

IV. RESULT AND DISCUSSION

1) It can be seen that by increasing the load from 10% MVC to 40% MVC, propagation waves with high amplitude values for both dominant and non-dominant arms were extracted. On the other hand, the number of propagating waves extracted per second from the dominant and non-dominant arms were different depending on the load applied.

For a load with 10% MVC, the number of propagating waves in the dominant arm was extracted more than in the non-dominant arm. On the other hand, when the load is 40% MVC, the number of propagating waves in the non-dominant arm was more than the dominant arm.

For 10% MVC, which is likely similar to a load which will be used even in daily life, it is speculated that there are many
MUs of the dominant arm that participate in the contraction activities compare to the non-dominant arm. For 40% MVC, which is an extraordinary load, in the non-dominant arm, it is considered that more Mus ignite than the dominant arm to bear the load. The results are shown in figure 3.

2. For 10% MVC load, it was confirmed that the distribution spreads relatively similar in the dominant arm among multiple subjects. In the case of a small load (10% MVC) that can be felt in daily life, the Mus of dominant arm participates in the activity, so it is possible that a certain distribution appears. On the other hand, it was possible that if a large load (40% MVC) was used that could not be felt in daily life, a large number of Mus which participate in muscle contraction might be seen on the dominant and non-dominant arms in order to be able to carry the excessive load. For 40% MVC load, the MU needs to perform contraction activity in a large block, so the distribution is considered to be large. The results are shown in figure 4.

3. About the time-dependent change of the propagation wave parameter extracted by 3ch propagation judgment of m-ch method, it investigated about the waveform feature in measurement of load 10% MVC for 30 seconds. The amplitude value, propagation velocity and wavelength on the vertical axis and time on the horizontal axis are shown in Figures 5 and 6 respectively. There was no tendency for the amplitude value to change with time.

It can be seen from Fig. 5 that the non-dominant arm tends to extract the propagating wave with a little faster propagation speed than the dominant arm, with respect to the temporal change in the propagation speed. In the non-dominant arm, it is considered that not only the slow Mus but also the fast Mus temporarily participated in the muscle contraction for a load of 10% MVC. Since fast MU is a muscle fiber that is more easily fatigued than slow muscle, the number of Mus participate in muscle contraction decreased after 20 seconds from the start of measurement. In the non-dominant arm which less frequently used as compared to the dominant arm, the MU of the fast muscle tends to participate in the activity over time for the load of 10% MVC.

As shown in Fig. 6, the wave length in non-dominant arm tended to be higher than the dominant arm. It has been thought that for a daily load of 10% MVC, the number of igniting MU of the non-dominant arm is higher than the dominant arm. In addition, since the wavelength of the non-dominant arm tends to increase in 10 to 20 seconds, the effect of muscle fatigue can be seen. It is assumed that in the non-dominant arm, which is used less frequently on a daily basis, the effect of fatigue tends to be more visible than the dominant arm, even with a 10% MVC load.

V. Conclusion

In this study, various loads were applied to the dominant arm and the non-dominant arm to obtain muscle potentials. The analysis method was performed in this research is Mukti channel method and it had been used to investigate the muscle contraction mechanism in the dominant arm and the non-dominant arm. In the methods used in other researched, only the size of the integrated electromyogram was compared, but according to the m-ch method, the type of motor unit active in the dominant arm and the non-dominant arm, and the tendency of motor units to participate in the contractile activity of each arm was clarified.

REFERENCES


Method to extract latent semantic components from noisy categorical time-series data applied to human sleep stage data

Ikuhiro Yamaguchi
Graduate School of Education
The University of Tokyo
Tokyo, Japan
ikuhiro@p.u-tokyo.ac.jp

Akifumi Kishi
Graduate School of Education
The University of Tokyo
Tokyo, Japan

Fumiharu Togo
Graduate School of Education
The University of Tokyo
Tokyo, Japan

Yoshiharu Yamamoto
Graduate School of Education
The University of Tokyo
Tokyo, Japan

Abstract—A method to extract latent semantic components from noisy categorical time-series data based on the Takens time-delay-embedding method and singular value decomposition is presented. A demonstration of this method to analyze a sleep stage time-series is demonstrated. The first component extracted by this method, i.e., the component with the largest singular value can be considered the circadian rhythm. The second component, which exhibits damping oscillation, can be interpreted as the ultradian rhythm, and matched with the moving-averaged $\phi_{i}$ which is an estimation of the cortico-thalamo-cortical loop strength calculated from the corresponding electroencephalogram data using the method previously reported. The sleep stage is treated as a nominal variable instead of an ordinal variable in this method; however, the quantitative variation of sleep state is extracted from the sleep stage time-series. We believe that this result suggests the validity and usefulness of both the methods, i.e., the method reported in the present study and the method reported in a previous study.

Index Terms—categorical time-series, latent semantic component, sleep, electroencephalography, cortico-thalamo-cortical loop

I. INTRODUCTION

Sleep is directly related to overall wellness and has been actively studied by medical scientists or medical engineers, using multifarious methods [1], [2]. However, in such methods, the categorization of sleep state into stages has been basic [1], and it is recognized that the dynamics of sleep stage transition is important [3]. The American Academy of Sleep Medicine (AASM) divides sleep into five stages; Wake, N1 (= Non rapid eye movement sleep 1), N2, N3, and REM (= rapid eye movement) [4]. From the viewpoint of signal processing, the sleep state stages can be interpreted as the reduction of high-dimensional data into low-dimensional data. The high-dimensional data is physiological, which includes the electroencephalogram (EEG), electro-oculography (EOG), and electromyogram (EMG). The low-dimensional data is qualitatively categorized data. Data reduction such as this is currently being actively researched in the field of machine learning or artificial intelligence [5].

However, further analysis after obtaining the sleep stage time-series is not easy because sleep stage data is categorical and not quantitative. That is, obtaining a sleep stage time-series has the advantage of dimensional reduction and the disadvantage that it is not quantitative. Therefore, a few quantification methods have been proposed. For example, in sleep restoration gain (SRG) [6], REM, N1, N2, and N3 are quantified as 0, 1/1.5, 1, and 1.5, respectively. Wake is quantified as -1.5 if the previous stage is not wake and -1 if the previous stage is wake. Using this method, we can quantize sleep quality. However, the method is arbitrary in the translation of categorical data into qualitative data. On the other hand, some probabilistic methods such as Markov model [3], [7]–[11] or spectral entropy [12]–[17] treat the sleep stage time series just as categorical and not quantitative data.

The Markov model that treats the sleep stage time-series as pure Markov chain was proposed at first [7]. However, the model could not reproduce the observations well, and an improved model that considers the time duration of each stage as a probabilistic process was proposed [8]. This model agrees reasonably with the observations, and it has been applied to studies on chronic fatigue syndrome [3], sleep apnea [9], chronic fatigue syndrome with or without fibromyalgia [10], ultradian REM sleep rhythm [11], etc. Recently, multiorder, Markov models were investigated to analyze sleep stage dynamics more accurately [11], [15].

For spectral entropy, an evaluation method using Walsh function or Haar function was proposed [12], and its relation with sleep fragmentation, daytime sleepiness [13], neonatal neurologic function [14], [16], sleep disorder [15], etc. have been reported.

However, ‘trends’ ( [18], [19] ) of sleep stage dynamics, such as oscillation with a period of approximately 90 min or gradual change from deep to light sleep, cannot be analyzed well by these methods. Hence, we propose a method to extract latent semantic components from noisy categorical time-series data based on Takens time-delay-embedding method [20] and singular value decomposition (SVD) [21]–[30]. While Fourier transform, moving averages (MA) [18], [19] or empirical mode decomposition (EMD) [31], [32] are suitable for quantitative data time-series decomposition, SVD appears to be suitable.
for qualitative data time-series decomposition, which is now actively studied in the field of natural language analysis [22]. In this study, we show the calculation process of our method in section II, and show an application example of the method in section III.

II. METHOD

A. Data Expression

First, following AASM, we divide sleep states into five stages: Wake, N1, N2, N3, and REM. We express these stages as (1 0 0 0 0), (0 1 0 0 0), (0 0 1 0 0), (0 0 0 1 0) and (0 0 0 0 1), respectively. Because we treat the stage data as nominal data, the order of this representation is arbitrary. The arbitrariness is ensured because the permutation of the data matrix columns is represented by an orthogonal matrix as described later. This expression is the same as that in [9]. Let X be a data matrix with row numbers corresponding to time, and row vectors corresponding to the expressed sleep stages. The following is an example.

\[
X = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
\vdots & & \ddots & & \vdots \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]

We express each column vector of X in lowercase bold;

\[
X = [x_1, x_2, x_3, x_4, x_5].
\]

We express the t-th row component of the column vector \(x_j\) as \(x_j^{(t)}\). In this study, we assume the sleep stage time-series data as a 24 h cyclic data with the time interval of 30 s, and set \(T = 24 \text{h} \times 60 \text{min} \times 60 \text{s} \div 30 \text{s} = 2880\).

B. Normalization

Next, we calculate the normalized data matrix Y as

\[
\bar{x}_j = \frac{1}{T} \sum_{t=1}^{T} x_j^{(t)}, \quad j = 1, 2, \ldots, 5.
\]

\[
\sigma = \left( \frac{1}{T} \sum_{t=1}^{T} (x_j^{(t)} - \bar{x}_j)^2 \right)^{1/2},
\]

\[
y_j^{(t)} = \frac{1}{\sigma} (x_j^{(t)} - \bar{x}_j),
\]

\[
Y = [y_1, y_2, y_3, y_4, y_5].
\]

C. Embedding

Next, we embed Y into a high dimensional phase space and obtain an extended data matrix Z using Takens time-lag method [20];

\[
Z_{(m)}^{(t)} = Y_{(t-m)}, \quad m = 1, 2, \ldots, M - 1.
\]

\[
Z = [Z_{(0)}, Z_{(1)}, \ldots, Z_{(M-1)}].
\]

In this study, we set \(M = 64\), corresponding to the 32-min time-window.

D. Singular Value Decomposition

We use the SVD [21]–[30];

\[
Z = USV^T,
\]

where \(U\) is \(T \times 5M\) orthogonal matrix, \(V\) is \(5M \times 5M\) orthogonal matrix, and \(S\) is \(5M \times 5M\) diagonal matrix with singular values \(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{5M} \geq 0\).

\[
S = \begin{bmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_{5M}
\end{bmatrix}.
\]

E. Extracting Latent Semantic Components

We term the j-th column vector of matrix \(U\), \(u_j\), the j-th latent semantic component [22];

\[
U = [u_1, u_2, \ldots, u_{5M}].
\]

They consist a basis of the phase space that satisfies the orthonormal condition:

\[
u_j^T u_j = \delta_{ij}.
\]

F. Denoising

We can denoise data matrix Z to \(\tilde{Z}\) by removing small singular-valued components as noise. In this study, we remove \(u_j, j \geq 3\);

\[
\tilde{Z} = USV^T.
\]

\[
\tilde{S} = \begin{bmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \lambda_{5M}
\end{bmatrix}.
\]

We can define the denoised data vector \(\tilde{y}_j\) as follows:

\[
\tilde{Z} = [\tilde{Z}^{(0)}, \tilde{Z}^{(1)}, \ldots, \tilde{Z}^{(M-1)}],
\]

\[
[\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_5] = \tilde{Z}_0.
\]

III. APPLICATION EXAMPLE

A. Latent Semantic Components

Fig. 1 shows an example of a sleep stage time-series, the first and second latent semantic components, and the moving-averaged cortico-thalamo-cortical loop strength \(c_2\), which is estimated from EEG obtained by the method previously proposed by us [17], [33]–[35]. The first and second latent semantic components can be interpreted as the circadian rhythm and ultradian rhythm, respectively. Note that the proposed method can extract some damping oscillation-like variation of the sleep state even if the method treats sleep stage data as nominal.
data not ordinal data. Therefore, they were termed as latent semantic components (LSC). We believe that the similarity between the second LSC and $c_2$ suggests the validity and practicality of both methods, i.e., the method reported in the present study and the method reported previously by us.

B. Denoised vectors

Fig. 2 shows the denoised vectors (Wake: $\tilde{y}_1$, N1: $\tilde{y}_2$, N2: $\tilde{y}_3$, N3: $\tilde{y}_4$, and REM: $\tilde{y}_5$) calculated from the data for Fig.1. During sleep (duration: 0 to 8 h), N1, N2, N3, and REM appear to be cyclic signals with the same period, approximately 90 min, and phase shift, i.e., they can be interpreted as signals belonging to the same cluster ‘Sleep.’ In contrast, Wake does not belong to the Sleep cluster. This result agrees well with previous findings;

- WSE2 is closer to CSE than WSE5 [17]. Here, WSE2 is Walsh Spectral Entropy calculated from binarized (Sleep/Wake) sleep stage time series, while WSE5 is calculated from original five stage data. CSE is a proposed spectral entropy that is calculated from EEG based on a cortico-thalamo-cortical loop model [17].
- The distribution of $c_2$ value obtained from 26 subjects could approximate by the Gaussian Mixture Model with two peaks corresponding ‘Sleep’ and ‘Wake’ [34].

![Fig. 1. Example of sleep stage time-series, first and second latent semantic components, and moving-averaged cortico-thalamo-cortical loop strength $c_2$ estimated based on an EEG.](image1)

![Fig. 2. Example of denoised vectors (Wake: $\tilde{y}_1$, N1: $\tilde{y}_2$, N2: $\tilde{y}_3$, N3: $\tilde{y}_4$, and REM: $\tilde{y}_5$) calculated from the data for Fig.1.](image2)

IV. DISCUSSION

A. Novelty of this study

The proposed method is based on a singular value decomposition (SVD) and time-lag embedding, that have been studied for many decades and applied in various fields such as finance, physiology, and genomics [21]–[30]. In sleep research, similar methods have been used to analyze the physiological data espicially EEG data. However, to the best of our knowledge, this is the first paper that proposes the application of such a method to the sleep stage time-series. SVD is basically the same as Latent Semantic Analysis (LSA) or Principal Component Analysis (PCA), and is sometimes applied in the pre-processing stage in machine learning or artificial intelligence. But there is no SVD application to sleep stage time-series, maybe because sleep stage data has some aspects of duality, as outlined below.

B. Input and Output

The technology for discriminating the sleep stage from the biomedical signal has been actively studied using machine learning and artificial intelligence. The goal is to output the sleep stage, and few studies have actively analyzed the sleep stage. Analyses performed in the medical field are mainly simple calculations such as those for sleep efficiency. The sleep stage seems to be becoming standard data in medical practice. We expect that the proposed method advances mathematical analysis in which sleep stage data is regarded as input.

C. Quantitative and Qualitative

Although the source data for determining the sleep stage is quantitative data measured by electronic devices, the sleep stage comprises categorical qualitative data. The sleep stage data looks like ordinal data like N1<N2<N3. However, the difference between Non-REM and REM is not quantitative, but completely qualitative. In this paper, we presented a method to quantify the sleep stage using the dynamics of its time-series, by referring to Latent Semantic Analysis (LSA). LSA is used in various fields and may be suitable for quantifying qualitative data as it is the starting point of topic model in natural language analysis. Verification of this quantification method would be an next issue.

D. Probabilistic noise and Deterministic trend

Although we described this as a denoising method in this paper, it is possible to utilize this as a detrending method that is important for exact theoretical analysis [18], [19]. This theoretical extension is the next issue.

E. Empirical data analysis and Theoretical modeling

Comprehensive study of sleep requires mathematical models that represent functions of neural circuits such as suprachiasmatic nuclei and corticothalamic loops [37]–[39]. Herein, observational data is analyzed, but it has also been shown that the analysis results are consistent with the corticothalamic model. This research could be useful in creating more reliable physiological and mathematical sleep models.
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Diffusion noise as a tool for in-vitro investigation of interacting biomolecules

M. Lechelon*, M. Gori†, M. Pettini*, Y. Meriguet†, A. Kudashova†, M. Sidore†, J. Torres†, L. Varani†
*Center of Theoretical Physics (CNRS UMR 7332),
Aix-Marseille University, Marseille, France
†Institute of Electronics and Systems (CNRS UMR 5214),
University of Montpellier, 860 rue St. Priest, 34095 Montpellier, France

Abstract—We present an experiment of the diffusion properties of biomolecules using fluorescence correlation spectroscopy. The experiments performed on R-Phycocerythrin (RPE) protein excited by optical means showed a clustering transition when the laser power exceeds a threshold value and for sufficiently high concentrations. We conclude that the behaviour of the diffusion noise found in theoretical and experimental results represent a very powerful tool to achieve a deeper understanding on the way biomolecules move and meet through long range electrodynamic interactions in living cells.

I. INTRODUCTION

Nowadays biomolecular interactions are mainly described on the basis of short-range interactions (approximately less than 10 Å) as physical contacts considering hydrophobic or electrostatic interactions, protrusion, planarity, accessible surface area and many other parameters [1]. However, recent theoretical and experimental achievements have given strength to the arguments supporting the idea that some sort of long-range distributed activity could be at work inside living cells [2].

As biomolecules move in an extremely noisy environment, they experience Brownian diffusion due to random collisions mainly with water molecules. However, the excellent efficiency and robustness of the complex machinery taking place in living cells can hardly be explained only by purely Brownian processes because random encounters alone would lead to average meeting times between molecules much larger than the observed ones. Moreover the maintenance of the cell functions is based on a precise timing with temporally driven patterns implying that the right biomolecules have to be at the right place, at the right time and following the right sequence.

Therefore a question deserving thorough investigation is related to the possible role of long-range interactions entering the protein-protein interactions schemes and eventually bringing new information about their association and the way they interact.

It is well known that, in several domains, the fluctuations in the positions of randomly moving particles and the associated diffusion coefficient represent an extremely sensitive probe to investigate their dynamics and the kinetic processes they undergo during their motion. We want here to stress that this strategy can be also applied to the following situation: a system composed of many interacting biomolecules moving in aqueous solution and undergoing random diffusion processes.

From the theoretical point of view this problem can be treated by numerically solving coupled Langevin equations through molecular dynamics simulations of a system composed of identical molecules considered as spherical charged particles moving in a viscous fluid and interacting through a pairwise potential [3].

In this paper we present and discuss experimental results obtained using fluorescence correlation spectroscopy to analyze the possible modifications in the brownian movement of biomolecules in aqueous solution when they are driven far-from-equilibrium condition by means of an optical excitation.

II. FLUORESCENCE CORRELATION SPECTROSCOPY

An interesting complement to the theoretical approach is represented by the experimental technique called FCS (Fluorescence Correlation Spectroscopy [4]) providing direct in-vitro measurements of the diffusion process of biomolecules by recording the fluctuations of fluorescence emitted in a confocal volume as shown in Fig. 1. The principle of the

![Fig. 1. Typical FCS setup where DM stands for dichroic mirror, P for pinhole and Det for detector. The excitation beam comes from the left, it is reflected by DM0 and then focused on the sample. The fluorescence light emitted in response to the excitation beam passes through DM0, then through the pinholes to improve the focus, and finally hits the detector Det 1. Adding a second DM such as DM1 (or a beam splitter) allows to cross correlate the signal, the technique being then called Fluorescence Cross Correlation Spectroscopy (FCCS).](image-url)
transport and flow, photophysical and photochemical transformations, chemical reactions or molecular aggregation. The signals of fluorescence recorded are analyzed after correlation, which is an efficient method to study the fluctuations, as the following second-order intensity correlation function:

\[ g_{ij}(t) = \frac{\langle F_i(t)F_j(t+\tau)\rangle}{\langle F_i\rangle\langle F_j\rangle} \quad (1) \]

with \( F_{i,j} \) the fluorescence signals. If \( F_i = F_j \) (signals recorded by the same detector) then equation 2 is referred to as auto-correlation function (ACF); if \( F_i \neq F_j \) then it is a cross-correlation function (CCF). The FCS techniques relies on the fluctuations of intensity, and as \( F(t) = \langle F \rangle + \delta F(t) \) with \( \langle F \rangle \) the average intensity and \( \delta F(t) \) the fluctuations of the signal, one often reports the related correlation function of fluorescence intensity fluctuations:

\[ G_{ij}(t) = \frac{\langle \delta F_i(t)\delta F_j(t+\tau) \rangle}{\langle F_i \rangle \langle F_j \rangle} \quad (2) \]

One can show that the analytical model for ACF of FCS essentially depends on the diffusion time of the tracer through the confocal volume \( \tau_D \), the number of molecules in the confocal volume \( N \), the shape of the confocal volume and particularly the structure parameter \( s \) describing the spatial properties of the detection volume \( s = \omega_T/\omega_z \) (where \( \omega \) are \( 1/e^2 \) radii of the sample volume directed perpendicularly to the optical axis and directed along the optical path). There are also many other experimental parameters that can change the FCS analytical model. In a first place, some phenomenon can have a fast fluorescence dynamic, and affect the ACF at short lag time. This is the case for antibunching, due to the delay between each light quantum emission and the time spent by the fluorophores in the triplet state, which is a period where the fluorescence stops. While antibunching is most of time not recorded in FCS acquisitions due to higher time resolution acquisition, triplet blinking can be seen, and the analytical model yields the following ACF:

\[ G(\tau) = \frac{1}{N} \left( 1 + \frac{T e^{-\tau/\tau_T}}{1 - T} \right) \frac{1}{\sqrt{1 + s^2\tau/\tau_D}} \quad (3) \]

with \( T \) the fractional population of the triplet state in the detection volume, and \( \tau_T \) the triplet lifetime. Many important physically-relevant fitted parameters can be sort out of the experiments based on the previous ACFs:

- The correlation time, obtained with the formula \( \tau_D = \frac{\omega_T^2}{4D} \), with \( D \) the diffusion coefficient of the observed dye.
- The average number of particles \( \langle N \rangle \) in the detection volume, which can be used to calculate the particle concentration in the sample given by \( G(0) = 1/\langle N \rangle \)
- The average sample intensity and the fluctuations behavior.

III. CHOICE OF THE PROTEIN

Experiments have been performed with a naturally fluorescent protein to avoid protein modification before use since it does not require a covalent labeling of fluorochromes. About the absorption and fluorescent properties, the choice mainly comes from experimental and technical constraints due to the laser used on the FCS device which excites at 488 nm. Considering these constraints we have selected a protein called R-Phycoerythrin (RPE) which is a light harvesting hexameric phycobiliprotein, found in bacteria, algae and plants. It belongs to a family of proteins called the phycobiliproteins (PBPs), making the phycobilisome (PBS), a macromolecular complex harvesting and conducting light with an efficiency of approximately 95% in the energy transfer process. What mainly differentiates the different PBPs are the quality and the quantity of their chromophoric groups [5].

IV. ANALYSIS OF THE TIME TRACES

We have performed experiments at different concentrations of RPE, also changing the laser power input. Different fluorescence traces are shown in figure 2, with three different laser powers: 33 \( \mu \)W (figure 2A), 76 \( \mu \)W (figure 2B) and 320 \( \mu \)W (figure 2C) for 14 average distances among the proteins as reported in the figure caption. The results clearly show some changes when using a different laser power. When the laser power is 33 \( \mu \)W (figure 2A) no intensity peak appears on the traces. When the power is 76 \( \mu \)W (figure 2B) sharp intensity peaks are visible for average distances starting from 650 \( \AA \) up to 750 \( \AA \). Finally, when the laser power is 320 \( \mu \)W (figure 2C) the traces show wide intensity peaks for average distances starting from 1050 \( \AA \) to 650 \( \AA \). The frequency and the width of the intensity peaks seem to increase with the concentration. This figure confirms that when the laser power is 33 \( \mu \)W the fluctuation patterns appear independent of the concentration. However, for the experiments performed with the laser powers of 76 \( \mu \)W and 320 \( \mu \)W, the traces at higher concentrations clearly show different fluctuation patterns starting at an average distance of 750 \( \AA \) (and below) for figure 2B and at 1050 \( \AA \) (and below) for figure 2C, respectively.

These first results based on the traces comparison at different concentrations and different laser powers indicate that...
large intensity peaks appear only when the laser power is above a threshold value and above a definite concentration. Similar patterns to the ones reported here for the fluctuation traces have been found in experiments involving formation of oligomers [6], and the large peak intensities have also been used by Perevoshchikova et al. [7] to develop a technique called Peak Intensity Analysis (PIA) to estimate the degree of binding of particles.

These results seems to indicate that several RPE molecules stick together; this would create clusters yielding an increased fluorescence emission coming from each individual molecule, whence the observed intensity peaks and long diffusion time given by the fitted CCF. These CCFs used for fitting and obtained from the cross correlation of the fluorescent traces visible on Fig. 2 are shown on Fig. 3.

To go even further in the confirmation of clusters formation we have recorded a video of the same solution of RPE with the laser power set at 320 µW, but using a CMOS camera (see Fig. 4). The video is again showing the formation of bright spots after few minutes of illumination.

The video directly confirms the formation of clusters as seen through the size of the bright spots. Indeed, some involve many pixels, making us think that some wide aggregates, bigger than the waist are diffusing in the solution. Moreover, the spots seem to follow a continuous path due to the small distances separating one bright spot followed through two subsequent images. We have claimed on the basis of what is reported in Fig. 4 to have observed the formation of clusters in the solution of RPE, when the laser power is high enough, in this case 320 µW.

### V. Analysis of the Variance

Further analyses have also been done to investigate about the threshold behavior of the cluster formation according to the distance among the molecules. To do so, the variance of the traces has been calculated for each sample with the average distance used in the previous sections, and with the laser power set at 33 µW, 76 µW and 320 µW (red squares, green disks and blue triangles respectively in Fig. 5). Panel A. shows the variance of the intensity. Due to the change of excitation power of the laser, the curves are shifted upward when the power increases, leading to a more difficult comparison. On panel B., we have subtracted to each curve the intensity variance measured at 2000 Å. It is interesting to note that the three curves overlap for the longer distances separating the molecules, from 2000 Å to 1050 Å.

The red and green curves also overlap up to 800 Å. These two values, 1050 Å and 800 Å have already been noticed in Fig. 2. Finally, we have assumed that the variance behavior or the red curve was the one expected for an homogeneous solution of RPE diffusion with Brownian motion and we have subtracted the values of the red curve to the corresponding...
values of the green and blue curves for similar average distances among the molecules, as seen on panel C. On this panel, the intensity variance increase is more visible at the distances previously mentioned, that is, 800 Å and 1050 Å, and in agreements with the analysis of the fluorescence traces made above.

VI. DISCUSSION AND CONCLUSIONS

Here, proteins diffusion in solution has been studied to investigate if long range electrodynamic interactions among these proteins can be strong enough to be detectable. Taking advantage of its properties, we have chosen a naturally fluorescent protein, the RPE. The results obtained show that when the laser power used with the FCCS device is low enough (experiments performed at 33 μW), the diffusion is Brownian from the lowest concentration (26 nM corresponding to an average inter-molecular distance of 4000 Å) up to the highest concentration tested (6.05 μM corresponding to an average distance between the proteins of 650 Å). However, when the laser power is increased (76 μW and 320 μW), some high intensity peaks show up on the fluorescence traces recorded for the highest concentrations (800 Å and 1050 Å respectively). This phenomenon is also accompanied with bright spots observed on videos. The correlation analysis of the traces thus shows diffusion times much longer than the ones expected for Brownian motion. These results are in agreement with the previously developed theoretical computations [8], and the only explanation that we can give of the presence of these intensity peaks and of the spots visible on the videos are clusters formed through the action of long-range electrodynamic interactions.

The evidence of aggregate formation is also supported by the observation of similar intensity peaks in experiments involving fluorescent probes binding nanoparticles, where large peaks of fluorescence are visible when multiple dyes bind to large particles [7].

A constructive criticism about the formation of clusters and against the explanation based on long-range electrodynamic interactions would be to relate the formation of aggregates to the RPE denaturation. However, in a first side, its oligomerization state makes its structural stability extremely high [5]. The loss of optical properties of the RPE gives information about the departure from its native conformation, either induced by heat or by chemical denaturation [9]. The pH is an important factor that can lead to protein denaturation, and RPE has been reported to be denatured at low pH (pH = 3) and hence to create irreversible aggregates, as studied by Ogawa et al. [10]. In our experiments involving RPE, pH measurements gave pH = 5 for a solution with intermolecular distance between the RPE proteins of 2000 Å, and the aggregation observed is reproducible and the clustering phenomenon occurs suddenly by crossing the threshold value of 1050 Å.

The temperature is another parameter that can lead to protein denaturation. In their article, Vaidya et al. [9] have shown that the RPE is experiencing a decrease of its fluorescent for temperatures higher than 62.8 °C. Considering the intensity analysis reported in Fig. 5, the intensity does not experience any decrease when reaching the highest concentration of RPE. In conclusion, our goal was to find experimental evidence of long-range electrodynamic interactions taking place between biomolecules. This work has been motivated by the poor understanding of the dynamics of intermolecular encounters in cellular signaling pathways, taking place with great precision and effectiveness both in time and space. Brownian motion, usually invoked to explain this encounters, fails to characterize such an organization due to its randomness leading to low efficiency. Whereas deterministic and selective forces acting at a long distance could accelerate the encounter between cognate partners of biochemical reactions.

The results showed a remarkable clustering transition of the proteins under two conditions: when the laser power exceeds a threshold value, and when the concentration of the protein is also above some threshold value. A sharp transition between the Brownian diffusion and the clustering of the protein molecules has been recorded depending on the average distance between the proteins. This is in excellent agreement with our preceding theoretical predictions and numerical simulations.

VII. ACKNOWLEDGMENTS

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REFERENCES

Abstract—The noise characteristics of industrial reed switches (relays with magnetic contacts) were studied. The studied relay samples had magnetic contacts that made with the use of Ni-Fe, Au and Ru coatings. It is shown that the type of metal coating of the contacts affects the noise parameters of the reed switches.

Keywords—low-frequency noise, power spectral density, metal contact, reed switch

I. INTRODUCTION

Imperfections and defects in the structure of various elements of modern electronic devices lead to the degradation of their parameters.

One of the most sensitive methods for studying the properties of materials and constructive elements of electronic devices which allow determining the nature of noise and the type of defects is the low frequency (LF) noise spectroscopy. It based on measuring the dependence of the power spectrum density (PSD) of noise on frequency.

The aim of the work was to study the effect of multiple-circuit relays with sealed magnetic contacts (reed switches) on the characteristics of low-frequency noise, as well as features of the surface structure modification of contact parts in the area of their electric contact.

II. SAMPLES AND EXPERIMENTAL EQUIPMENT

Two experimental sets with magnetic contacts consisted of 10 reed switches MCA-14103 were taken as experimental samples. One of them consisted of reed switches with different contact coatings Au-Ru or Fe. The other one had 10 reed switches based on permalloy contacts consisting of 50% Fe and 50% Ni.

Reed switches had the following geometrical dimensions:
- cylinder length is 14 mm;
- diameter of the cylinder is 2.3 mm;
- reed switch length is 44.7 mm;
- diameter contact details is 0.55 mm.

Reed switches were subjected to multiple from 2000 to 10000 switching cycles.

Reed switches with different number of switching were first investigated by the method of low-frequency noise spectroscopy. After that the glass case of the reed switches was destroyed. The contacts were placed in a vacuum and the specific features of the modification of the structure of the contact surface at the sites of current flow were examined by scanning electron microscopy (SEM).

The measurements of the low-frequency noise spectra were carried out using an automated complex for the study of low-frequency noise spectra using a low-noise preamplifier adapted for the study of low-resistance samples with a gain in the range of 10-1000 and with a constant current through the reed switch 0.1-100 mA. The magnitude of the PSD was measured at a frequency of 1 Hz. The choice of the gain provides the necessary sensitivity of the installation, which allows to obtain a PSD of low-frequency noise in the range of $10^2$-$10^4$ Hz. The mode of operation was chosen experimentally by the type of dependence of the PSD noise. The functional diagram of the measurement unit is shown in Fig. 1.

Aging by repeated short-circuiting-disconnecting the relay was used with specially designed device (Fig. 2) which made it possible to close the contacts of the reed switch installed in it with a switching frequency of up to 40 Hz at a direct current $I = 0.5$ A and a voltage at the load $U = 5$ V, specified number of times.

The structure of the contact surface before and after testing, and the size of the contact area of the cathode and
anode, was studied by using a JEOL 6610 LV scanning electron microscope.

III. EXPERIMENTAL RESULTS AND DISCUSSION

The parameters of the low-frequency noise spectra of the reed switches of the set No. 1 were monitored without their switching tests.

The technical characteristics of the investigated reed switches and the parameters of the low-frequency noise spectra are presented in Table 1 and typical PSD-dependences are shown in Fig. 3.

<table>
<thead>
<tr>
<th>№</th>
<th>$F_a$, А</th>
<th>$R_c$, Ω</th>
<th>$F_r$, А</th>
<th>Coating</th>
<th>$\text{PSD}_{V^2/Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13</td>
<td>0.08</td>
<td>7</td>
<td>Au-Ru</td>
<td>$2.05 \times 10^{-8}$</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>0.08</td>
<td>7</td>
<td>Au-Ru</td>
<td>$7.34 \times 10^{-8}$</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0.16</td>
<td>6</td>
<td>Fe</td>
<td>$5.22 \times 10^{-8}$</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>0.12</td>
<td>5</td>
<td>Fe</td>
<td>$8.91 \times 10^{-8}$</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>0.11</td>
<td>7</td>
<td>Fe</td>
<td>$2.43 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

In the table 1: $F_a$ is the actuation force; $R_c$ is the contact resistance; $F_r$ is the power of release. PSD magnitudes at the frequency of 1 Hz are presented.

The most interesting part of the LF noise spectra is in the range of 0.01–1 Hz. In this range a noise has the $1/f^\beta$ type. The degree of $\beta$ according to different authors may be in the range from 0.6 to 2.5 and it is possible to discuss about the nature of the low-frequency noise [1-3]. The obtained experimental data were approximated using the linear regression method and the parameter $\beta$ was determined. It was found that $\beta$ values varied for different samples of the set No.1 from 0.4 to 1.6.

The magnitude of the PSD at a fixed frequency (for example 1 Hz) we can conclude that the reed switches coated with noble metals (samples 1 and 2 of table 1) "noisy" less intensely than samples with Fe coated contacts. The PSD of these samples differs by more than an order of magnitude.

The value of the power density of the low-frequency noise correlates with the features of the surface structure of the samples in accordance with the vacancy model of G.P. Zhigal’skii [4]. Coatings of contacts of relays with a smaller amount of metal grains were characterized by a higher intensity of low-frequency noise.

The parameters of the low-frequency noise spectra of reed switches of the set No. 2 were investigated according to another method. Initially the parameters of the set of 10 initial samples were measured. Then 9 reed switches were subjected to switching tests. After testing the spectra of the low-frequency noise as well as the structure of the contact spots of reed switches were investigated again.

Fig. 4 presents examples of the experimental spectra of the low-frequency noise of the initial samples 1, 2, 8.

The contacts of the studied samples from the set No.2 are made of Fe – Ni alloy. The value of the PSD at the fixed frequency (for example 1 Hz) we can conclude that the PSD subjected to switching tests reed switches as a rule an order of magnitude higher than the original.

For each measured sample the contact surface was examined using the SEM (Figures 6-8). The study of the surface made it possible to understand the nature of the change in the PSD due to the different area of contact spots of the anode and cathode. The area of the contact spots of the anode and cathode increases significantly with an increase in the number of switching cycles (Table 2).
Fig. 4. The spectra of the low-frequency noise of the initial samples 1, 2, 8 of the set No. 2 and the approximation of the spectra by the method of linear regression.

Fig. 5. presents examples of the experimental spectra of the low-frequency noise samples of samples 1, 2, 8, subjected to aging.

The results of the study of samples of the set No. 2 are presented in table 2. In the table 2: $S_{\text{cont. cat}}$, $S_{\text{cont. anode}}$ are the areas of contact spots obtained and measured using SEM.

<table>
<thead>
<tr>
<th>№</th>
<th>Initial</th>
<th>After testing</th>
<th>Number of commutation</th>
<th>$S_{\text{cont. cat}}$ μm²</th>
<th>$S_{\text{cont. anode}}$ μm²</th>
<th>PSD V²/Hz</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.64 \times 10^{-9}$</td>
<td>1.02</td>
<td>10000</td>
<td>10.47</td>
<td>9.60</td>
<td>$1.42 \times 10^{-6}$</td>
<td>2.97</td>
</tr>
<tr>
<td>2</td>
<td>$1.78 \times 10^{-8}$</td>
<td>0.65</td>
<td>2000</td>
<td>5.75</td>
<td>6.76</td>
<td>$3.68 \times 10^{-6}$</td>
<td>1.72</td>
</tr>
<tr>
<td>3</td>
<td>$7.93 \times 10^{-10}$</td>
<td>0.31</td>
<td>2000</td>
<td>4.79</td>
<td>4.62</td>
<td>$1.93 \times 10^{-8}$</td>
<td>0.37</td>
</tr>
<tr>
<td>4</td>
<td>$3.21 \times 10^{-9}$</td>
<td>1.21</td>
<td>4000</td>
<td>–</td>
<td>–</td>
<td>$2.94 \times 10^{-8}$</td>
<td>1.58</td>
</tr>
<tr>
<td>5</td>
<td>$1.97 \times 10^{-10}$</td>
<td>1.23</td>
<td>2000</td>
<td>5.87</td>
<td>5.45</td>
<td>$8.53 \times 10^{-9}$</td>
<td>1.34</td>
</tr>
<tr>
<td>6</td>
<td>$3.27 \times 10^{-10}$</td>
<td>0.54</td>
<td>10000</td>
<td>16.58</td>
<td>17.74</td>
<td>$5.76 \times 10^{-10}$</td>
<td>1.09</td>
</tr>
<tr>
<td>7</td>
<td>$2.93 \times 10^{-10}$</td>
<td>2.02</td>
<td>4000</td>
<td>11.67</td>
<td>10.77</td>
<td>$2.31 \times 10^{-10}$</td>
<td>1.53</td>
</tr>
<tr>
<td>8</td>
<td>$5.64 \times 10^{-10}$</td>
<td>1.83</td>
<td>4000</td>
<td>6.81</td>
<td>6.22</td>
<td>$1.91 \times 10^{-8}$</td>
<td>1.27</td>
</tr>
<tr>
<td>9</td>
<td>$6.52 \times 10^{-10}$</td>
<td>1.04</td>
<td>10000</td>
<td>4.24</td>
<td>3.72</td>
<td>$1.83 \times 10^{-8}$</td>
<td>1.48</td>
</tr>
<tr>
<td>10</td>
<td>$1.28 \times 10^{-9}$</td>
<td>1.58</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
It is interesting to note that after aging on the contact coatings growths of various sizes appeared having a rounded shape with a hole inside (anodes) or with a ring-shaped formation (cathodes). The nature of growths is connected with the processes of point melting of the electrode material during switching cycles.

![Fig. 6. SEM images of the anode of sample 1 of the set No. 2 (a-general view of the anode, b-artifacts of the contact spot, c-diameters of growths of the contact spot)](image-url)

![Fig. 7. SEM images of the cathode of sample 2 of the set No. 2 (a-general view of the cathode, b-artifacts of the contact spot, c-diameters of growths of the contact spot)](image-url)

![Fig. 8. SEM images of the anode of sample 8 of the set No. 2 (a-general view of the anode, b-artifacts of the contact spot, c-diameters of the growths of the contact spot)](image-url)

The areas of the cathode and anode contact spots for specific samples approximately coincide. The increase in the number of switching leads to an increase in the area of contact spots with growths.

### IV. CONCLUSIONS

The results obtained in the work indicate an unambiguous relationship between the characteristics of the spectra of the low-frequency noise from the manufacturing techniques and the characteristics of the reed switches operation.

In this regard, the method of spectroscopy of low-frequency noise can serve as a convenient tool for diagnosing and predicting the reliability of reed switches.

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An FPGA Based Practical Implementation of Stochastic Resonance
For Image Enhancement

Sumit Kumar, IEEE, Member, Sudha Chauhan,
Gaurav Sundaram, MauSam Chouksey, Rajib Kumar Jha, IEEE, Senior Member
Indian Institute of Technology Patna

Abstract—In this paper, a non-linear non-dynamic stochastic resonance (SR) based technique is proposed for enhancement of dark and low contrast images. Noise-enhanced signal processing theory is applied to a low contrast image to improve the contrast. Insufficient illumination is the major cause of low contrast of the image, which can be stated as internal noise. This internal noise is neutralized by the addition of some pre-calculated external noise. The low contrast image is added to different frames of random noise and is thresholded repeatedly against a fixed parameter. After that, averaging produces a high contrast (enhanced) image. Noise-induced resonance is obtained at a particular optimum noise intensity. This optimum intensity is obtained by varying the noise intensities. Performance of the proposed technique is investigated for Gaussian noise. Quantitative evaluation of the performance is done in terms of color enhancement, contrast enhancement factor, and perceptual quality measure.

Furthermore, a hardware implementation of the proposed SR algorithm using NEXYS 4 DDR Artix-7, which is a low power FPGA device is done to show and validate the performance of the proposed algorithm in a practical environment. Power consumption and resource utilization of the proposed algorithm is also addressed. Finally, three different quantitative parameters i.e., contrast enhancement factor (F), color enhancement factor (CEF) and perpetual quality metric (PQM) are calculated to compare the hardware and software result of the proposed method.

Index Terms—Image Enhancement, FPGA.

I. INTRODUCTION

Image [1], [2] enhancement improves the interpretability or perception of information in images for human viewers, provide better input for other automated image processing techniques. Insufficient illumination causes many images to have a very low dynamic range of intensity. Hence, these images are processed before being displayed. Techniques such as gamma correction, histogram equalization, high pass filtering, homomorphic filtering, low pass filtering, etc., have focussed on the enhancement of gray level images in the spatial domain [3], [4]. These methods have also been employed for color image enhancement in the RGB space. Jobson et al. [5] has reported retinex theory that also leads to good contrast enhancement of an image. Nevertheless, the requirement of filtering with multiscale Gaussian kernels and post-processing stages for adjusting colors makes it computationally intensive. Another technique has also been reported in RGB space which uses equalization of the 3-D histograms [6].

Noise (here low illumination) is usually considered to be a trouble that decreases the signal-to-noise ratio (SNR) of non-linear systems. On the contrary, SR is a phenomenon in which external noise is added for enhancement rather than obstructing the system performance. Usually considered undesirable, noise can be constructively used in different application such as image processing, biomedical etc.

The first experimental work on visualization of stochastic resonance (SR) has been reported [7]–[13]. They report the outcome of a psychophysics experiment which shows that the human brain can interpret details present in an image contaminated with time-varying noise and the perceived image quality is determined by the noise intensity and its temporal characteristics. Jha et al. [14] used SR to enhance dark images by deriving a specific condition for threshold using Gaussian noise.

Field Programmable Gate Array (FPGA) with various hardware resources effectively fill the gap between application specific integrated circuit(ASIC) and microprocessors. FPGA has higher performance than microprocessors. Moreover, FPGA has higher programmability, flexibility, low cost and less development time than ASIC, although it consumes 9–12 times more power than ASIC. ASIC has been significantly replaced by FPGA in electronic industries, networking area to minimize the cost and time to market. Memory plays a critical role in FPGA for most applications like image processing, medical imaging, aerospace and defense systems, computer vision, signal processing, speech recognition, bioinformatics, cryptography and growing range of other areas. The processors with this application use traditional on-chip or off-chip memories. Since on-chip memory implemented inside FPGA has no external connections on the circuit board, it provides maximum access speed from memory and highest throughput. The use of on-chip memory is expected to increase continuously for enhancing the performance of future generations portable devices and high- performance processors [15].

Organization of the paper: Section II deals with the basic theory of non-dynamic stochastic resonance. Section III and Section IV deal with the software and hardware approach for the proposed method respectively. In the end, the conclusion has been presented in Section V.

II. NON DYNAMIC STOCHASTIC RESONANCE

In ordinary consciousness, the word noise relates to obstruction. It was a traditional belief that the presence of noise
makes the system worse. But, recent studies have shown that in non-linear systems, noise can induce more ordered regimes, which cause the signal-to-noise ratio (SNR) to increase and amplification of weak signals [16]. In other words, noise can be used to enhance weak signals. Stochastic resonance is a phenomenon where the addition of noise to the input signals amplifies the output signals of some non-linear systems. More technically, SR occurs if the SNR, input/output correlation has a well-marked maximum at a fixed noise level [17].

A system should possess three basic properties to exhibit SR. They are a non-linearity in terms of the threshold, a source of additive noise, and a sub-threshold signal similar to signal with small amplitude. This phenomenon often occurs systems with threshold-like behavior. The behavior of the SR mechanism reveals that at lower noise intensities the weak signal doesn’t cross the threshold. Hence the output is a very low SNR. The output is dominated by the noise for high noise intensities which also leads to a low SNR. But, for moderate noise intensities, the noise allows the signal to cross the threshold. Hence maximum SNR at some optimum noise level is obtained.

### III. SOFTWARE APPROACH

The proposed algorithm has the following steps. The steps as shown through Fig. 1 should be applied to all the three planes (R, G and B) of a colored image in parallel.

Step 1:

1) A very low contrast image $P_{RGB}(x, y)$ is taken as an input image.
2) N frames of random noise, $\zeta(x, y)$, of mean zero and standard deviation $\sigma_\zeta$ are produced. Each of the noise frames is added to the low contrast image $P_{RGB}(x, y)$ to give N different noisy low contrast images (for single noise standard deviation ($\sigma_\zeta$)).
3) Thresholding is done for each noisy image $P_i$ using a fixed threshold (taken as the mean ($\mu_\zeta$) of input image itself). Enhanced image of good contrast is obtained when we take an average of all the thresholded frames ($P''_i$). Performance metrics F, CEF, and PQM are calculated for this output.
4) Standard deviation of the noise is increased by a unit and Step 1 to Step 3 is repeated. Values of performance metrics are analyzed. When CEF+I-F becomes maximum within the constraint that PQM is close to 10, the process is stopped.

### IV. HARDWARE APPROACH

As the trend of programmable hardware chip for a particular application has increased significantly, we implement and show the proposed algorithm on low power FPGA NEXYS 4 DDR Artix - 7 device. It may be put to use for various other applications. The feasibility of the proposed stochastic resonance algorithm is also measured. Our focus is more on the accuracy of the proposed method. We have also mentioned about resources used during the implementation. However, our results of hardware implementation are optimized in terms of error but not in terms of resources.

Here, we mainly focus on the data path while implementing the proposed algorithm on the FPGA. We are not concerned about control signals in much detail. This hardware implementation is shown for a single color plane (P). We take N (here, N is 30) noise frames. Here, additional expenses of resources occur due to the processing of all the input data in parallel. The input data are fetched into register pixel by pixel at every clock event (Positive edge triggering). When the addition of the image pixel intensities with the noise frames is done at every clock event, it is sent to the comparator, which performs thresholding against a fixed parameter (mean of the input image, $\mu_\zeta$). The 30 (for a single plane) parallel additions are accepted in FPGA due to the flexibility of the clock management. The hardware component utilization of the module for the proposed algorithm is discussed in Table II.

### TABLE I: The parameters for symmetric Gaussian noise.

<table>
<thead>
<tr>
<th>Noise</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ_ζ</td>
<td>25</td>
</tr>
<tr>
<td>F</td>
<td>125.8494</td>
</tr>
<tr>
<td>CEF</td>
<td>125.3829</td>
</tr>
<tr>
<td>PQM</td>
<td>8.4206</td>
</tr>
</tbody>
</table>

### TABLE II: Device utilization summary report.

<table>
<thead>
<tr>
<th>Primitive and Black Box Usage</th>
<th>Logic Utilisation</th>
<th>Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>BELS</td>
<td>1172</td>
<td></td>
</tr>
<tr>
<td>the GND</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>LUT2</td>
<td>990</td>
<td></td>
</tr>
<tr>
<td>LUT5</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>LUT6</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>VCC</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FlipFlops/Latches</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FDRE</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Clock Buffers</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>BUFGP</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>IO Buffers</td>
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<td></td>
</tr>
<tr>
<td>IBUF</td>
<td>1395</td>
<td></td>
</tr>
<tr>
<td>OBUF</td>
<td>63</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE III: The quantitative parameters for different bit-width.

<table>
<thead>
<tr>
<th>Bit-width</th>
<th>F</th>
<th>CEF</th>
<th>PQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8.0638</td>
<td>1.7143</td>
<td>15.0511</td>
</tr>
<tr>
<td>5</td>
<td>16.6190</td>
<td>17.5675</td>
<td>12.5715</td>
</tr>
<tr>
<td>6</td>
<td>27.7151</td>
<td>24.9691</td>
<td>11.6162</td>
</tr>
<tr>
<td>7</td>
<td>32.1034</td>
<td>26.7281</td>
<td>11.4358</td>
</tr>
<tr>
<td>8</td>
<td>32.1608</td>
<td>26.7675</td>
<td>11.4414</td>
</tr>
<tr>
<td>9</td>
<td>32.1311</td>
<td>36.7298</td>
<td>11.4352</td>
</tr>
</tbody>
</table>
Fig. 1: P represents one color plane, i.e., R, G or B. \( \zeta_i(x, y) \) is the random noise with zero mean and standard deviation as \( \sigma_i \). These noises are added to the color plane to produce N different noisy images. The noisy low contrast images obtained are indicated by \( P_i \). Further, each of the \( P_i \)'s is thresholded against the mean \( \mu_o \) to produce different \( P'_i \). Finally, all the \( P'_i \)'s are averaged to get the increased contrast image for the particular plane P. This algorithm is implemented for each of the planes, R, G and B. Results of each plane are concatenated to produce the final output image.

Fig. 2: The size of the image is taken as 256 x 256. Adder block is represented by \( A_i \) and comparator block is represented by \( C_i \). Different bits are chosen for integer and fractional part of the noise frames and input image.
V. Conclusion
A non-dynamic stochastic resonance-based technique has been investigated in this paper for the enhancement of dark images. By treating a low contrast image as subthreshold and adding random noise followed by hard-thresholding and averaging, the contrast of the image was found to remarkably increasing. Addition of symmetric noise is used. Gaussian distribution is used for symmetric for image improvement.

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References
Cosmic Nonstationarity of the Coherent Gravidynamic Quantum 1/f Effect

Peter H. Handel
Department of Physics and Astronomy
and Center for Nanoscience
University of Missouri - St. Louis. USA
corresponding author, handel@umsl.edu

Erika Splett
Department of Physics and Astronomy
Visiting International Scholar
University of Missouri - St. Louis. USA
erikasplett@yahoo.de

II. DERIVATION OF THE COHERENT QED Q1/FE FROM THE ASYMPTOTIC PROPAGATOR

For N electrons in a Fermi sphere shifted in momentum space by a vector \( \mathbf{p}_0 \) and occupying N/2 orbitals \( \mathbf{e}^{\mathbf{p}} \), the asymptotic propagator derived by these authors [2]-[6] can be reduced for large time components of \( x' - x \), to the non-relativistic form [7]

\[
-i<\Phi_\mathbf{p}|\mathbf{T} \mathbf{y}_\mathbf{p}(\mathbf{x}')\mathbf{y}_\mathbf{p}(\mathbf{x})|\Phi_\mathbf{p}> = \delta_{ss'} G_a(x'-x) = (i\sqrt{V})\sum \left\{ \exp\left[\mathbf{p}(\mathbf{r}-\mathbf{r}') - \mathbf{p}^2/2m(\hbar/\mathbf{f})\right] \frac{1}{n^2} \right\}
\]

Here we have used Eq. (1). We now consider a beam of charged fermions, e.g., electrons, represented in momentum space by a vector \( \mathbf{p}_0 \) and directly in a intuitive semiclassical way. Like all quantum 1/f effects in this paper, it is both an infrared-divergence-and-coherence phenomenon. The asymptotic QED propagator, valid at large times and distances or low frequencies, includes the Coulomb field in the notion of particle. It also predicts in a particular instability, similar to the effects of “dark energy.”

Keywords—1/f noise, quantum 1/f noise, dark energy, coherent quantum 1/f effect, quantum graviodynamics

Abstract—The Coherent Gravidynamical Quantum 1/f Effect is derived both electrically from an asymptotic QED propagator and directly in a intuitive semiclassical way. Like all quantum 1/f noise it is both an infrared-divergence-and-coherence phenomenon. The asymptotic QED propagator, valid at large times and distances or low frequencies, includes the Coulomb field in the notion of particle. It also predicts in a particular instability, similar to the effects of “dark energy.”

1. INTRODUCTION

Starting first from the asymptotic propagator of the coherent picture of Quantum-Electrodynamics (QED), we derive the QED coherent quantum 1/f effect (Q1/FE) and study the nature of its nonstationarity. This, more realistic, “coherent states” picture of QED was introduced by Kibble, Zwanziger et al. in the 1960’s. It included for the first time the long-range Coulomb field in the unperturbed hamiltonian of the charged particle (electron), obtaining a QED propagator for the first time, but only in asymptotic form, for large times and distances, i.e. at low frequencies. This propagator was proven by us to reproduce our earlier semiclassical derivation of the universal 1/f spectrum of the coherent Q1/FE with an added feature: a non-integrable factor \( c=1/137 \) in the 1/f spectral density. Here \( c=e^2/\hbar \) c=1/137.

In this paper we show that this nonstationarity, limited to the cosmic time since the big bang, may correspond to a series of expansion, accelerated expansion, higher order expansion, etc., terms. Based on the similarity shown by Weinberg [1], between the QED and QGD infrared divergence domains, we conjecture the same result to apply for the QGD low-frequency domain. This hints to a similar QGD series of expansion, accelerated expansion, acceleration of the acceleration, etc., terms, in the cosmic event space.

II. DERIVATION OF THE COHERENT QED Q1/FE FROM THE ASYMPTOTIC PROPAGATOR

For N electrons in a Fermi sphere shifted in momentum space by a vector \( \mathbf{p}_0 \) and occupying N/2 orbitals \( \mathbf{e}^{\mathbf{p}} \), the asymptotic propagator derived by these authors [2]-[6] can be reduced for large time components of \( x' - x \), to the non-relativistic form [7]

\[
-i<\Phi_\mathbf{p}\mathbf{T} \mathbf{y}_\mathbf{p}(\mathbf{x}')\mathbf{y}_\mathbf{p}(\mathbf{x})|\Phi_\mathbf{p}> = \delta_{ss'} G_a(x'-x) = (i\sqrt{V})\sum \left\{ \exp\left[\mathbf{p}(\mathbf{r}-\mathbf{r}') - \mathbf{p}^2/2m(\hbar/\mathbf{f})\right] \frac{1}{n^2} \right\}
\]

The equality is always satisfied when decoherence took place, scrambling the phases. The first term can be expressed in terms of the particle density of spin s, n=2/N=V

\[
<i\Phi_\mathbf{p}\mathbf{T} \mathbf{y}_\mathbf{p}(\mathbf{x}')\mathbf{y}_\mathbf{p}(\mathbf{x})|\Phi_\mathbf{p}> = (n2)^2 + \delta_{ss'} G_a(x'-x),
\]

The “relative” autocorrelation function \( A(x-x') \) describing the normalized pair correlation independent of spin is obtained by dividing by \( n^2 \) and summing over s and s'
space by a sphere of radius $p_\circ$, centered on the momentum $p$, which is the average momentum of the fermions, with $p_\circ$ parallel to $r-r'$. The energy and momentum differences between terms of different $p$ are large, leading to rapid oscillations in space and time which contain only high-frequency fluctuations. The low-frequency and low-wavenumber part $A_0$ of this relative density autocorrelation function is given by the terms with $p=p'$:

$$A_0(x-x') = \frac{1}{1} - \frac{1}{2N^2} \sum_{n,p} \delta(p(p-r')/\hbar) - (m^2 + p^2)^{-1/2}(t')/(c/\hbar) \equiv 2\pi$$

Here we have used the mean value theorem, including the $2\pi$ power as a slowly varying function of $p$ and neglecting $p_\circ$ in the coefficient of $\tau = t-\tau'$, with $z=m_\circ r'_\circ$. This is the general asymptotic result valid for large $\theta = |p(p-r')/mc^2|$. The correlations propagate along the beam with a group velocity given by the average velocity $p_\circ/m$ of the particles in the beam, and with the phase velocity of $c^2/v^2$. Using [35], we obtain from Eq. (6) the form

$$A_0(x-x') = \frac{1}{N} - \frac{1}{2N^2} \sum_{n} \cos((2\pi N^2)/mc^2) \{mc^2/\hbar \}^{2\pi N^2/(\cos(\omega) \omega)}$$

in which the fractional power could have been neglected in the integrand for all practical purposes except for the theoretical question of the integrability of the $1/\omega$ spectrum and stationarity. According to the Wiener-Khintchine theorem, the coefficient of the cos gives the spectral density. To get it for the fractional fluctuations $\delta n/\bar{n}$, we also divide by the constant term $N^2$. Eq. (7) for the coherent Quantum Electrodynamical chaos process in electric currents becomes

$$S_0(x') \equiv \frac{1}{N} - \frac{1}{2N^2} \sum_{n} \cos((2\pi N^2)/mc^2) \{mc^2/\hbar \}^{2\pi N^2/(\cos(\omega) \omega)}$$

The fractional autocorrelation of current fluctuations $\delta j$ is obtained by multiplying Eq. (4) on both sides with $(ep_\circ/m)^2$, and dividing by $(en_\circ/m)^2$ which is the square of the average current density $j$, instead of just dividing by $n^2$. So it is the same as the fractional autocorrelation for quantum density fluctuations in the outgoing current. Indeed, for current density fluctuations $\delta j$ we include a $\hbar/\pi mV$ in front of each of the two $\psi$ operators in Eq. (1), a factor $pp'/p_\circ^2$ in Eq. (4) after the summation signs, a factor $p_\circ^2$ in the first form of Eq. (6), a factor $p_\circ^2$ in the second form, and no changes in Eqs. (7)-(8). Eq. (8) becomes

$$S_0(j) \equiv \frac{1}{N} - \frac{1}{2N^2} \sum_{n} \cos((2\pi N^2)/mc^2) \{mc^2/\hbar \}^{2\pi N^2/(\cos(\omega) \omega)}$$

This result coincides with our earlier theoretical result for coherent quantum 1/f noise if we replace $N$ with $N^2$. Correlations are defined only for $N \geq 2$. The validity of this equation is restricted to low frequencies and wave-numbers. This equation is in excellent agreement with mobility and diffusion fluctuations 1/f noise in large electronic solid-state devices.

Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations. Most of the conventional 1/f fluctuations, are also in the mobility, but some of these are also found in the recombination speed or tunneling rate, being perceived and usually interpreted, as 1/f fluctuations in the concentration of carriers. This is how the quantum theory of fundamental 1/f noise solves the age-old controversy between those claiming 1/f noise in semiconductors was a carrier number fluctuation and those considering it a mobility fluctuation.

All integration and summations go up to infinity. Our result could be of cosmologic interest. Using identity [8],

$$\theta \omega = - \frac{1}{2\pi (\omega \omega)} \sum_{n=0}^{\infty} (\omega \omega) 2\pi \cos(\omega \omega) d\omega$$

with arbitrarily small cutoff $\omega_0$, with infinity as the upper limit, we obtain from Eq. (7) for the autocorrelation function of fractional current or density fluctuations the exact form

$$A(x-x') = 1 + \left[ \frac{\omega_0}{\omega_0 N} \right] \sum_{n=0}^{\infty} (\omega \omega) 2\pi \cos(\omega \omega) d\omega$$

This shows a $\omega^{-1/2}$ power spectrum and a $1/\omega$ dependence of the spectrum of fractional $n$ and $j$ fluctuations. We neglect the curly bracket in the denominator which is close to unity for very small $\omega_0$. Eq. (9) for the coherent QED chaos process in electric currents can thus be written also in the form

$$S_0(j) \equiv \frac{1}{N} - \frac{1}{2N^2} \sum_{n} \cos((2\pi N^2)/mc^2) \{mc^2/\hbar \}^{2\pi N^2/(\cos(\omega) \omega)}$$

This result derived directly earlier [9], [10], is in excellent agreement with the measurements [11]-[20], in large devices such as large n$^2$p Hg1-xCd$_x$Te infrared detector diodes. It is also close to the empirical value of 0.002/\omega observed earlier by Hooge [20] in semiconductors and metals, after he understood the universal turbulence theory of 1/f noise. Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations.

III. NONSTATIONARITY

Consider, e.g., an infinite beam of particles of mass $m$ in cosmos, denoting $\omega_0 \rightarrow 2\pi T_\omega$, where $T_\omega$ is the 13.7$^2$ years of the universe. Noticing that $\alpha < -1$, and that the finite age $T$ of the universe provides a natural cutoff $\omega_0 \rightarrow 1/T$, we re-write Eq. (11) in the form

$$A(x-x') \equiv \frac{1}{N} - \frac{1}{2N^2} \sum_{n=0}^{\infty} (\omega \omega) 2\pi \cos(\omega \omega) d\omega$$

This result coincides with our earlier theoretical result for coherent quantum 1/f noise if we replace $N$ with $N^2$. Correlations are defined only for $N \geq 2$. The validity of this equation is restricted to low frequencies and wave-numbers. This equation is in excellent agreement with mobility and
fluctuations. To find the spectral density of these inescapable fluctuations which are known to characterize any quantum state which is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger's definition of coherent states.

The coherent quantum 1/f effect will be derived in three steps: first we consider a hypothetical world with just a single mode of the gravitational field coupled to a beam of material particles. Considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle-density (or concentration) which arise from the nonstationarity of the coherent state. Then we calculate the amplitude with which this one mode is represented in the field of an electron, according to electrodynamics. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the gravitational field be characterized by the wave vector $q$, the angular frequency $\omega_0$ and the polarization $\lambda$. Denoting the variables $q$ and $\lambda$ simply by $q$ in the labels of the states, we write the coherent state of amplitude $|z_q>$ and phase $\arg z_q$ in the form

$$|z_q> = \exp[-(1/2)|z_q|^2] \exp[z_q \omega_0] |0> = \exp[-(1/2)|z_q|^2]$$

$$= \sum_{n=0}^{\infty} \left( \frac{z_q^n}{n!} \right) |n>.$$

Here $z_q^n$ is the creation operator which adds one energy quantum to the energy of the mode. Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n> = (2^n n!)^{-1/2} \exp[-x^2/2] H_n(x) e^{int \tau}.$$  

This yields for the coherent state $|z_q>$ the representation

$$\Psi_q(x) = \exp[-(1/2)|z_q|^2] \exp[x^2/2] \sum_{n=0}^\infty \left( \frac{z_x e^{i\omega t} |n!(2^n n!)^{1/2} H_n(x)\right.$$}

In the last form the generating function of the Hermite polynomials was used. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time $t$ or the phase $\omega t$ is, for $|z_q|^2 < 1$,

$$P_q(\tau, t) = \langle |\Psi_q|^2, |\Psi_{q+t}|^2 \rangle = \sum_{n=0}^\infty \left( (|z_x e^{i\omega t}|n!(2^n n!)^{1/2} H_n(x)$$

Integrating over $x$ from $-\infty$ to $\infty$, we find the autocorrelation function

$$A_q^2(t) = (2)^{-1/2} \left( 1 + 2|z_q|^2 \cos \omega t \right).$$

This result shows that the probability distribution contains a constant background with small superposed oscillations of frequency $\omega$. Physically, the small oscillations in the total probability describe self-organization or bunching of the particles in the beam. They are thus more likely to be found in a measurement at a certain time and place than at other times and places relative to each other along the beam. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator which is also an energy eigenstate, and therefore stationary and free of oscillations. Note also the presence of four single-particle wave functions, because the
two-particle wave function without interactions is a product of two single-particle wave functions. We now determine the amplitude $z_q$ with which the gravitational field mode $q$ is represented in the correct definition of the physical particle. The simple way to do this, like in the QED case that was treated first, is to let (in QGD) a bare particle also dress itself, this time through its interaction with the gravitational field, i.e. by performing first order perturbation theory with the non-relativistic interaction Hamiltonian

$$H = m \phi,$$  \hspace{1cm} (20)

where $\phi$ the scalar gravitational potential. This corresponds to a Fourier expansion $-4\pi G m/q^2$ of the gravitational potential $-G/m$ of a material particle in a box of volume $V$, and multiplication with a squared gravitonic "wave function" $(\hbar c q V)^{-1}$. This way we obtain

$$| z_q |^2 = \pi G (m/q)^2 (\hbar c q V)^{-1}. \hspace{1cm} (21)$$

Considering now all modes of the gravitational field, we obtain from the single-mode result of Eq. (5)

$$B(t) = C \sum q (1 + 2 | z_q |^2 \cos \omega_q t) = C \{ 1 + \eta + 2 | z_q |^2 \cos \omega_q t \} \int | z_q |^2 \cos \omega_q t \hspace{1cm} (22)$$

Here we have again used the smallness of $z_q$ and we have introduced a constant $C$ proportional to the squared velocity of the particles in the beam. Using Eq. (21) we obtain

$$B(t) = C \{ 1 + 4 (V/2^{3/2}) \int | z_q |^2 \cos \omega_q t \} \int | q |^2 \cos \omega_q t \hspace{1cm} (23)$$

Here $B = G m^2/c^2$ is $10^{9} m^2/\text{gram}^2$ replaces in our case the fine structure constant. The first term in curly brackets is unity and represents the constant background, or the d.c. part of the mass current density defined by the motion of the beam of particles through vacuum. The autocorrelation function for the relative (fractional) density fluctuations, or for the fractional mass-current density fluctuations in the beam of material particles is obtained therefore by dividing the second term in curly brackets by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener-Khintchine theorem, the coefficient of cosort is the spectral density of the fluctuations, $S_j | \eta |^2$ for the particle concentration, or $S_j$ for the current density $j = e(k/m) | \eta |^2$

$$S_{j \eta} | \eta |^2 | \eta |^2 = S_{j \eta} | \eta |^2 = 2 (\beta/\pi N) = 2 G m^2/c^2 \pi N c$$

$$\approx 4 \times 10^7 \text{m}^3/\text{pN gram}^2.$$

(24)

Here we have included the total number $N$ of material particles of mass $m$ that are observed simultaneously in the denominator, because the noise contributions from each particle are independent. For example, for $m = 10^{-6} \text{g} = 1 \mu \text{g}$, we get about $10^3$/pN from Eq. (24). This is similar to the coherent QED $1/f$ result calculated above.

**V. DISCUSSION**

The results obtained in the last section show that the new coherent Gravodynamic Quantum $1/f$ Effect (QGD $1/f$) is hard to be observed on atomic particles, but is easy to observe as a new form of $1/f$ noise in beams of mesoscopic aggregates (almost involving the squared Avogadro number in $\beta$) or even in macroscopic flows of matter. The new effect differs from the well known earlier conventional form, because it is present in any current of matter, and not only as a result of scattering, as we know, was the case for the conventional Gravodynamic Quantum $1/f$ Effect (QGD $1/f$). The latter was a property of the physical quantum mechanical cross sections that we had introduced as part of a new aspect of quantum mechanics, that we called quantum $1/f$ noise. We now understand that in the gravidynamical case, just as in the electrodynamical or lattice-dynamical (piezoelectric) cases discussed elsewhere, the observed quantum $1/f$ noise represents macroscopic quantum fluctuations including both coherent and conventional contributions. Simple formulas allow for calculating both the coherent and conventional quantum $1/f$ effect, combining them (see Gravidyn in these Proc.)
Temporal stability measurements of a cooled infrared type II superlattice (T2SL) focal plane array detector

Vignesh AROUNASSALAME  
DOTA  
ONERA  
Palaiseau, FRANCE  
vignesh.arounassalame@onera.fr

Jean NGHIEM  
DOTA  
ONERA  
Palaiseau, FRANCE  
jean.nghiem.xuan@gmail.com

Maxence GUENIN  
DOTA  
ONERA  
Palaiseau, FRANCE  
maxence.guenin@onera.fr

Eric COSTARD  
IRnova AB  
Electrum 236  
SE 16440 Kista, Sweden  
eric.costard@ir-nova.se

Philippe CHRISTOL  
I.E.S, Univ. Montpellier  
CNRS  
F-34000 Montpellier, France  
christol@ies.univ-montp2.fr

Isabelle RIBET-MOHAMED  
DOTA  
ONERA  
Palaiseau, FRANCE  
isabelle.ribet@onera.fr

Abstract – Type II Superlattice detectors (T2SL) showed promising achievements thanks to recent breakthroughs in its fabrication process. Following these evolutions, T2SL IDDCA (Integrated Detector Dewar Cooler Assembly) are now commercially available and their performances are evaluated with more advanced figures of merit to compare them to other detector technologies. Residual Fixed Pattern Noise (RFPN) is used to evaluate Focal Plane Array (FPA). However, this measurement is strongly dependent on the way to define defective pixels and some of them can have a flickering signal due to random telegraph signal noise. In this paper, a T2SL MWIR 320x256 pixels IDDCA is studied. We compare two data processing for defective pixels detection and the results obtained on RFPN over time. Then, we study RTS pixels with focus on their number of appearances and their classification.

Keywords—Infrared Photodetector, T2SL, FPA RFPN, RTS

I. INTRODUCTION

Infrared detection knows a growing expansion in various markets, from aircraft enhanced vision systems to nondestructive control of skin cancer [1]. This is due, in part, to the continuous progress in the detector technology.

T2SL detectors emerged as a new field of infrared detection for high-performance imaging applications. One of its advantages is a tunable cut-off wavelength thanks to its particular structure of “Superlattice”. The good performances of the structure enabled to move on to the “system” stage. At the FPA level two important figures of merit are used to evaluate the detector: modulation transfer function (MTF) and stability over time. MTF describes how well a detector can reproduce spatial frequencies [2], whereas stability over time indicates how long time the calibration used is valid.

The stability over time is currently estimated through the RFPN. However, this figure of merit quickly shows its limits because of the conditions of application. Indeed, depending on the criteria used to set aside defective pixels, the results can change quickly, hence the importance of using common foundations in order to have pertinent comparisons between two different detectors. This problem is perfectly represented with some pixels affected by random telegraph signal (RTS) noise. The signal delivered by these pixels can oscillate between two or more levels at certain times, creating a particularly harmful blinking effect for image quality [3].

In this paper, a T2SL MWIR 320x256 pixels IDDCA (Integrated Detector Dewar Cooler Assembly) provided by IRnova is studied. We first describe our experimental protocol to evaluate the temporal stability of a FPA. We present the results of the evolution of the RFPN over time. Then we focus on random telegraph signal noise, and present the data processing and the experimental results.

II. RESIDUAL FIXED PATTERN NOISE

A. Principle of the RFPN

The Fixed Pattern Noise corresponds to the spatial fluctuations of the signal delivered between the different pixels composing a FPA when it receives a flux of uniform irradiance. The fixed pattern noise can find its origin in two different parts of the pixel: the first one is the detection circuit (the part responsible of the conversion of photons into electrons) because of differences in cut-off wavelengths for example. The second one is the Read Out Integrated Circuit (ROIC, the part making the link between the photodiodes and the electronics processing the signal) for example if the output amplifiers have different offsets. The Residual Fixed Pattern Noise is calculated after a linear two-point calibration [4] is realized on the signal in order to reduce the Fixed Pattern Noise. This calibration is done by using two extended blackbodies at different temperatures. The corrected signal for one pixel (i,j) $S'_{i,j}$ is calculated using the two coefficients of the linear correction $G_{i,j}$ and $O_{i,j}$:

$$\begin{align*}
S'_{i,j}(\phi) &= G_{i,j}S_{i,j}(\phi) + O_{i,j} \\
S'_{i,j}(\phi_1) &= (S(\phi_1)) \\
S'_{i,j}(\phi_2) &= (S(\phi_2))
\end{align*}$$

(1)

where $S_{i,j}(\phi)$ is the signal delivered by the pixel (i,j) for an incident power equal to $\phi$ minus the electrical offset of the
pixel. $\phi_1$ and $\phi_2$ correspond to the reference incident powers of the two blackbodies used. $(S(\phi_1))$ (respectively $(S(\phi_2))$) represents the spatially averaged signal at the incident of power $\phi_1$ (respectively $\phi_2$). The two correction coefficients are calculated from (1) assuming that each pixel has a linear response with the incident power:

$$G_{i,j} = \frac{(S(\phi_2)) - (S(\phi_1))}{S_{i,j}(\phi_2) - S_{i,j}(\phi_1)}$$

(2)

$$O_{i,j} = (S(\phi_1)) - G_{i,j}S_{i,j}(\phi_1)$$

(3)

The two-points correction (TPC) is very effective at the time it is implemented, the corrected image is perfectly uniform at the two reference incident powers $\phi_1$ and $\phi_2$. However, the problem is to know if the correction used is appropriate for a long time. That is why, the Residual Fixed Pattern Noise is calculated. It indicates the effectiveness of the reduction of the Fixed Pattern Noise by its definition itself:

$$RFPN(\phi) = \sqrt{\frac{1}{N} \sum_i \sum_j (S'_{i,j}(\phi) - (S'_{i,j}(\phi)))^2}$$

(4)

where $N$ is the total number of pixels of the FPA.

The effectiveness of the correction is not straightforward, because there is no absolute definition. One can use a relative definition to the temporal noise, which is defined as the temporal fluctuations of the signal delivered by a pixel. Indeed, fixed pattern noise and temporal noise are in general quadratically summed [5]. Therefore, the criterion that the RFPN must be inferior to the temporal noise in order to get a good image quality is reasonable.

B. Data processing

The RFPN is relevant only if it is used on pixels whose behavior follows a Gaussian statistic. But that is not the case for defective pixels. That is why, before evaluating the RFPN, a data processing has to be realized in order to detect the defective pixels and exclude them from the calculation of the RFPN. The RFPN is represented with the Well Fill (WF) of the detector. The WF can be defined as a function of the level of fullness of the ROIC capacity. Beyond 100% it is saturated and cannot be exploited. It is calculated from the incident power with the relation:

$$WF(\phi) = \frac{(S(\phi))}{(S_{sat})}$$

(5)

where $(S_{sat})$ corresponds to the average signal when the saturation is reached on the detector. For each pixel the electrical offsets have already been subtracted of the signal.

At this point, the analysis of the measurement consists of three main stages (these are successive steps):

- The detection of defective pixels
- The application of the two-points correction
- The calculation of the residual fixed pattern noise

As it was underlined in the introduction, the RFPN is a figure of merit subjected to important variations depending on the conditions used for the detection of defective pixels. As a way to highlight this problem, we will compare in the following two sets of conditions to detect defective pixels and compare the results obtained as a consequence on the RFPN.

The first set in TABLE 1, which will be further referred as reference algorithm, is applied on the averaged raw image at 50% of the well fill. This algorithm has four quantities exploited: continuous level, temporal noise, responsivity and noise equivalent temperature difference (NETD). For each pixel, if one of these quantities is too far from the corresponding average value, then it is set aside. The exact criteria and the number of pixels falling in each category are summarized in TABLE 1. The total number of defective pixels with this algorithm is 41, which corresponds to an operability of 99.95%. This total is not equal to the sum of the number of pixels in each category, because one defective pixel can have multiple defects and fall in the corresponding criteria.

TABLE 1 – Criteria used to classify one pixel as a defective one (reference algorithm), $(S)$ is the spatially average signal, $\sigma_i$ is the temporal noise of the pixel and $(\sigma)$ its spatially averaged value, $R_{ij}$ is the responsivity of the pixel and $(R)$ its spatially averaged value, NETD$_{ij}$ is the NETD of the pixel and $(NETD)$ its spatially averaged value.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Number of pixels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous level</td>
<td>$</td>
</tr>
<tr>
<td>Noise</td>
<td>$</td>
</tr>
<tr>
<td>Responsivity</td>
<td>$</td>
</tr>
<tr>
<td>NETD</td>
<td>$</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

The second algorithm, which will be called advanced algorithm, is based on the same principle, except that the two-point correction is applied before the detection of the defective pixels (except for the noise which is still applied on the raw image) and the first criterion on the continuous signal is different.

TABLE 2 – Criteria used to classify one pixel as a defective one (advanced algorithm), $(\sigma_i')$ is the spatial standard deviation of the continuous corrected signal in the FPA.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Number of pixels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous level</td>
<td>$</td>
</tr>
<tr>
<td>Noise</td>
<td>$</td>
</tr>
<tr>
<td>Responsivity</td>
<td>$</td>
</tr>
<tr>
<td>NETD</td>
<td>$</td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>
The advanced algorithm detects 8 more pixels, the 41 others are not necessarily the same as the ones detected in the reference algorithm. Indeed, applying the two-point correction before the detection of defective pixels cancels the dome effect which is due to a variation in the geometrical throughput between the center and the corner of the FPA. Thus, this effect which does not belong to the FPA problems hides some of the defective pixels and classifies some as defective while they are not.

Once the defective pixels have been set aside, the residual fixed pattern noise can be calculated on the corrected images (application of the two-point correction) thanks to equation (4).

C. Results and discussion

Fig.1 presents the temporal evolution of the ratio between the RFPN and the Temporal Noise (TN) at 50% well fill. For both algorithms, the RFPN/TN increases with time, but the advanced algorithm manages to keep the ratio lower than one, meaning than the temporal noise remains the predominant noise.

These results clearly show that the temporal stability of the T2SL MWIR FPA is excellent, even after seven weeks. They also stress the importance of the data processing phase with the criteria used to rule out defective pixels. Indeed by removing only 8 pixels, the ratio RFPN/TN decreased by 25%. The reference algorithm could not detect them, because their continuous levels were close to the average signal at 50% of the well fill mainly because of the dome effect. The application of the TPC enabled to detect them.

III. RANDOM TELEGRAPH SIGNAL NOISE

Pixels are considered as exhibiting random telegraph signal (RTS) noise when their signal oscillates between at least two levels, while the received power and operating conditions do not change. RTS noise can be harmful for image quality since the resulting blinking effect cannot be suppressed by TPC. That is why, it is considered as major problem that different technologies have to face [6-7].

A. Experimental set up and data processing

Acquiring data to study RTS noise requires long temporal acquisitions under fixed operational conditions (integration time, blackbody temperature...). A quick study shows that recording cubes of 5000 images with an integration time of 4 ms and a frame time (irreducible due to the detector used) of 16 ms is a good compromise between RTS pixels detection and data processing complexity.

In order to study the effects of RTS noise on the RFPN, the RTS data was acquired at the same time that the RFPN acquisition data. The blackbody was scanned in temperature between 10°C and 58°C with a step of 3°C. For each temperature, a cube of 5000 images was acquired, and a cube of 256 images was extracted from this big cube. Cubes of 5000 images were analyzed by an algorithm [8] for RTS detection while cubes of 256 images were used for calculating the RFPN. Then, pixels were defined as affected by RTS noise, if they were detected by both algorithms. Such measurements were repeated 24 times, representing a cumulated acquisition time of more than 9 hours. Some of the measurements were made within the same cooling, others after allowing the detector to return to room temperature.

B. Results and discussion

Measurements show that the number of RTS pixels of the MWIR T2SL 320x256 pixels FPA is very low. It varies between 0 and 10 pixels detected by acquisition of 5000 images. TABLE 3 presents a classification of the 52 pixels RTS detected out of 192 cubes, while Fig.2 shows the temporal signal of a RTS pixel. For each RFPN measurement, only the first 8 temperatures were exploited for RTS detection, to ensure that RTS pixels would not be hidden by an increasing photon noise.

<table>
<thead>
<tr>
<th>Classification</th>
<th>Number of pixels</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 level – RTS pixels</td>
<td>19</td>
</tr>
<tr>
<td>Spikes</td>
<td>15</td>
</tr>
<tr>
<td>Atypical (low frequency fluctuations)</td>
<td>18</td>
</tr>
</tbody>
</table>

TABLE 3: Classification of pixels detected affected by RTS noise

Fig.2- Signal as a function of time for one RTS pixel
Furthermore, we studied the cumulative number of RTS pixels as a function of the measurement number, in order to observe if a stabilization of this number of pixels happens.

In Fig. 3, instantaneous and cumulative number (by cumulative we mean the number of pixels which flickered at least one time from the beginning of the measurements) of RTS pixels are reported. The cumulative number of RTS pixels increases rapidly and then slows down after 125 measurements where some stabilizing plates can be observed.

After analyzing the RTS pixels detected by the advanced algorithm, a study was made in order to observe if some RTS pixels might have been not detected but have an effect on the RFPN. Therefore, we added to the list of defective pixels, the remaining pixels detected by the algorithm for RTS detection but not by the advanced algorithm. Fig. 4 shows that there is no difference observed on the RFPN/TN ratio by adding the remaining RTS pixels to the list of defective pixels. It proves that the algorithm does not need improvements.

IV. Conclusion

In this paper, we described our protocol to evaluate the stability over time of an FPA and to count up/classify pixels with random telegraph noise. We presented the results obtained for a T2SL MWIR 320x256 pixels IDCA provided by IRnova. The stability over time is excellent over seven weeks. The observation made on the criteria used demonstrated the importance to define and state clearly the criteria used for the selection of operational pixels.

Furthermore, the study of RTS pixels enabled a classification in distinguished categories of the behavior of pixels affected by RTS noise. The count up of RTS pixels needs further measurements in order to conclude on the existence of a stabilization of the number of cumulated RTS pixels.

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Characterization methods of low frequency RTS noise in cooled infrared detectors

Maxence GUENIN  
Département Optique et Techniques  
Associées ONERA  
Palaiseau, FRANCE  
maxence.guenin@onera.fr

Sophie DERELLE  
Département Optique et Techniques  
Associées ONERA  
Palaiseau, FRANCE  
sophie.dereille@onera.fr

Laurent RUBALDO  
Development & Production  
Sofradir  
Vieurey-Voroize, France  
laurent.rubaldo@sofradir.com

Isabelle RIBET-MOHAMED  
Département Optique et Techniques  
Associées ONERA  
Palaiseau, FRANCE  
isabelle.ribet@onera.fr

Marcel CAËS  
Département Optique et Techniques  
Associées ONERA  
Palaiseau, FRANCE  
marcel.caes@onera.fr

Abstract— Two methods of detection and characterization of blinking pixels are presented and compared. The first one is based on the time-scaled signal and the other is based on the power spectral density of the signal. These methods are then applied on a temperature dependent measurement. It is then shown that the number of RTS pixels and the blinking frequency follow a Boltzmann behavior.

Keywords— Focal Plane Arrays, Infrared, Random Telegraph Signal, Low Frequency Noise, Spectroscopy

I. INTRODUCTION

Within the frame of High Operating Temperature (HOT) infrared focal plane arrays (FPAs), low frequency noise, otherwise called 1/f noise [1], is one of the main issues to tackle in terms of long term stability of the image quality. It contributes to the degradation of the images quality. And then it decreases the time before a two-point correction is needed. Studies are led to uncover the predominant low frequency contributor to the degradation of RFPN in visible CMOS [2] and infrared FPAs [3][4][5][6]. Therefore, obtaining a physical understanding of the main sources of 1/f noise, such as Random Telegraph Signal (RTS), is critical to improve the detectors image stability.

A RTS pixel, usually called “blinking pixel” has its temporal signal fluctuating between two or multiple stable states. It is characterized by the amplitude jump between these states, and the mean state lifetime. If we define the states as: \([1], [2], ..., [N]\), the blinking frequency is defined as follows:

\[
f_c = \sum_{i=1}^{N} \frac{1}{\tau_i}
\]  

with \(\langle \tau_i \rangle\) the mean lifetime of the level \(i\). Fig.1 shows an example of a bi-stable RTS pixel, with its associated amplitude and mean lifetimes. Since the lifetime distribution for one single pixel is random, we can also assign it a Gaussian distribution with a standard deviation \(\sigma_{\tau}\). The amplitudes and lifetimes greatly differ from one blinking pixel to another, which makes them difficult to detect systematically.

In the first part of this paper, our RTS pixels detections and characterization methods and algorithms are described. The first method is based on Pruned Exact Linear Time (PELT) algorithm [7]. It is used to detect and characterize blinking pixels. The second method is Low Frequency Noise Spectroscopy (LFNS) [4], which is, in our case, solely used for characterization. In a second part, preliminary Arrhenius plot results obtained with both methods are presented and discussed.

Fig. 1. Typical bi-stable RTS signal over time and its different properties..
we attribute each point of the signal to one of the states with a “State Reduction” (SR) algorithm we developed. It uses the following criterion on the pixel’s non-RTS Gaussian noise: if the difference between two states is lower than \( m \) times the white noise of the signal, these two levels are equivalent to a state with a level equal to their mean value. The coefficient \( m \) is empirical, and may be adjusted. However it is most of the time close to 2. The lifetime distribution of each state can then be obtained, with its mean value and standard deviation. Fig. 2a) and b) shows PELT changepoint fit in two types of cases: high jump amplitude-to-noise ratio and low jump amplitude-to-noise ratio. We can see that in both cases, the PELT curves (in red) match well the bi-stable and multi-stable telegraph signal (in blue). Fig. 2c) shows 2b) associated histogram, and highlights the difficulty to fit Gaussians on this histogram.

This PELT method is independent from state lifetimes, which gives as much weight to short-lived states as to long-lived states and enables the detection of RTS pixels with unbalanced lifetimes (for example: \( T_{down} \gg T_{up} \)). But it also makes the spike-typed signals (very short and random pulses) more detectable and therefore brings false alarms. The other main associated drawback is its slightly higher tendency to detect changepoints along low frequency drift. This effect is mainly corrected by our SR algorithm but errors, such as detecting very slow drift noise as RTS, can still be noted, as the confirmation of the algorithm remains visual.

In order to confirm the veracity of the mean lifetime values extracted with PELT method, Low Frequency Noise Spectroscopy (LFNS) [10][11][12] is systematically applied on samples with a sufficient amount of images (min. 5k images, typ. 50k images). The LFNS spectrum is the Power Spectral Density (PSD) spectrum multiplied by the frequency. As mentioned in the introduction, RTS noise is part of the “1/f\(^\alpha\)” low frequency sources of noise of the spectrum, where \( \alpha \) mainly ranges between 1.0 and 2.0. It can be shown that RTS noise has a Lorentzian spectrum with a corner frequency \( f_c \), which is the blinking frequency. It follows the following equation in the case of bi-stable RTS noise [13]:

\[
S_{RTS}(f) = \frac{A^2}{f_c^2(t_1 + t_2)[1 + (\frac{f}{f_c})^\alpha]}
\]

with \( A \) the RTS amplitude and \( t_1, t_2 \) the respective mean lifetimes for each stable state. We can deduce that, in this case, \( \alpha_{RTS} = 2.0 \). Thus it is represented in LFNS by an intense peak centered on \( f_c \), while the 1/f noise is flatlined by the f-normalization. The area of the peak is proportional to the square of the jump amplitude. This technique has the advantage of emphasizing the high-alpha sources of noise against the low-alpha ones. The main drawback is that this technique is not a self-sufficient detection technique, as we cannot distinguish RTS noise from Generation-Recombination (GR) noise. Both of them have Lorentzian spectra and therefore have the same signature in LFNS. In general, this method is used to study GR noise and carrier lifetime, another way of reducing low frequency noise in semiconductors. In our case, Low Frequency Noise Spectroscopy is a good technique to determine precisely an RTS pixel blinking frequency, and is robust to 1/f low frequency noise, compared to temporal analysis.

Fig.3 presents a low frequency noise spectrum of a real pixel. It is fitted with a Lorentzian+white noise model and a value of blinking frequency is extracted. The PSD is estimated using the Welch periodogram method [14].
It is important to note that this blinking frequency is not representative of the whole infrared array. With a typical signal of 5k frames, the measurable blinking frequency ranges from 0.1Hz to 10Hz. Likewise, this Lorentzian+white noise model is only strictly suitable in the case of bi-stable RTS pixels, which have a Lorentzian PSD. The case of multi-stable RTS pixels is more complex. If the number of states is three, the system contains two related systems. They are either two independent systems with, coincidentally, the same amplitude, which brings us back to the aforementioned case, or two strongly correlated systems. The PSD of such a system is unclear and is not considered at the moment. This is the reason why only bi-stable pixels, which are predominant, will be considered in this paper.

III. EXPERIMENTAL RESULTS

Experimental study has been realized on a 640x512 px², 15μm pitch, blue Mid-Wave Infrared (MWIR) p-on-n HgCdTe R&D FPA from Sofradir [3]. Its cutoff frequency is $\lambda_c = 4.2\mu m$ at 140K which is in the middle of the MWIR band. The FPA is integrated in a custom liquid nitrogen continuous flow cryostat which enables control of the sensor temperature. The sensor is polarized in reverse and placed in front of a blackbody with fixed temperature (typ. 25°C). A series of 7 measurements of 50K images with the detector temperature ranging from 120K to 155K has been performed, during the same thermal cycle. The framerate being 99.8 fps, the measurement timespan is approximately 501 s. For each measurement, each pixel with a sufficient noise is treated with the PELT algorithm. When a RTS pixel is detected, its parameters are calculated (jump amplitude, mean lifetime), the LFN spectrum is computed and the LFNS blinking frequency is extracted. The proportion of RTS pixels detected for each measurement is shown in figure 4a). It is calculated by normalizing the number of RTS pixels by the highest one of the sequence (which is recorded at 155K). An exponential model is fitted with $R^2=0.9988$. Likewise, it is confirmed by the linear fit of the curve: $\ln(N_{RTS})=f(1000/T)$ with $R^2=0.9977$ in fig. 2b).

We now have a confirmation that the number of RTS pixels inside a single MWIR blue FPA has a Boltzmann behavior. In parallel, it is also shown in figure 5 that blinking frequency is activated by temperature. Fig. 5b) shows superposed LFN spectra for the PELT-detected RTS pixel n°182113 whose temporal signal is shown in figure 5a). The temporal signal has been truncated to only 500 frames for visual convenience purposes, but the LFN spectra have been evaluated with the full signal, i.e. 50K images.

![Fig. 4. a) Arhenius curve $N_{RTS}=f(T)$ of the number of RTS pixels and its exponential fitted model. b) $\ln(N_{RTS})=f(1000/T)$ curve and its linear fitted model.](image)

The blinking frequency and the area of the peak appear to increase with the temperature, as expected. Each spectrum is systematically fitted with our Lorentzian+white noise model. Fig.5c) shows the evolution of the normalized blinking frequency as a function of temperature for the same pixel. In the same way as the number of RTS pixels, it follows an Arhenius law, which is coherent with the literature [3].

LFNS is a very straightforward and efficient way to obtain corner frequencies, but is limited by the experimental conditions. The highest obtainable frequency is linearly related to the ROIC. On the opposite, the lower measurable frequency is linear with the actual length of the experiment. However, in the case of standard 640x512 pixels FPAs, the amount of data resulting in very long experiments is huge and sometimes cannot be afforded. The minimum frequency is then a compromise between a windowing of the FPA and the amount of statistics on RTS pixels needed.
considered a true RTS. LFNS can also be used to study GR noise due to its similar spectral signature.

The validity of these methods has been demonstrated in the case of an Arrhenius law study. In parallel, this study confirmed the Boltzmann behavior of the number of RTS pixels and of the blinking frequency of an individual pixel. The next step in the study is to apply these methods to the whole RTS pool of pixels in the infrared FPA in order to estimate amplitude, mean lifetime and activation energy distributions. Those methods will be used to study the RTS phenomenon in cooled infrared FPA under different stress situations.

REFERENCES


CONCLUSION

In this paper, we used two complementary methods, one based on the analysis of the temporal signal using the PELT algorithm, and the other based on the analysis of the power spectral density using the LFNS method. The PELT changepoint method is a very efficient RTS detection method, and extracts amplitudes, lifetimes, and blinking frequency for bi-stable and multi-stable blinking pixels. To avoid false alarms, Low Frequency Noise Spectroscopy is applied in order to confirm the blinking frequency obtained with PELT (only for bi-stable pixels for now). A pixel that has gone through both algorithms with consistent values is
Pulse Detection With A Multi-State System

Roy M. Howard
School of Electrical Engineering, Computing and Mathematical Sciences
Curtin University
GPO Box U1987, Perth, Western Australia, 6845, Australia.
r.howard@curtin.edu.au

Abstract—A multi-state system with potential for pulse detection is proposed. The system is based on utilizing a linear and a quantized feedback path and exhibits integration without the usual droop inherent in practical integrators. The integrator time constant is infinite. This holds provided the output rms noise level, at the three sigma level, is less than half the quantization level used. Effectively, the multi-state system acts as an integrator for a pulse input but acts as a broadband amplifier for the input noise. For pulse detection, it is shown that the performance is similar to that of a matched filter detector for the case of white noise. The structure is such that it enables sub-threshold detection of a pulse train.

Keywords--multi-state system, quantized feedback, pulse detection, matched filter, white noise.

I. INTRODUCTION

Complex systems are diverse in nature and their modelling is system dependent. Many complex systems are naturally modelled as multi-state systems with their operation being defined by dynamics between states with various feedback paths defining the options for state transitions, e.g. [1]. Cooperative behaviour, as found in self organizing systems, e.g. the decision making model [2], is based on modelling interactions between a collection of entities where local outcomes are dependent on connected state levels.

A useful model for a one-dimensional multi-state system is defined by the differential equation

\[ \frac{dx}{dt} = \sin(2\pi x(t)) \] (1)

which comprises, as illustrated in Fig. 1, of a sequence of stable states interleaved by metastable states. A similar multi-state system, based on the use of a quantizer, is illustrated in Fig. 2. This paper examines the potential such a multi-state system has for pulse detection when half the quantization level is greater than three times the rms output noise level. For such a case, the multi-state system acts as an integrator for a pulse input but acts as a broadband amplifier for the input noise. A result of this is that there is effective signal integration without the droop inherent in practical integrators and which requires additional circuitry to eliminate [3]. For pulse detection, it is shown that the performance is similar to that of a matched filter detector for the case of white noise. The structure enables sub-threshold detection of a pulse train and automatic counting of distinct pulses in a pulse train.

II. MULTI-STATE SYSTEM

Consider a multi-state system shown in Fig. 2. The differential equation characterizing the system is

\[ x^{(1)}(t) = -k_{hf} x(t) + k_{hf} k_h \left[ x(t) + \frac{0.5\Delta}{\Delta} \right] + k_h [s(t) + n(t)] \] (2)

where \( x \) is the input signal and \( n \) accounts for the noise at the input. The feedback transfer function is illustrated in Fig. 3 and, for the normalized case defined by \( \Delta = 1 \), there are stable points at \( x(t) = \ldots -1, 0, 1, \ldots \).

For small signal variations around a stable point, use of the Laplace transform, for the zero noise case, yields

\[ X(s) = \frac{GS(s) + x(0) / p}{1 + s / p}, \quad G = 1 / f_{B'} p = k_{hf} k_h \] (3)

Thus, for small signal operation around a stable operating point: gain \( = 1 / f_{B'} \), \( f_{3dB} = p / 2\pi = k_{hf} k_h / 2\pi \) (Hz) and the gain-bandwidth product is \( \text{gbwp} = k_h / 2\pi \). Importantly, for an impulsive input with an area of \( A_p \), the expected step change in the output signal is \( k_h A_p \).

![Fig. 1. Relationship between \( x(t) \) and \( x'(t) \) for the multi-stable system specified by (1).](image)

![Fig. 2. Quantizer based multi-state system.](image)
III. SYSTEM ANALYSIS

Of interest is the response of the system to a pulse input which is of sufficiently short duration such that it acts as an impulse. A half sine pulse defined according to (normalized input amplitude case)

\[ s(t) = \sin(2\pi f_o t)[u(t) - u(t - D)], \]  

(4)

with a duration of \( D = 1/2f_o \) and an area of \( 1/\pi f_o \), is used.

A. System Pulse Response

The response of a first order system with a transfer function, e.g. [5], has been used. Thus:

\[ H(s) = \frac{G}{1 + s/p}, \quad f_{3dB} = \frac{p}{2\pi} G = \frac{1}{f_B}. \]  

(5)

to the pulse defined by (4) is

\[ x(t) = G x_1(t) u(t) + G x_1(t - D) u(t - D) \]  

(6)

where

\[ x_1(t) = \frac{2\pi f_o p}{p^2 + 4\pi^2 f_o^2} e^{-p t} + \frac{p}{2\pi f_o} \sin(2\pi f_o t) - \cos(2\pi f_o t). \]  

(7)

The pulse response is shown in Fig. 4 for the case of \( f_o = 1 \).

A measure of when the system cannot follow the input pulse, i.e. a measure of when the input signal acts as an impulse, is

\[ f_{3dB} \leq f_o/2 = 1/4D. \]  

(8)

B. Quantizer Modelling

An ideal quantizer, with a resolution of \( \Delta \), has a transfer function defined by

\[ q(x) = \Delta \left[ \frac{x + 0.5\Delta}{\Delta} \right]. \]  

(9)

To facilitate simulation, the following continuous approximation to the quantizer transfer function, over the range \(-n\Delta\) to \(n\Delta\), is used:

\[ q_{\Delta}(n, x) = -n\Delta + \sum_{i=-n+1}^{n} \frac{\Delta}{2} \left[ 1 + \tanh \left( \frac{x + 0.5\Delta - i\Delta}{\Delta_q} \right) \right], \]  

(10)

where \( \Delta_q = \Delta/100 \). The overall feedback transfer function is shown in Fig. 3.

C. Noise Characterization

The input noise signal is assumed to be white over the interval \([0, f_{max}]\), with an average power of \( A_n^2 \), and is defined according to [4]

\[ n(t) = \sqrt{\frac{\Delta N}{2\pi}} \sin(2\pi f_o t + \phi_k) \]  

(11)

where the frequencies \( \{f_1, ..., f_N\} \) are chosen at random from the interval \([0, f_{max}]\) and \( \{\phi_1, ..., \phi_N\} \) are chosen at random from \([-\pi, \pi]\). The power spectral density of such a noise signal is approximated by

\[ G_n(f) = \frac{A_n^2}{2f_{max}}, \quad -f_{max} < f < f_{max}. \]  

(12)

D. Minimum Detectable Pulse Area

First, an input pulse with a short duration, such that it acts as an impulse, is assumed. Its area is denoted \( A_p \). For this case the output signal level is

\[ \text{output signal level} = k_h A_p = 2\pi \cdot \text{gbwp} \cdot A_p. \]  

(13)

Second, the noise variance at the output, for the case of white noise, and small signal operation around a set operating point, is

\[ \sigma^2 = \int_{-\infty}^{\infty} G_n(f)|H(f)|^2 df = 2G_n(0) \cdot \frac{\pi k_h \text{gbwp}}{2 \pi} \]  

(14)

where the noise equivalent bandwidth result for a single pole transfer function, e.g. [5], has been used. Thus:
\[
\sigma = \sqrt{G_n(0)} \cdot \sqrt{f_{3dB}} \cdot \text{gain} = \sqrt{G_n(0)} \cdot \frac{\text{gbwp}}{\sqrt{f_{3dB}}} \quad (15)
\]

Consistent with (13) and (15), for gbwp fixed and a fixed output signal level, the output rms noise level increases according to the \(\sqrt{\text{gain}}\) and decreases according to \(1/\sqrt{f_{3dB}}\). Hence, improved performance is obtained, for gbwp fixed, by decreasing the gain and increasing \(f_{3dB}\).

The minimum detectable pulse area, defined by the situation where the output level equals the quantization level, and the rms noise level equals the half the quantization noise level divided by three (the three standard deviation case) implies:

\[
\left\{ \begin{array}{l}
2\pi \cdot \text{gbwp} \cdot A_p = \Delta \\
\frac{\pi G_n(0)}{\sqrt{3}} \cdot \frac{\text{gbwp}}{\sqrt{f_{3dB}}} \leq \frac{\Delta}{6} \\
A_p \geq \frac{\Delta}{2\pi \cdot \text{gbwp}} \geq \frac{6}{2\pi \cdot \text{gbwp}} \left[ \frac{\pi G_n(0)}{\sqrt{f_{3dB}}} \cdot \frac{\text{gbwp}}{\sqrt{f_{3dB}}} \right]
\end{array} \right. \Rightarrow (16)
\]

Hence, the minimum detectable pulse area is

\[
A_p(\min) = \frac{3}{\sqrt{\pi}} G_n(0) f_{3dB}. \quad (17)
\]

For a fixed noise level, and fixed gain-bandwidth product, the minimum detectable pulse area is inversely proportional to the square root of the system bandwidth and can be decreased by increasing the system bandwidth (decreasing the system gain).

However, as the bandwidth is increased, a point is reached where the system no longer acts impulsively with respect to the input pulse. For a pulse duration of \(D\) seconds this is specified by (8). Assuming the maximum bandwidth is \(f_{3dB} = 1/4D\), it follows that the minimum detectable pulse area is

\[
A_p(\min) = \frac{6}{\sqrt{\pi}} G_n(0) D. \quad (18)
\]

E. Matched Filter Pulse Detection

Consider the case of pulse detection by a matched filter, e.g. [6], where the matched filter impulse response is defined by

\[
h(t) = s(T - t) \quad (19)
\]

for the case of the input signal \(s\) being a pulse. The output of the matched filter detector is

\[
x(t) = \int_{-\infty}^{\infty} s(\lambda) h(t - \lambda) d\lambda = \int_{-\infty}^{\infty} s(\lambda) s(T - t + \lambda) d\lambda. \quad (20)
\]

At the time \(t = T\), the matched filter output is a maximum and equals the pulse energy in the interval \([0, T]\), i.e.

\[
x(T) = \int_{0}^{T} s^2(\lambda) d\lambda = \text{pulse energy}. \quad (21)
\]

For the case of white noise, with a power spectral density of \(G_n\), the noise variance at the filter output is

\[
\sigma^2 = \int_{-\infty}^{\infty} G_n(f) |h(f)|^2 df = G_n(0) \int_{0}^{\infty} |h(t)|^2 dt = G_n(0) \cdot \text{pulse energy}
\]

where Parseval’s relationship has been used. With the minimum detectable pulse energy being defined as the energy when the output signal level is equal to the rms noise level, it follows that:

\[
\text{pulse energy} \geq \sqrt{G_n(0) \cdot \text{pulse energy}} \Rightarrow \text{pulse energy} \geq G_n(0)
\]

For the case where the signal level is equal to three times (or more) the rms noise level, it follows that

\[
\text{pulse energy} \geq 9G_n(0). \quad (24)
\]

F. Comparison of Pulse Detection Approaches

Consider the case of a rectangular pulse with a duration of \(D\) seconds and a height of \(A\), consistent with a pulse area of \(AD\) and a pulse energy of \(A^2D\). For such a case and using the situation of the output level being three times the rms signal level:

\[
AD \geq 6/\sqrt{\pi} \cdot \sqrt{G_n(0) D} \quad \text{multi-state system}
\]

\[
\Rightarrow A^2D \geq 36/\pi \cdot G_n(0)
\]

\[
A^2D \geq 9G_n(0) \quad \text{matched filter detection}
\]

Hence, for this case, the multi-state system yields slightly poorer performance, in terms of minimum detectable signal pulse energy, in comparison to a matched filter detector.

IV. RESULTS

Results are presented for the normalized case defined by: gain = 1, \(f_{3dB} = 1\), gbwp = 1 and unit quantization level, \(\Delta = 1\). For this case, the gain for the pulse area, as defined by (13), is \(2\pi\). The input pulse duration is assumed to be such that the system acts impulsively. Consistent with (8), \(D = 1/8f_{3dB}\) and \(f_0 = 4f_{3dB}\) is assumed. Consistent with (4) and (13), the input pulse amplitude is \(\pi f_0 \Delta / k_h = f_0 \Delta / 2\text{gbwp}\) for an input signal level of \(\Delta\).

The input noise is assume to be white over the frequency range \(-10f_{3dB} < f < 10f_{3dB}\), i.e. \(f_{\text{max}} = 10f_{3dB}\). The input
rms noise level, for the case of the output rms noise level being equal to \(\Delta/6\), then is

\[
A_n = \frac{\Delta}{6} \frac{\sqrt{2}}{4\pi} \frac{f_{\max}}{f_{3\text{dB}}} \frac{g_{\text{bw}}}{p}
\]

which follows from (12) and (15). The number of sinusoids comprising each noise signal is assumed to be 1000.

A. Pulse Detection

One example of the detection of a single pulse is shown in Fig. 5. The pulse amplitude is \(A_p/\Delta\) and this level is consistent with an output level of unity.

The detection of a pulse train, with random times as defined by a Poisson point process with a rate of \(\lambda = 0.1\), is shown in Fig. 6.

B. Sub-Threshold Detection

The detection of a pulse train, which is embedded in a white noise signal, is illustrated in Fig. 7 for the case of pulse amplitudes equal to 0.5\(\pi f_o/k_h\) and consistent with an individual output pulse level of \(\Delta/2\). This level is below the level of detection in the absence of noise.

C. Notes

For separated pulses in a pulse train comprising of equal area pulses, the structure is an automatic counter of the number of pulses. In general, the output is consistent with the integral of the input signal but without the droop inherent in a practical integrator. The time constant is infinite. When the input pulse area is not consistent with a stable output level, there is a transient to the closest stable output level. The time constant associated with this transient is defined by the small signal bandwidth and is \(\tau = 1/p = 1/k_{f_{3\text{dB}}}\). For the case where the input pulse area does not precisely correspond to one of the quantized output levels, a quantization error in the output exists. A quantizer, based on a sample and hold circuit, yields very similar performance provided the sample rate is greater than the system bandwidth.

V. CONCLUSION

A multi-state system, based on a linear and a quantized feedback path, with potential for pulse detection, has been proposed. Provided half the quantization level is greater than three to four times the rms output noise level, the system acts as an integrator for impulsive input pulses but like a broad-band amplifier for the input noise. The result is pulse integration without droop. For pulse detection, it is shown that the performance is similar to that of a matched filter detector for the case of white noise. The structure is such that it enables sub-threshold detection of a pulse train.

VI. REFERENCES


Noise and Linearity of High-Speed SiGe HBT Cells in CE and CB Configuration

P. Sakalas$^{1,3,4}$, Member IEEE, A. Mukherjee$^1$, M. Schröter$^1$, Senior Member IEEE

$^1$CEDIC, Technische Universität Dresden, 01062 Dresden, Germany
$^3$FRL, Semiconductor Physics Institute of Center for Physical Sciences and Technology, 10257 Vilnius, Lithuania
$^4$BPTI, Baltic Advanced Technology Institute, Saulėtekio al. 15, LT-10224 Vilnius, Lithuania

Abstract -- High-frequency (h. f.) noise and linearity of high-speed power cells consisting of advanced SiGe heterojunction bipolar transistors (HBTs) in common-emitter (CE) and common-base (CB) configuration were investigated. The cells features optimized metallization interconnections to reduce parasitics. DC, RF, and nonlinear large-signal characteristics as well as noise parameters were measured, simulated and analyzed. The observed low noise and output power linearity of the SiGe HBT power cells in CB operation makes them suitable for low-noise and high-frequency power applications.

Index-Terms: Compact modeling, heterojunction bipolar transistor, linearity, noise parameters, SiGe HBT.

I. INTRODUCTION

Recent advances and predictions in SiGe HBT technology [1],[2],[3] have enabled the realization of ultra h. f. low noise amplifiers, transceivers and MIMO radars [4]-[14]. Both the power amplifier and the low noise amplifier (LNA) are important components in RF transceivers. In order to increase power gain and linearity, a cascode connecting transistors in CE and CB configuration is usually used [15], with a SiGe HBT based LNA operating at 130 GHz [16]. A record frequency of 245 GHz LNA was realized in 500 GHz (DotFive) SiGe technology with a CB configuration [14]. G-band low-noise amplifiers were realized in both CE and CB operation [17], which achieved the lowest ever noise figure at 140-210 GHz frequencies [17]. LNA with CE exhibit better noise performance compared to stacked CE and CB operation LNA. The noise behavior of SiGe HBTs in different configurations was investigated in [18],[19]. It was found that the noise of first generation SiGe HBTs in CB configuration is nearly the same as in CE configuration for frequencies below 2 GHz but is higher at frequencies above 6 GHz, especially at higher collector current density [18]. The difference was explained with the noise correlation effect. In addition, the base resistance has a significant impact on $N_{\text{min}}$ in the CB case. However, a simplified noise model ignoring the base resistance for the first generation SiGe HBTs yields nearly same $N_{\text{min}}$ [19] in CE and CB configuration. The circuit design with high speed SiGe HBTs in CE, CB configuration requires exact models which captures linearity and noise behavior.

In this work, fourth generation multifinger high-speed SiGe HBT cells with optimized (i.e. reduced) base resistance in CE and CB configuration were measured, analyzed and compared using a compact model that includes the impact of noise correlation. Due to the multifinger HBT structure and respective smaller base resistance a larger $f_{\text{max}}$ and lower noise figure ($N_F$) are expected. Measured noise parameters in a wide frequency band from 8 GHz to 50 GHz, linearity and the associated analysis of a double-emitter high-speed cell in CE and CB configuration are presented.

II. DUT AND MEASUREMENT SETUP

A. HBT technology and SiGe Cell Layout Design

The investigated SiGe cells consisted of two CBEBEBC devices connected in length direction, combined into a single cell with an emitter area $A_E=2 \times 2 \times 0.13 \mu m \times 10.16 \mu m$. They were fabricated in the IHP SG13G2 process, which offers seven metal layers [20]. The high-speed cell layout was realized with two HBTs, separated by a shallow trench and combined into a single cell. The layout is shown in Fig. 1. The collectors and bases of the individual devices, were connected locally and terminated with Topmetal2 of the process at the east and west side respectively. The emitter contacts, which are parallel to each other, were connected towards the vertical direction up to Topmetal2 and finally grounded from both north and south sides.

![Fig. 1: Layout of embedded in RF pads 2xCBEBEBC SiGe HBT cell with grounded CE configuration.](image-url)

B. Measurement equipment

On-wafer DC, RF (0.1-67 GHz), and nonlinear characteristics were measured with a PNA-X 5247A and HP4142 SMU. High-frequency noise parameters were measured with Maury Microwave Automated Tuner System ATS 5.21 07. Circuit simulations were done with...
III. RESULTS AND DISCUSSION

A. DC and RF characteristics

Output IV characteristic for the complete SiGe cell in CE configuration are shown in Fig. 2. Avalanche multiplication at $V_{BE} = 0.74$ V starts at $BV_{CEO} = 1.6$ V. Model parameters for both CE and CB SiGe HBTs were extracted from a set of measurements over temperature and using special tetrode structures. For $J_C(V_{CE})$ excellent agreement between model and measurement is obtained. Exceeding $BV_{CEO}$ with the collector bias turned out to be beneficial in terms of output power and RF performance for cascode power circuits [4],[23]. Therefore, $V_{CE} = 1.8$ V was set as quiescent bias. Load circles with an incident RF power of -9.4 dBm are shown for both $V_{CE} = 1.5$ and 1.8 V. In both cases, the avalanche region is entered dynamically. Investigations show only a negligible impact of the avalanche effect on distortion. $J_C$ increase (c. f. Fig. 2 at $V_{CE} > 1.6$ V) is more due to the thermal selfheating which is a competitive mechanism to avalanche multiplication: more avalanche current turns more selfheating resulting to less avalanche current.

![Graph](Fig. 2: Collector current density of the cell versus CE voltage with $I_B$ drive (1 uA to 501 uA, 50 uA). Lines are HICUM.)

Forward Gummel plots at $V_{CE}=1.5$ and 1.8 V are presented in Fig. 3. The dips in absolute value of $J_B$ at $V_{CE} = 1.8$ V correspond to current reversal due to avalanche multiplication in the base-collector region. The time dependent incident $V_{BE}$ wave at 10 GHz is also shown in order to establish a correspondence with the bias operating range.

![Graph](Fig. 3: Gummel plot at $V_{CE} = 1.8$ V, 1.5 V and $V_{BE}$ wave at 10 GHz of incident voltage with quiescent $V_{BE}$ = 0.84 V.)

Output IV characteristics of SiGe HBT in CB configuration with $I_E$ drive are given in Fig. 4. The current gain cut-off frequency $f_T$ (for CE configuration only) and the maximum frequency of oscillation $f_{max}$ versus current density are given in the Fig. 5. The $f_{max}$ is similar for the CE and CB devices. The collector current wave in response to the incident power at 10 GHz is also shown in Fig. 5 to indicate current density swing ranges. The excellent agreement of the model with DC, RF and nonlinear characteristics in the forward active region enabled further detailed analysis of the noise behavior.

![Graph](Fig. 4: Collector current density versus collector base voltage of CB SiGe HBT with emitter current drive: -50 mA to -5 mA, 5 mA step.)

![Graph](Fig. 5: Current gain cut-off frequency $f_T$ and $f_{max}$ over collector current density for a SiGe HBT CE and CB cells. Collector current wave of incident power at 10 GHz is shown for CE.)
one dB compression point for CE SiGe HBT at \( V_{CE}=1.8 \) V and \( V_{BE}=0.85 \) V: 1dB = -7dBm and for CB configuration more than +5 dBm.

Fig. 7: Output power of fundamental (2 and 10 GHz) and harmonic frequencies versus input power of the CB cell at \( V_{CB}=0.9 \) V and \( V_{EB}=-0.93 \) V.

Fig. 8: Output power of fundamental (2 GHz) and harmonic frequencies versus \( P_{in} \) of the CE cell at \( V_{CE}=1.8 \) V, \( V_{BE}=0.85 \) V.

Fig. 9: \( J_c(P_{in}) \) at 2 and 10 GHz for the CB and CE cells, biased (CB) with \( V_{EB}=-0.93 \) V, \( V_{CB}=0.9 \) V, \( V_{CE}=1.8 \) V, \( V_{BE}=0.85 \) V (CE). Fig. 8 shows quiescent and dynamic current (at 2 and 10 GHz) versus input power. Frequency dependence is observed which is more pronounced in CE configuration devices.

C. High-frequency noise behavior

High-frequency noise parameters of the CE and CB cells were measured in a frequency band of 8 to 50 GHz. \( N_{F_{\min}} \) is presented in Fig. 10. The best in terms of low noise, bias points were selected to present frequency dependent noise data for CE and CB configuration devices. The comparison of \( N_{F_{\min}} \) is conditional since noise figure depends not only on the level of internal noise sources like collector current shot noise but also on transfer function or gain [21]. Therefore there is not much sense to keep same collector current in CE and CB devices and having different gain and thus transfer function to transfer noise source (collector current shot noise) to the input as defined in \( N_{F_{\min}} \) calculation. As expected for high-speed devices, fairly low noise is observed. CB devices show lower noise at lower frequencies due to lower resistance at their input port (emitter resistance) compared to the base resistance for CE device. The corresponding noise resistance \( R_n \) is around 5 \( \Omega \) (CE) and 4 \( \Omega \) (CB) over the measured frequency band. Such low noise resistance enables an easy impedance matching of the SiGe power cell for obtaining low-noise performance over a wide bias range. The frequency dependence of \( N_{F_{\min}} \) for CB operation is very close to that of the CE case as was observed in [18]. The impact of shot noise correlation on \( N_{F_{\min}} \) for this high-speed technology is negligible up to frequencies beyond 60 GHz.

Fig. 10: Minimum noise figure \( N_{F_{\min}} \) versus frequency for SiGe CE and CB power cells. \( R_{BX}=6 \) \( \Omega \), \( J_c=41 \) mA/\( \mu \)m\(^2\) (for \( V_{BE}=0.86 \) V).

The second (upper) set of curves in Fig. 10 shows the current density dependence of \( N_{F_{\min}} \). For a fixed frequency of 10 GHz, \( N_{F_{\min}} \) within the current density range of 2...10 mA/\( \mu \)m\(^2\) is lower for the CB case. Noise due to avalanche multiplication was found to be negligible.

IV. CONCLUSIONS

Linearity and high-frequency noise of SiGe HBTs in common-base and common-emitter configuration were investigated. A high-speed power cell with CB configuration exhibited high linearity for 10 GHz input power and a low third harmonic (below 60 dBm at \( P_{in}=0 \) dBm) at the cost of lower transducer power gain (4 dB) compared to a SiGe HBT in CE configuration and 25 dB power gain. Impact of avalanche multiplication on harmonic distortion was found to be negligible despite quiescent operation of about 0.2 V beyond \( BVD_C \). A dispersion of the third harmonic for both CE and CB devices was observed.

Noise parameters for a wide frequency band of 8-50 GHz were measured for both CE and CB devices. Selected bias points for frequency dependent noise parameters were used for relative comparison. A lower
$NF_{\min}$ for the CB device at lower millimeter wave frequencies was observed. The low noise figure along with a low noise resistance and relatively fair linearity enables the use of high-speed SiGe HBTs not only for low-noise applications but also for power amplifiers in cascades at millimeter wave frequencies.

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Implementing software defined noise generators

Robert Mingesz*
Department of Technical Informatics, Interdisciplinary Excellence Centre, University of Szeged
Szeged, H-6720, Hungary
mingesz@inf.u-szeged.hu

Dénes Faragó
Department of Technical Informatics, Interdisciplinary Excellence Centre, University of Szeged
Szeged, H-6720, Hungary
dfarago@inf.u-szeged.hu

Abstract—In the last few decades, the research field related to noise and fluctuations became more and more important and the usage of the results is increasingly promising. In addition to simulations, testing and validation of implemented systems should also be performed on real physical systems. These measurements require noise generators with specific distribution and spectral properties. It is obvious, that the most convenient method to generate such noise is by using digital signal processing. The randomness of these sources is provided by pseudo-random number generators, generators that have limited cycle length/time. Of course, signal generators capable of noise generation are commercially accessible, however, these solutions have limitations. In our previous publication, we presented a Digital Signal Processor based solution capable of generating $1/f$ shaped noises. As a result of advances in technology these days there are many user-friendly and high-performance solutions that provide us new possibilities in noise generation. To ensure proper period length, first, we implemented different pseudo-random number generators on the FPGA based hardware. Then, we adapted and extended our algorithm to the new hardware, as a result, we could significantly increase the sampling frequency, the available bandwidth, the D/A resolution and the number of channels. The noise generator is developed in the LabVIEW programming environment and can run on any FPGA based NI platforms, but it can be ported to any other FPGA based hardware. The new instrument uses a flexible filter arrangement to combine filters of different frequencies to produce more generic spectra up to five decades. The implemented solution is open-source, thus, anyone can use, customize or make further developments of the design for their personal needs.

Keywords—$1/f$ noise, noise generator, FPGA, DSP, digital filter, software defined instrument

I. INTRODUCTION

There are several research fields where noise plays a prominent role. For example, in the case of Fluctuation enhanced sensing (FES), the noise from the sensors carries substantial information about the gases or other substances to be detected [1], in the case of Kirchhoff-Law-Johnson-Noise (KLJN) secure key distribution, noise plays an essential role in encrypted communication [2]. In order to test such systems and to develop the necessary electronic equipment, there is a need for reliable high-performance noise generators that produce an analog signal with appropriate parameters.

Several methods exist for implementing noise generators. If white noise production is required, a combination of Zener or noise-diodes and amplifiers can comfortably deliver superior signal quality up to high frequencies. To produce $1/f$ noises, there are numerous, even completely analog solutions. However, this approach has drawbacks, therefore, in most cases, a mixed-signal solution is chosen where an algorithm produces the desired noise, which is then converted to the desired signal source by digital to analog (D/A) converters. Many of noise generators based on these solutions are commercially available, but most of them have several limitations.

Our approach was to find a solution, that is based on off-the-shelf hardware, thus it can be easily reproduced. Our experiments required an uninterrupted, continuous noise source, with substantially long cycle length. The required sampling frequency was over 100 kHz, up to 1 MHz, and the spectral distribution of the noise had to be more general than a $1/f$ noise.

II. CYCLE LENGTH AND QUALITY OF PSEUDO-RANDOM-NUMBER GENERATORS

True random generators are ideal in many ways to provide sources for random noise generation. However, realizing true random generators is a complicated and expensive task. In addition, noises generated using true random sources cannot be reproduced and the same experiment cannot be performed again. In contrast, algorithm-based pseudorandom number generators can be implemented easily using a small amount of resources. Moreover, the same random sequence can be reproduced by resetting the algorithm with the same beginning state (seed). Compared to the true random generators, their disadvantage lies in their deterministic nature and the finite length of the produced number series.

In today's world, long-term measurements are necessary in numerous cases where a short cycle length can cause unpleasant problems. Commercially available arbitrary waveform generators (AWG) can generate arbitrary signals, but they generate noise with limited and often short cycle length due to finite memory or the weakness of the random generator contained therein.

Colored noise with arbitrary spectral shape can be produced using FFTs. However, the produced signal is definitely finite, and seriously limited by the available memory, currently, it is impractical to create continuous noise with more than $2^{32}$ points.

Currently, there are many high-quality pseudorandom number generators that can produce long enough cycle lengths that are suitable for most of the applications. If even longer sequences are required, then different random generators can be combined. This method not only increases the cycle length but also increases the independence of the elements in the sequence of numbers, thereby improving the uniformity and quality [3].

While the cycle length of generators is given in samples, in real measurements it is important to compare them to applicable measurement times. A simple comparison is done in TABLE I.

*Corresponding Author

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### TABLE I. COMPARISON OF CYCLE LENGTHS WITH APPLICABLE MEASUREMENT TIMES

<table>
<thead>
<tr>
<th>Cycle length</th>
<th>Sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 kHz</td>
</tr>
<tr>
<td>65536 points</td>
<td>0.65 s</td>
</tr>
<tr>
<td>(16 bit pseudorandom generator)</td>
<td></td>
</tr>
<tr>
<td>8 million points</td>
<td>80 s</td>
</tr>
<tr>
<td>(typical AWG)</td>
<td></td>
</tr>
<tr>
<td>128 million points</td>
<td>21 min</td>
</tr>
<tr>
<td>(AWG with extended memory)</td>
<td></td>
</tr>
<tr>
<td>4 billion points</td>
<td>12 hours</td>
</tr>
<tr>
<td>(2128)</td>
<td></td>
</tr>
<tr>
<td>64 bit LCG or Xorshift pseudorandom number generators</td>
<td>5.9E−6 years</td>
</tr>
<tr>
<td>2128 points</td>
<td></td>
</tr>
<tr>
<td>128 bit Xorshift generators</td>
<td>1.1E+26 years</td>
</tr>
</tbody>
</table>

As shown in TABLE I, short cycle noise generators will repeat the generated signal in a very short period of time, which is unacceptable for most of the measurements. It will be even more problematic if we use more than one signal source for our measurement at the same time, the result of the measurements can be simply incorrect. Using 64-bit or 128-bit generators significantly decreases the chance of such artifacts. The quality of the random generators, the randomness of the generated number series, may be critical in most cases, especially for cryptographic applications. Quality of generators is usually verified by statistical tests. One of the most widespread statistical test collection is the Diehard tests [4][4], with an open source implementation of Dieharder [5]. We tested our implementations using this set of tests, the results are shown in TABLE II.

### III. REAL-TIME NOISE GENERATION HARDWARE

Since the cycle length of FFT based noise generators are severely limited for most of the desired applications, the only possibility is to create a continuous noise stream in real time. For this, we first generate a source of random numbers, then we form the spectral shape using a filter bank, then we apply a proper D/A conversion.

While a personal computer has enough processing power to perform this task, since it is not designed to be a real-time system, it is not suitable for continuous noise generation. Additionally, the bandwidth limit between the PC and the D/A card can also cause problems. For this reason, we are looking for devices that are designed to be real-time systems. The most suitable solutions:

- **Microcontrollers**: currently available 32-bit ARM Cortex M4 and M7 microcontrollers can easily handle 32-bit floating-point numbers, have clock frequencies over 200 MHz, while they also have multiple built-in D/A converters with sampling frequencies over 1 MHz.

- **Digital signal processors (DSP)**: these devices are especially designed for real-time signal processing; our previous noise generator was based on an Analog Devices DSP [6]. These days, more and more DSP functionalities are built in high-performance microcontrollers or FPGAs.

### Field-programmable gate arrays (FPGA)

Field-programmable gate arrays (FPGA): these devices have a large number of logic gates suitable for implementing high-speed digital devices. Modern FPGAs also contain a high number of DPU slices. For this reason, it is especially suitable to generate multiple streams of real-time noise data.

Our goal was to find an off-the-shelf solution, that can provide enough processing power at an affordable cost. We implemented our solution based on a National Instrument FPGA hardware (NI USB7856R) [7]. The hardware can be conveniently programmed in LabVIEW environment, the performance of the hardware implementation can be easily compared to simulation and theoretical results.

The NI USB7856R is a USB based instrument containing a Kintex-7 160T FPGA. It has eight 16-bit D/A converters, capable of sampling frequencies up to 1 MHz. The FPGA has 101,400 LUTs and 600 DSP slices. The maximum clock frequency is 200 MHz, in our application we used a 40 MHz clock. Our solution is compatible with other types of NI multifunction reconfigurable I/O devices (USB, PCIe, and PXIe based) and cRIO devices.

![Fig. 1. Block diagram of the noise generator hardware](image-url)

### IV. IMPLEMENTING RANDOM NUMBER GENERATORS

There are many well-known pseudo-random generators, we selected the following ones to be implemented on our hardware: Xorshift [8], Linear congruential generators (LCG) [9], linear feedback shift registers [10], and Mersenne Twister [11]. We implemented these generators as LabVIEW code, this solution provides a handful of possibilities of using the generators in different, LabVIEW compatible FPGA based platforms, as well as in other real-time systems or on traditional personal computers.

Most of the random generators can be implemented in different word lengths resulting in different cycle lengths. The generators were tested using the previously mentioned Dieharder tests, the results, as well as the resource usages are summarized in TABLE II.

The LCGs, while widely popular, are not the best performing generators. Their bit quality is mostly caused by their hyperstructure as well as the shorter cycle length of their least significant bits [9]. However, in our application, since we are using only the most significant bits, this may not cause significant problems.

The linear feedback shift register may seem a good solution considering its cycle length and resource usage; however, it requires multiple cycles in order to generate a single random number.

The Xorshift generator (Fig. 2) is generally a high-quality generator with limited resource usage.

Implementing the Mersenne Twister is not so straightforward; at the same time, it passes all the tests and has an astonishingly long cycle length. A long initialization...
sequence is also needed, but since it has to be done only once, it can be implemented on the host computer sparing a lot of valuable FPGA resources.

Note: none of the implemented random generators are cryptographically secure, for this reason, they should not be used in live cryptographic applications.

TABLE II. QUALITY, CYCLE LENGTH AND RESOURCE REQUIREMENTS OF IMPLEMENTED PSEUDORANDOM NUMBER GENERATORS

<table>
<thead>
<tr>
<th>Pseudorandom Number Generator</th>
<th>Passed Dieharder Tests</th>
<th>Maximum Cycle Length</th>
<th>FPGA resource utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear congruential generator</td>
<td>3/17</td>
<td>$2^{64}$</td>
<td>Slices: 812 DSP48s: 10</td>
</tr>
<tr>
<td>Linear feedback shift register</td>
<td>12/17</td>
<td>$2^{1936} - 1$</td>
<td>Slices: 716 DSP48s: 0</td>
</tr>
<tr>
<td>Xorshift</td>
<td>15/17</td>
<td>$2^{128} - 1$</td>
<td>Slices: 737 DSP48s: 0</td>
</tr>
<tr>
<td>Mersenne Twister</td>
<td>17/17</td>
<td>$3^{207} - 1$</td>
<td>Slices: 356 DSP48s: 0</td>
</tr>
</tbody>
</table>

Fig. 2. Labview implementation of the 64-bit Xorshift pseudorandom number generator.

All the implemented random generators produce a uniformly distributed number. When using noises, we generally need a Gaussian-distributed noise. To achieve this distribution, we can use different algorithms. One of them is the well-known Box-Muller transform. The main drawback of this method is that it requires the usage of floating-point arithmetic and several functions that are hard to be implemented on FPGAs. Another method to approximate Gaussian distribution is to sum a number of uniformly distributed noises. In our implementation, we summed 12 random numbers to produce a single normally-distributed number.

V. ACHIEVING THE DESIRED SPECTRAL SHAPE

In the previous implementation, we used first-order low-pass filters to generate $1/f^\alpha$ noises. However, to perform a wide variety of experiments, it is indispensable to generate more generic spectra. Using strictly low-pass filters limits the possibilities, therefore we implemented generic components that can serve as high-pass, low-pass or band-pass filters. These are composed of two traditional IIR filters that are connected in series. The desired spectral shape can be achieved by applying a bank of these filters on the Gaussian random number source, as visualized in Fig. 3.

These filters can be easily parameterized by their cut-off frequencies and amplitudes. In order to achieve the desired spectrum, these parameters must be optimized. In our setup, the cut-off frequencies are first equally distributed over the relevant frequency band. Then we can select which type of filters do we want to use: low-pass, high-pass or band-pass. In the next step, we provide a set of starting values for filter amplitudes. The LabVIEW’s built-in differential evolution-based global optimization function [12] can be used to optimize only the filter amplitudes or both the amplitudes and cut-off frequencies.

The solution uses only the analytically calculated transfer functions. The goodness function of the fit is the absolute difference between the desired and the realized power spectra in the selected frequency range. In our previous solution, we selected a more complex function, of course, there is also the possibility to consider other aspects here.

Fig. 3. Obtaining the desired spectral shape using a bank of generic filters (low-pass, high-pass or band-pass)

The optimization is done on the host PC and may take several minutes, depending on the number of filters and the desired shape. During the optimization, we can consider the $\sin(x)/x$ effect of the D/A conversion.

Fig. 4. Filter setup for achieving a $1/f^\alpha$ noise.

Fig. 5. Filter transfer multiplied by $f^{\alpha/2}$ to emphasize the difference between the desired and realized filter.
In Fig. 4 we can see the result of such an optimization when the desired output was a pink noise with α=0.8 using only low-pass filters. In the next figure, we multiplied the spectrum with $f^{-0.8}$ to emphasize the difference between the desired and realized filter. We can observe that the filters with most significant roles are the first and last one.

VI. PERFORMANCE ANALYSIS

In our hardware tests, we used a 128-bit Xorshift pseudo-random generator with a filter bank containing 10 filters. To save DSP resources, a single filter component implemented on the FPGA fabric is used to sequentially calculate the result of the filter bank. This filter, depending on filter coefficients, can serve as a first or second order low-pass, high-pass or band-pass filter. The used hardware has a maximum sampling frequency of 1 MHz, the implementation was fast enough to provide the required data rate. Higher filter count or higher data rate can be achieved by increasing the parallelism in the hardware. The resource usage on the NI USB-7856R is summarized in TABLE III. Based on these results, we can observe that up to 6 parallel noise generators can be implemented without further optimization.

TABLE III. DEVICE UTILIZATION OF OUR CURRENT SETUP ON THE NI USB-7856R

<table>
<thead>
<tr>
<th>Resource</th>
<th>FPGA Resource Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Slices</td>
<td>16.1% 4084 out of 25350</td>
</tr>
<tr>
<td>Slice Registers</td>
<td>9.2% 18610 out of 202800</td>
</tr>
<tr>
<td>Slice LUTs</td>
<td>7% 7091 out of 101400</td>
</tr>
<tr>
<td>Block RAMs</td>
<td>0.3% 1 out of 325</td>
</tr>
<tr>
<td>DSP48s</td>
<td>5.8% 35 out of 600</td>
</tr>
</tbody>
</table>

We extensively tested the implementation using software simulations and real measurements. As a demonstration, in Fig. 6 we generated a signal that has a corner in the spectrum at 13 kHz. In Fig. 7 we demonstrate the measured spectrum of the generated analog signal.

VII. CONCLUSION

We realized an efficient mixed-signal noise generation system based on the widely known method of applying a filter bank to modify the shape of a noise. The implementation can support up to six independent noise outputs at a 1 MHz sample rate. The parameters of filter banks are set precisely by an optimization algorithm, resulting in spectral shapes that could not be achieved otherwise.

The implemented software is open source and is created in the widespread LabVIEW programming environment. The code can be used on a wide range of NI FPGA devices, and beside the implemented algorithm it can be easily supplemented with further unique functions, which provide new possibilities like a mixture of noise and deterministic signals, synchronous excitation, and measurement.

Further information and the source code are available on our webpage[13].

REFERENCES

Radon Transform and Dynamic Stochastic Resonance based Technique for Line Detection from Noisy Image

Rajib Kumar Jha\textsuperscript{1}, IEEE, Senior Member, Badal Soni\textsuperscript{2}, Sumit Kumar\textsuperscript{1}, IEEE, Member, Vivek S. Verma\textsuperscript{3}

\textsuperscript{1}Indian Institute of Technology Patna, \textsuperscript{2}National Institute of Technology Silchar, \textsuperscript{3} A. K. Garg Engg. Coll. Ghaziabad

(jharajib, soni.bada88, sumitphd13, viveksv10) @gmail.com

Abstract—The Radon transform is an important transform to detect line feature from the noisy image. Radon transform can transform two-dimensional images (with noisy or disturbed lines) into a domain of possible parameters of line, where each line in the image will give a peak position at the corresponding parameters of the line. It has led to many line detection applications within image processing, computer vision, earthquake engineering etc. When the lines are subjected to very high background noises, Radon transform alone is not so effective. Here, in this paper, we propose dynamic stochastic resonance (DSR) based Radon transform for weak line extraction. The DSR is an iterative process that tunes the coefficient of Radon transform so that we may get the enhanced lines of the image. We compare our proposed method with the results of the Gaussian low pass filter. The proposed technique adopts local adaptive processing, and it significantly enhances the line feature of an image. Experimental results are also given to show the effectiveness of the proposed method.

Index Terms—Radon Transform, Dynamic Stochastic Resonance, Noise, Denoising.

I. INTRODUCTION

The extraction of the lines of an image is one of the basic need in the image processing application where line features are closely related to the edges of an image. The edge is defined as a boundary between two regions with relatively distinct gray levels. Guido et al. [1] introduce an algorithm to detect a line which is based on weighted minimum mean square error. This algorithm involves the use of matrices and uses a set of matrix operations such as transpose and multiplications. Marco et al. [2] developed a set of algorithms to detect a line based on a general formulation of a combinatorial optimization problem. In the algorithm, a lot of expensive operations like multiplication, power and exponent functions are used. However, algorithms perform faster. Guru et al. [3] proposed an algorithm based on small eigenvalue analysis to extract straight line segments in an image. The algorithm depends on scanning the input image from the top left corner to the bottom down the corner with a moving mask matrix using the small eigenvalue analysis and pixel connectivity. Also, this algorithm is improved by Lee et al. [4] and it consists of two significant steps. The first step is labeling the edge of the image. The second step is applying the principal components analysis for each labeled edge. However, Kazuhito et al. [5] proposed a high-speed line detection method using the Hough transform in the local area, which consists of thinning and thresholding as a major operation. There are many existing techniques adopted to improve the performance of the Hough transform. Dahyot et al. [6] introduced the concept of statistical Hough transform, where he extended the formulation to continuous kernel estimates. Furthermore, Dahyot et al. discussed the robustness and insensitive of the estimated density to noise and the choice of the origin of the spatial coordinates [6].

Here, in this paper, we propose dynamic stochastic resonance which can implement the automatic search and fill peak points. However, this algorithm has a high demand for image preprocessing; otherwise, it causes the mass error accumulation. Kyrki et al. [8] discussed an improved line detection algorithm combining the local and the global detection which improved the speed effectively and minimizes memory occupation. But this algorithm had weak robustness [9].

Here, we take a noisy line image, and after subtracting mean value from image pixels, we apply Radon transform and then convert it into the frequency domain using DCT. After that, we apply the DSR iterative equation. This DSR system enhances the energy of the noisy line of the image. It also suppresses the noise because the parameter of the DSR system has been calculated with the condition of maximization of SNR.

Organization of the paper: The rest of the conceptual part is arranged as follows. Section II deals with the basic mathematical framework. Implementation, results and discussions are presented in Section III. We make concluding remarks in Section IV.

II. PRELIMINARY MATHEMATICS

Here, in this section, we discuss the basic mathematics of dynamic stochastic resonance (DSR) and Radon transform.

A. Dynamic Stochastic Resonance

The concept of SR was first proposed to explain the recurrence of the ice age by Benzi et al. [10]. The mechanism of SR is based on the addition of a moderate amount of noise. It was traditionally believed that the presence of noise could only make a system worse. However, recent studies have shown that in non-linear systems, noise can induce more ordered regimes and increase the signal-to-noise ratio (SNR), and noise can be used to play a productive role in enhancing the weak input signal [11]–[13]. A concept of dynamic stochastic resonance (DSR) that uses noise to improve the performance of a system has been used for different image and signal processing applications such as image enhancement [14], [15], edge detection [16], de-noising [17], encryption [18], and watermarking [19]–[22]. The SR mechanism shows that at lower noise intensities the weak signal is unable to cross the threshold, thus giving
a very low SNR, for large noise intensities the output is dominated by noise, also leading to a low signal to noise ratio.

The bistable model is conventionally used by the physicists to explain DSR phenomenon. A classic one-dimensional nonlinear dynamic system that exhibits SR is modeled with the help of Langevin equation of motion which is given in Eq. (1).

\[ m \frac{d^2x(t)}{dt^2} + \gamma \frac{dx(t)}{dt} = -\frac{dU(x)}{dx} + \sqrt{D} \xi(t). \]  

Eq. (1) describes the motion of a particle of mass \( m \) moving in the presence of friction (\( \gamma \)). The restoring force is expressed as the gradient of bistable potential function \( U(x) \). \( D \) is the noise variance and \( \xi(t) \) is an additive zero mean stochastic fluctuation (noise). If the double well system is heavily damped, the inertial term \( m \frac{d^2x(t)}{dt^2} \) can be neglected. \( U(x) \) is a bistable quartic potential given in Eq. (2).

\[ U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4. \]  

where, \( a \) and \( b \) are positive bistable double-well parameters. The \( x_m = \pm \sqrt{\frac{a}{2}} \) are separated by a barrier of height and \( \Delta U = \frac{a}{2} \), when the \( \xi(t) \) is zero. Eq. (1) can be normalized by the constant term \( \gamma \). Now, the addition of a periodic input signal \( B \sin(\omega t) \) to the bistable system makes it time-dependent whose dynamics are governed by Eq. (3).

\[ \frac{dx(t)}{dt} = -\frac{dU(x)}{dx} + B \sin(\omega t) + \sqrt{D} \xi(t), \]  

where \( B \) and \( \omega \) are the amplitude and frequency of the periodic signal respectively.

The most common quantifier of SR is SNR whose expression for DSR as derived in [23] is given below.

\[ \text{SNR} = \left[ \frac{4a}{\sqrt{2}(\sigma_{b}\sigma_{a})} \right] \exp\left( \frac{-a}{2\sigma_{a}^{2}} \right). \]  

where \( \sigma_{b} \) is the standard deviation of the noise in the SR-based system and \( \sigma_{a} \) is the internal noise standard deviation of the original bistable system. In [20], these double well parameters \( a \) and \( b \) have been obtained by maximization of the SNR expression of DSR. This gives \( a = 2\sigma_{a}^{2} \) for obtaining optimum SNR. To ensure that the low contrast signal is subthreshold, a condition for the value of parameter \( b \) has been derived. The value of \( b \) is obtained as

\[ b < \frac{4a^3}{27}. \]  

For computational implementation of DSR in digital images, Eq. 3 is discretized into Eq. (6).

\[ x(n + 1) = x(n) + \Delta t \left[ (ax(n) - bx^3(n)) + \text{Input} \right]. \]  

Note that \( \text{Input} = B \sin(\omega t) + \sqrt{D} \xi(t) \) denotes the sequence of the input signal and noise with the initial condition being \( x(0) = 0 \). The final stochastic simulation is obtained after the certain number of iterations.

B. Radon Transform

The Radon transform computes the line integrals from multiple sources along parallel paths in a certain direction. \( g(q, \rho) \) is the line integral of the image intensity \( f(x, y) \) along a line \( l \) and that is the distance from the origin, and it is the angle from X-axis to the normal direction of the line.

\[ g(\theta, \rho) = \int f(x, y) dl. \]  

Radon transform is defined in continuous domain as follows.

\[ R(\theta, \rho) = \int f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy, \]  

where \( f(x, y) \) is the grayscale image and \( \delta(x) \) is given as follows.

\[ \delta(x) = \begin{cases} 1, & k = 0, \\ 0, & k \neq 0. \end{cases} \]  

We can say that Radon transform detects the line by integrating image pixels in all possible direction in an image. Let the integral of the image function \( f(x, y) \) along the line \( \rho = x \cos \theta + y \sin \theta \) is denoted by \( R(\theta, \rho) \). Each line will give the peak in the transform domain which corresponds to a line that is brighter than the background. Thus, line detection in the image is simplified as the detection of peaks and through transform domain.

III. Implementation, Results & Discussion

The steps of line detection are as in Algorithm 1. The experiments are carried out to evaluate the performance of proposed technique of line detection. We have taken an image and then we add different salt & pepper noise for making the original noisy image. For the simulation, all experiments are conducted on MATLAB 2014b with 4GB RAM and Intel(R) Core(TM)i3-4130 CPU 3.40 GHz. The parameters of the DSR are taken as follows. \( m = 10^{-5} \), \( a = 2m((\text{std}(R(\theta)))^2), \) \( t = 0.002, \) and \( b = (4(a^3))/27). \)

A. Quantitative Parameters

We consider noise variance (NV), noise mean value (NMV) and noise standard deviation (NSD). Furthermore, we consider mean square difference (MSD) as other crucial parameter.

1) Noise mean value (NMV), noise variance (NV) and noise standard variance (NSV) can be mathematically expressed as follows.

\[ \text{a) NMV} = \sum_{i,j} I_{\text{out}}(i,j), \]

\[ \text{b) NV} = \sum_{i,j} (I_{\text{out}}(i,j) - \text{NMV})^2, \]

\[ \text{c) NSD} = \sqrt{\text{NV}}, \]

where \( I_{\text{out}} \) is the output image with dimension \( I \times J \). A lower value of variance gives a clearer image although it depends on the intensity.

2) Mean square difference (MSD) indicates the average difference of the pixels throughout the image where \( I_{\text{out}} \)
Algorithm 1 Proposed DSR based Line Detection

1: procedure

GRAY SCALE LINE IMAGE IS TAKEN AND ADD THE NOISE INTO THE IMAGE (X)

2: Mean value of image pixels is calculated, and this value is subtracted by every pixel of the image so that we can make the image rotation invariant.

\[ f^u(x', y') = f(x, y) - \text{mean}(f(x, y)), \quad (10) \]

where \( f(x, y) \) is the gray scale image and \( f^u(x, y) \) is the processed image after subtracting mean value from original.

3: Radon transform is applied on the processed image \( f^u(x, y) \) for different \( \theta \) angles given by following equation,

\[ R'(\theta) = \int_{-\infty}^{+\infty} f^u(x' \cos \theta - y' \sin \theta, x' \sin \theta + y' \cos \theta) dy'. \quad (11) \]

Here, \( R'(\theta) \) is the Radon transform coefficients.

4: Discrete cosine transform (DCT) is calculated after Radon transform for converting image pixel into frequency domain

\[ R_{dct}'(\theta) = DCT(R'(\theta)) \quad (12) \]

5: Now applying the DSR iterative equation on DCT coefficient for enhancing the energy of line pixel and reducing the noise effect by DSR based double well system. Putting Radon transform in Eq. 6, we can write the equation as follows.

\[ R_{dsr}(n+1) = R_{dsr}(n) + (\Delta t) \times [\alpha R_{dsr}(n) - b R_{dsr}^3(n) + R_{dct}'(\theta)] \quad (13) \]

6: This is an iterative process. Here, a and b are double good parameter selection of these parameters given above and \( n \) is the number of iteration and we consider \( n = 25 \). \( R_{dsr}(0) = 0 \) is considered.

7: end procedure

is the output image and \( I_{in} \) is the original image with salt & pepper noise. It can be defined as follows.

\[ MSD = \frac{\sum_{i,j} (I_{out}(i,j) - I_{in}(i,j))^2}{I \times J} \]

Fig. 1: Four different input images with distinct edges information. (a) Test-1, (b) Test-2, (c) Test-3, (d) Test-4.

Fig. 2: Images (a) & (b) represent the noisy input images, however, (c)–(f) are the output images. (a) var=0.3, (b) var=0.5, (c) output of LPF for (a) image, (d) output of LPF for (b) image, (e) output of Proposed DSR for (a) image, (f) output of Proposed DSR for (b) image.

Fig. 3: Images (a) & (b) represent the noisy input images, however, (c)–(f) are the output images. (a) var=0.3, (b) var=0.5, (c) output of LPF for (a) image, (d) output of LPF for (b) image, (e) output of Proposed DSR for (a) image, (f) output of Proposed DSR for (b) image.

B. Discussion

Silent feature of our proposed line detection method discuss here. In Fig. 2 and Fig. 3, we have shown the outcomes of the proposed method and [24] when the input images are noisy images. The Fig. 2(e) and Fig. 2(f) are the proposed output for Fig. 2(a) and Fig. 2(b) noisy inputs. It can be seen that our proposed method can detect and extract the lines clearly with compare to [24]. Similarly, we can discuss for the Test-2 images, as shown in Fig. 3.

TABLE I: different assessment parameters results on noise (variance)= 0.30

<table>
<thead>
<tr>
<th>Image</th>
<th>MSD</th>
<th>NMV</th>
<th>NV</th>
<th>NSD</th>
<th>MSD</th>
<th>NMV</th>
<th>NV</th>
<th>NSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test-1</td>
<td>167.13</td>
<td>201.25</td>
<td>0.002</td>
<td>0.0490</td>
<td>205.36</td>
<td>195.93</td>
<td>0.0005</td>
<td>0.0235</td>
</tr>
<tr>
<td>Test-2</td>
<td>164.91</td>
<td>205.21</td>
<td>0.002</td>
<td>0.0437</td>
<td>206.37</td>
<td>201.05</td>
<td>0.0005</td>
<td>0.0225</td>
</tr>
<tr>
<td>Test-3</td>
<td>173.68</td>
<td>202.01</td>
<td>0.001</td>
<td>0.9335</td>
<td>215.62</td>
<td>198.03</td>
<td>0.0006</td>
<td>0.0250</td>
</tr>
<tr>
<td>Test-4</td>
<td>167.02</td>
<td>201.10</td>
<td>0.002</td>
<td>0.0457</td>
<td>216.92</td>
<td>193.33</td>
<td>0.0010</td>
<td>0.0315</td>
</tr>
</tbody>
</table>
The quantitative parameters such as MSD, NMV, NV, and NSD can be studied in Table I. Our proposed method has much better capability compared to others to detect the lines in a noisy environment. The features and discussion of the benefits of DSR have been provided below.

1) Mechanism of Dynamic Stochastic Resonance for Line Detection from noisy image: The basic mechanism of DSR is to improve the performance of traditional line detector. The DSR iteration tunes the DCT based Radon transform coefficients. It increases the energy of line pixels present in the image. Our technique is to detect line from heavy background noise images. Noise may occur in digital images for several reasons. The signal is usually corrupted by noise which is generally salt & pepper noise or maybe additive Gaussian noise. In this noise scenario, the Radon transform is unable to detect lines from the image. Therefore, in our technique, we used the combination of Radon transform and dynamic stochastic resonance. The coefficients are tuned in such a way that the energy of the information part increases and the effect of noise is suppressed. For this, we used an adaptive process for selection of parameters of DSR, which is calculated by maximization of SNR. DSR iterative equation given in Eq. 6 tunes the DCT coefficients of Radon transform.

IV. CONLUSION

We have seen from results that proposed line detection algorithm works well in case of heavy background noise. The results of the proposed algorithm have been compared with conventional Gaussian Low pass filter (LPF) on real-life images with noise. It is to be noted that on all the real-life images considered; the proposed algorithm produced fairly good results on different noise intensity values.

In the current line detection scenario, lines can be considered to be a weak signal as it is statistically invisible when the image is subjected to noise. The image apart from the lines can be considered as noise. Here, stochastic fluctuation (noise) can be given to the transformed coefficients of the noisy image so that its distribution varies in such a way that at some optimum noise density, the Radon coefficient can jump from the noisy state to the enhanced state. The results suggested that the Radon coefficients get enhanced and would easily be detected the line features of the image. The experimental results in Table I, Fig. 2 and Fig. 3 show that our method is very effective in-line detection for many images.

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REFERENCES


Suprathreshold Stochastic Resonance Characterization for Gamma Noise with Watermarking Application

Sumit Kumar 1, IEEE, Member, Nancy Chauhan 2, Rajib Kumar Jha 1, IEEE, Senior Member
1 Indian Institute of Technology Patna, 2 Indian Institute of Information Technology Una
Email:(sumitphd13,nancychnl,jharajib@gmail.com)

Abstract—Noise plays a constructive role in a lot of non-linear applications. Many non-linear systems perform better when some calculated external noise is added. This phenomenon is called stochastic resonance (SR). When a parallel array of SR is used, it is termed as Suprathreshold Stochastic Resonance (SSR). Many of the systems and models where SR is effectively observed are non-linear systems with a single threshold value.

In this paper, the effect of SSR using Gamma noise has been reported, and the expression for correlation has been derived. Furthermore, the same concept has been used in a watermarking application, where Gamma noise is added as the signature in the cover image. Gamma noise is later added externally to the watermarked image which maximizes the cross-correlation at a particular value of the variance of the SR noise. The obtained results suggest that the maximum value of cross-correlation is attained for a certain amount of added noise.

Index Terms—SSR, Watermarking, Gamma noise.

I. INTRODUCTION

Stochastic resonance (SR) is a phenomenon that is said to occur when the presence of noise benefits the optimal output of some non-linear systems. The concept of SR was proposed by Benzi et al. [1], where SR was loosely discussed as an event in which the presence of an optimal amount of noise leads to an increment in signal-to-noise ratio.

Collins et al. [2] discuss the FitzHugh-Nagumo neuron model characterized by aperiodic stochastic resonance (ASR). They mention a measure based on the cross-correlation coefficient to characterize ASR. A lot of work including the study of leaky integrate-and-fire and Hodgkin-Huxley neuron models has been done using SR with a cross-correlation measure to find the optimal noise level for a considerable number of neurons [3]. SSR has also been demonstrated in integrate-and-fire neurons in the context of both input correlations and for other noise-based enhancement effects (cite Durrant et al. 2011) [4], along with saturation [5]. Furthermore, SSR concept has also been used for SR in visual perception enhancement, and the findings on using a particular noise have also been discussed [6].

Stocks et al. [7] show that SR can occur where Shannon’s average mutual information measure between input and output of the array gets optimized under the condition that all thresholds are set to be the same value. Most importantly, SR is said to have happened regardless of whether the input signal is entirely subthreshold or not. This is the specialty of SSR which justifies the name “Supra-threshold”. Similarly, more work has been shown in McDonnell et al. [8], Kumar et al. [9]–[11]. In this paper, the concept of SSR [12] has been discussed, and the expression for the cross-correlation coefficient for Gamma signal and Gamma noise distribution has been derived. Subsequently, the same concept is applied to watermark signal detection. [13]

The paper is organized as follows. In Section II, the expression for the cross-correlation coefficient for Gamma distributed signal and Gamma distributed noise has been derived. Section III deals with proposed method for watermark embedding and extraction. The results are discussed in Section IV. Finally, the conclusion of the paper is given in Section V.

II. BASIC MATHEMATICS

Here, the exact analytical expression of the cross-correlation coefficient for Gamma noise has been derived. Suprathreshold stochastic resonance (SSR) was described by Stocks et al. [14].

A. Mathematical Model for SSR and Single Threshold

In this method, the output depends on the threshold value i.e., it becomes one if input signal X and added noise ζ crosses the threshold Δ, otherwise it becomes zero. Mathematically, the thresholded signal \( Y_i \) is given as

\[
Y_i = \begin{cases} 
1 & \text{for } X + \zeta_i > \Delta, \\
-1 & \text{for } X + \zeta_i < \Delta,
\end{cases}
\]  

(1)

Considering \( Y_{\text{norm}} := \frac{1}{N} \sum_{i=1}^{N} Y_i \). The normalized signal \( Y_{\text{norm}} \) becomes

\[
Y_{\text{norm}} = \frac{1}{N} \sum_{i=1}^{N} \text{sgn}(X + \zeta_i),
\]

(2)

where \( \text{sgn}(\cdot) \) is a sign function.

B. Calculation of correlation

The probability that the output of any thresholded signal is 1 (i.e., the sum of signal and noise is greater than \( \Delta \)) is denoted by \( P_{1|x} \) for a known signal, x. Here, it is assumed that the threshold value is considered as \( \theta \) and the noise distribution is identical. So, \( P_{1|x} \) is given as follows.

\[
P_{1|x} = \begin{cases} 
\int_{\theta}^{\infty} f_\zeta(\omega) d(\omega), & \text{continuous } \zeta \\
\sum_{k=\theta}^{\infty} P(\zeta = k), & \text{discrete } \zeta,
\end{cases}
\]

(3)

where \( f_\zeta(\cdot) \) is the pdf of the noise. Now, we want to calculate \( E[Y_{\text{norm}}|X = x] \). When \( Y_{\text{norm}} := \frac{1}{N} \sum_{i=1}^{N} Y_i \) from Eq.2, the expected value of \( Y_{\text{norm}} \) given x is

\[
E[Y_{\text{norm}}|X] = \frac{1}{n} E \left[ \sum_{i=1}^{n} \text{sgn}(X + \zeta_i) \right]
\]

(4)
Since all noises are independent and identically distributed, so
\( E[Y_{\text{norm}}|X] = E[\text{sgn}(X + \zeta_i)] \) \hspace{1cm} (5)
\( E[Y_{\text{norm}}|X] = (1P_{|x|} + (-1)(1 - P_{|x|}) = (2P_{|x|} - 1) \) \hspace{1cm} (6)

The cross-correlation (\( \rho \)) between X and \( Y_{\text{norm}} \) can be found by
\[ \rho = \frac{E[XY_{\text{norm}}] - E[X]E[Y_{\text{norm}}]}{\sqrt{\text{var}[X]\text{var}[Y_{\text{norm}}]}} \] \hspace{1cm} (7)
\[ E[xY_{\text{norm}}] = \int_{-\infty}^{\infty} x E[Y_{\text{norm}}|X] P(x) dx . \] \hspace{1cm} (8)

\[ E[Y_{\text{norm}}|X] = 2 \int_{-\infty}^{\infty} x P(x) P_{|x|} dx - E[x] \] \hspace{1cm} (9)

For discrete \( x \) the integral in above expression will be replaced by summation over the entire domain of \( x \).

C. Correlation for Gamma Distributed Signal

The cross-correlation is computed between \( Y_{\text{norm}} \) and \( x \). The pdf of watermark signal taken as Gamma distributed noise is \( P_{\zeta}(\cdot) \). The pdf of the random noise which is used in detection of embedded watermark signal is also Gamma distributed and is given by \( P_{\zeta}(\cdot) \). So,

\[ f_x(x) = \begin{cases} \frac{1}{\Gamma(a)} e^{-x} x^{a-1}, & 0 \leq x \leq \infty, a > 0 \\ 0, & \text{ elsewhere} \end{cases} \] \hspace{1cm} (10)

\[ f_{\zeta}(\zeta) = \begin{cases} \frac{1}{\Gamma(b)} e^{-\zeta} \zeta^{b-1}, & 0 \leq \zeta \leq \infty, b > 0 \\ 0, & \text{ elsewhere} \end{cases} \] \hspace{1cm} (11)

\[ P_{|x|} = \int_{-\infty}^{\infty} \frac{1}{\Gamma(b)} e^{-\zeta} \zeta^{b-1} d\zeta = \frac{1}{\Gamma(b)} \sum_{k=0}^{b-1} e^{-(\theta-k)(\theta-x)} k! \] \hspace{1cm} (12)

The expected value of \( P_{|x|} \) over the signal distribution can be given as below.

\[ E[P_{|x|}] = \int_{0}^{\infty} P_{|x|} \frac{1}{\Gamma(a)} e^{-x} x^{a-1} dx \\
= \frac{1}{\Gamma(a)} \frac{1}{\Gamma(b)} \sum_{k=0}^{b-1} e^{-\vartheta} \theta^{k} x^{a-1} dx, \] \hspace{1cm} (13)

where \( \alpha = \int_{0}^{\vartheta} (1 - \frac{x}{\vartheta})^k \theta^k x^{a-1} dx, \)
\[ \alpha = \int_{0}^{\vartheta} (1 - \frac{x}{\vartheta})^k \left( \frac{x}{\vartheta} \right)^{a-1} dx. \] \hspace{1cm} (14)

Let \( \frac{\vartheta}{\theta} = t. \) Furthermore, it can be written as
\[ \alpha = \theta^{k+a} \Gamma(k+1) \Gamma(a) \Gamma(k+a+1) \] \hspace{1cm} (15)

Then, \( E[P_{|x|}] \) can be written as follows.
\[ E[P_{|x|}] = \frac{1}{\Gamma(b)} \sum_{k=0}^{b-1} e^{-\theta} \theta^{k+a} \] \hspace{1cm} (16)

Now, substituting Eq. 16 in Eq. 18, we get
\[ E[y|x] = E[\text{sgn}(x + \zeta_i)] = 2P_{|x|} - 1. \] \hspace{1cm} (17)

Similarly,
\[ E[y] = E[E[y|x]] = 2E[P_{|x|} - 1] \] \hspace{1cm} (18)

Now we need to calculate \( E[y^2|x] \).
\[ E[y^2|x] = \frac{1}{N^2} \left[ N + (N-1)(2P_{|x|} - 1)^2 \right] \] \hspace{1cm} (19)

Similarly, we can calculate \( E[xy] \) as below.
\[ E[xy] = 2 \int_{0}^{\infty} (xf_x(x)P_{|x|}) dx - E[x] \] \hspace{1cm} (21)

The results clearly show that for increasing number of parallel arrays, the cross-correlation value \( \rho \) is increasing.

III. PROPOSED METHOD FOR WATERMARK DETECTION

The embedding and extraction steps are shown in Fig. 2 and Fig. 3 respectively.

A. Watermark embedding

Here, the watermark embedding technique has been discussed in detail. Initially, the original image is taken of size 256 x 256 as shown in Fig. 4. The size of the watermark signal is also taken to be 256 x 256. The algorithm used for watermark embedding is given below.

1) Three level DWT coefficient of LL bands are evaluated.
2) A watermark signal that is Gamma distributed is generated with a certain seed value.
3) T1 is initialized, and the watermark is embedded in those pixels whose intensity is higher than T1.
4) Finally, the inverse wavelet transform is taken to get the watermarked image. The size of the watermarked image is same as that of the original image.
B. The Proposed algorithm using SSR and single threshold

For watermark detection, a 3-level DWT transform is applied to image. Significant coefficients are selected and correlated with the original watermark X. For the selection of DWT coefficients, a threshold $T_2 > T_1$ is chosen. This is done because correlation is calculated only for all those coefficients where watermarks were added at the time of embedding. Besides, external noise will have increased the values of these pixels and so, a threshold higher than $T_1$ is needed.

The proposed algorithm is discussed below.

1) A 3-levels DWT transform is done on the watermarked image to get $V_d$.

2) Gamma distributed noise is added to those DWT coefficients which are higher than the second threshold value $T_2$.

3) The third threshold value (say M) is taken as the mean of those DWT coefficients of the watermarked image which are higher than the second threshold value $T_2$. The noise added DWT coefficients (say U) is then compared with the third threshold value (M). If U is less than M, then the corresponding coefficient is replaced by zero; otherwise, it is replaced by 255. So, a thresholded image $V_d''$ is obtained.

4) Similar to Step-1 and Step-2, Gamma distributed noises are added 30 times to the 3-level DWT coefficients of the distorted watermarked image. This is done because the addition of noise 30 times to the 3-level DWT coefficients improves the detector response. Increasing the number of times that noise is added further gives no improvement in detection and the detector response saturates. However, if noise is added less than 30 times to the 3-level DWT coefficients, the detector response is found to be deteriorating.

5) The 30 threshold images, $V_d''$, are summed and averaged to get a single frame $V_d''$. The mathematical equation for this is given in Eq. 2, where N is the number of times the noise is added to the DWT coefficients. These are

$$\sum_{i=1}^{30} V_d'' = \frac{1}{30} \sum_{i=1}^{30} V_d'' $$

6) Correlation Z is calculated between $V_d''$ coefficients and original watermark X using Eq. 23, given below.

$$Z = \frac{1}{A} \sum_{i=1}^{M} |V_d''| $$

7) A fourth threshold S is defined using Eq. 24 given below, where A runs over all coefficients with values $> T_2 > T_1$. A is the number of such coefficients, M is the difference of size of the image to the size of LL band of the image, and Z is the correlation value.

$$S = \frac{\alpha}{2A} \sum_{i=1}^{M} |V_d''| $$

8) Compare correlation value Z with the fourth threshold of S. Watermark is said to be detected in the image if $Z > S$. A higher value of the ratio of correlation to the threshold($\frac{Z}{S}$) provides more authenticity to the detection of watermark. It has been shown in Fig. 5.

IV. RESULT & DISCUSSION

In this section, the results of watermark detection application with the insertion of external noise are discussed. The software platform used was MATLAB 2014b with 8GB RAM and Intel(R) Core(TM)i3-4130 CPU @ 3.40 GHz. The watermark signal was embedded into the wavelet coefficients whose value were greater than $T_1$. Since the value of $T_1$ was taken such that $T_1 > 0$, it may be said that it loosely followed a shifted version of Gamma distribution. Assuming that the signal is Gamma distributed, Gamma noise was selected as

$$ V_d'' = \left[ \frac{1}{30} \sum_{i=1}^{30} V_d'' \right] $$

where $V_d''$ and $V_d''$ are the individual pixel values in the watermark X. The size of the $V_d''$ and X are the same as the original image size.
Table I: Comparative correlations of proposed method with others for Lena image.

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</thead>
<tbody>
<tr>
<td>Correlation (Z)</td>
<td>32.88</td>
<td>84.1</td>
<td>0.816</td>
<td>3.5</td>
<td>1.62</td>
<td>166.0408</td>
</tr>
<tr>
<td>Threshold (S)</td>
<td>9.76</td>
<td>14.0</td>
<td>0.41</td>
<td>1.73</td>
<td>0.45</td>
<td>16.4476</td>
</tr>
<tr>
<td>Ratio (Z/S)</td>
<td>3.53</td>
<td>6.07</td>
<td>1.99</td>
<td>2.02</td>
<td>3.6</td>
<td>10.0952</td>
</tr>
</tbody>
</table>

SSR noise. In the Table I, the correlation (Z), threshold (S) and ratio \( (\frac{Z}{S}) \) for some of the other known techniques has been compared with the results for the proposed method. The proposed method gives us a correlation value (Z) that is significantly higher when compared with Dugad et al. [15] and Jha et al. [16]. It is also seen that the ratio between correlation coefficient and threshold \( (\frac{Z}{S}) \) is higher for the proposed method. So, it can be said that the proposed method works suitably well when compared to others.

V. Conclusion

A new method of using SSR with Gamma noise is developed, and the expression for cross-correlation has been derived. For different values of noise variance (i.e., \( \zeta \)), cross-correlation coefficient values have been plotted. The plot shows a maximum of cross-correlation coefficient at a particular value of \( \sigma \) of stochastic resonance noise. Here, \( b \) is the variance of the externally added Gamma noise. Furthermore, this concept has been applied to enhance watermark detection. The authenticity of watermark detection improves in the presence of SSR noise which confirms the benefits of SSR. SSR is tested on different attacked watermarked images, and the correlation is found to be 4-5 times higher than the threshold. This demonstrates that the SSR method which is noise based improves the correlation in watermark detection. The detector response proves the robustness of SSR based watermark detection technique.

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References

Effect of Clustering on Fluctuations in Binding Activity of Sugar Chains to Influenza Viruses

Toshio Kawahara, Hiroaki Hiramatsu, Yuhsuke Ohmi, Toshimitsu Hayashi, Yasuo Suzuki
College of Life and Health Sciences
Chubu University
Kasugai 487-8501, Japan

Nongluk Sriwilaijaroen
Faculty of Medicine
Thammasat University
Pathumthani 12120, Thailand

Shin-ichi Nakakita
Life Science Research Center
Kagawa University
Kagawa 761-0793, Japan

Yohei Watanabe
Graduate School of Medical Sciences
Kyoto Prefectural University of Medicine
Kyoto 602-8566, Japan

Kenzo Maehashi
Institute of Engineering
Tokyo University of Agriculture and Technology
Tokyo 184-8588, Japan

Takao Ono, Yasushi Kanai, Kazuhiko Matsumoto
The Institute of Scientific and Industrial Research
Osaka University
Osaka 567-0047, Japan

Abstract—The spread of influenza A virus variants requires continuous monitoring given the high mutation rate. The surveillance of such mutations is essential and therefore highly sensitive sensor systems are needed. Host changes can be observed as changes in the bonding between virus protein and host sugar chains. We have investigated the dispersion of sugar chains such as sialylglycopeptide (SGP) and sialoglycopolymer, to name a few. These represent potential candidate molecules in biosensors for detecting influenza virus host specificity. The clustering of sugar chains might enhance and stabilize the binding activity and the relative errors in binding activity fluctuations will be addressed here.

Index Terms—influenza virus, host specificity, sugar chain, neoglycoprotein, biosensor

I. INTRODUCTION

A worldwide outbreak of influenza is the most feared potential public health emergency of international concern [1]. The spread of influenza A virus variants requires continuous monitoring given the high mutation rate of the virus [2]. Many infections in humans have been reported and this might be due to the potential of these viruses to mutate which results in a change of host from birds to humans [3], [4]. Since the surveillance of such mutations is essential, the development of highly sensitive sensor systems is needed. We have developed a biosensor using nano-carbon materials which can detect the preferred host of the virus [5], [6].

Host changes can be observed as a change in the bonding between virus protein and host sugar chains. We have developed several processes to fabricate biosensor systems. One of these utilizes an immobilizing process for the binding of specific molecules onto the device surface, such as those found in avian and human influenza viruses [7]–[10]. For biosensors with host specificity for influenza viruses, we used sugar chains with different coordination of terminal sialic acid and penultimate galactose, such as α(2-3)-linkage sialic acid for an avian influenza virus and α(2-6)-linkage sialic acid for a human virus.

In this paper, we will discuss about the dispersion of sugar chains such as sialylglycopeptide (SGP) [11], and sialoglycopolymer [12], [13] to name a few. These represent potential candidate molecules for use in biosensors for the detection of influenza viruses, and we have investigated virus host specificity using an enzyme-linked immunosorbent assay (ELISA). Using bovine serum albumin (BSA) localization, we have obtained a stable distribution. The clustering of sugar chains might also enhance and stabilize the binding activity. The relative errors in binding activity fluctuations will also be addressed here. Compared with single sugar chains of SGP, clustered molecules of sialoglycopolymer and SGP-BSA display smaller relative errors.

II. EXPERIMENTS

Sugar chains of SGP (molecular weight (MW) ∼2.8 kDa), sialoglycopolymer (MW ∼3500 kDa), and neoglycoprotein of SGP-BSA (MW ∼66 kDa) were used. For SGP, glycopeptide
powder was purified to monomer by reversed-phase HPLC. For sialoglycopolymer, sialyllactosamine was attached to polymer chains of γ-polyglutamic acid by chemoenzymatic synthesis [14]. The exchange ratio of sugar chains was 51.5%, and the structures of the synthesized sialoglycopolymer were confirmed by 1H and 13C NMR analysis [15]. For SGP-BSA, the amino group of SGP was attached to sulfhydryl groups of BSA using m-maleimidobenzoyl-N-hydroxysulfosuccinimide ester [16]. The average number of SGP groups attached to each BSA molecule was approximately 10, as determined by HPLC analysis of purified fluorescently-labeled N-glycan.

The properties of the binding between viruses and sugar chains were examined by ELISA. Solutions of sugar chains at concentrations from 1000 ng/mL to 15.6 ng/mL with twofold serial dilution by PBS were spread onto 96-well plates (Nunc MaxiSorp flat-bottom, Thermo Fisher Scientific) for sialoglycopolymer and SGP-BSA. One thousand nanograms per milliliter of sialoglycopolymer corresponds to 997 pM of sialic acid, and that of SGP-BSA to 279 pM. SGP was used at concentrations ranging from 5 ng/mL to 78 pg/mL, and 5 ng/mL of SGP corresponds to 3.49 pM of sialic acid. Following incubation of the sugar chains for 2 hours, viruses comprising 32 HAU of A/California/07/2009 (H1N1) and 6 HAU of A/mallard/Hokkaido/24/2009 (H5N1), a non-pathogenic avian influenza virus, were fed into the wells. The viruses were adsorbed overnight at 4°C. Bound viruses were inactivated and fixed with 10% formalin in PBS buffer (3.7% formaldehyde) at room temperature for 30 min. Using rabbit anti-human influenza A and B polyclonal antibodies (Takara Bio) and Simple Stai MAX PO (Nichirei Bioscience), the addition of o-phenylenediamine (Wako Pure Chemical) produced a yellow color. The binding activity was measured by the absorbance at 490 nm using a Model 680 Microplate Reader (Bio-Rad).

AFM (AFM5010, Hitachi High-Tech Science, Tokyo, Japan) was used for the observation of sugar chains. For the SGP and sialoglycopolymer, the stock sample (1 mg/mL) was diluted to 10 ng/mL or 1 ng/mL with double-distilled water and then 1 μL of the samples was observed on a freshly cleaved mica surface in air using a self-sensing micro-cantilever PRC-DF40P instrument (Hitachi High-Tech Science) with resonance frequency of 477.2 kHz. For the observation of sugar chains of SGP-BSA in buffer solution, the stock sample of the sugar chains (1 mg/mL) was diluted to 1 μg/mL with double-distilled water, and 100 μL of the samples was spread onto a freshly cleaved mica surface in the dish. Micro-cantilevers of SI-DF3 (Hitachi High-Tech Science) with resonance frequency of 27 kHz and spring constant of 1.7 N/m were used.

III. RESULTS AND DISCUSSION

Figure 1(a) shows the binding activity of SGP to avian influenza virus of H5N1. SGP was directly attached onto the ELISA plate by the terminal amino groups. Avian type receptor of α(2-3)-linked sugar chains has a higher binding activity compared to human type receptor of α(2-6)-linked sugar chains. In this case, however, we observed large fluctuations in the binding activity. For example, the standard deviation of the binding activity at an SGP concentration of 3.49 pM was 0.115 with an average absorbance of 0.131. At times we observed no binding activity of α(2-3)-linked sugar chains to H5N1, and where control experiments confirmed the presence of active viruses. Furthermore, the relative errors as shown in Fig. 1(b) also show a wide distribution, where over 0.4 (40%) relative errors could be observed. The standard deviation of the relative errors is 0.170 for the binding activity of α(2-3)-linked sugar chains to H5N1, and 0.114 for that of α(2-6)-linked sugar chains.

An AFM image of SGP (10 ng/mL) spread onto mica is shown in Fig. 2(a). Several molecules are present with wide height distribution between 0.3 nm and 3.9 nm, as shown in Fig. 2 (b). This might result from molecular conformational differences of the SGP. SGP has multiple amino groups that can potentially bind to the substrate, and differences in the extent of binding could generate the conformational variety
observed. When the molecules adopt a lying conformation, which can be observed with heights of less than ca. 1 nm, the active sites of terminal sialic acid seems to be too close to the substrate and the binding could be shielded. The standing conformation of SGP, which can be observed with heights of ca. 3 nm, could be associated with greater binding activity. Figure 2(b) shows the spatial size distributions at ca. 15 nm and 20 nm. This might result from the conformational flexibility of long molecules of SGP. Thus, SGP shows large fluctuations, which could relate to the observed fluctuations in binding activity.

Our attention shall now turn to the clustered structures such as sialoglycopolymer and SGP-BSA. The binding activity of α(2-6)-linked sialoglycopolymer to influenza viruses is shown in Fig. 3(a). Only the human influenza viruses of H1N1 have high activity. The fluctuation in binding activity seems to be lower with small error bars. Figure 3(b) shows the relative errors of binding activity. The standard deviation of the relative errors is 0.071 for the binding activity of α(2-6)-linked sugar chains to H1N1, and 0.169 for H5N1. Therefore, the standard deviation of the binding activity between H5N1 and α(2-6)-linked sugar chains is reduced to 0.071 compared to SGP (0.114).

Figure 4(a) shows an AFM image of sialoglycopolymer (1 ng/mL) on mica. Uniform dispersion can be achieved. Furthermore, the small side chains of sialyllactosamine can easily adopt a standing conformation. As a result, most of the molecules have a height of ca. 0.6 nm as shown in Fig. 4(b). Partially clustered sugar chains might have higher binding activity, and there is the possibility that many free sugar chains would remain during the binding experiments.

Finally, we show the binding activity of SGP-BSA in Fig. 5(a). As the range in the number of sialic acids is the same in Fig. 3 (a), the binding activity could be enhanced in SGP-BSA. The fluctuations in binding activity seem to be reduced with small error bars. Figure 5(b) shows the relative errors of the binding activity. The standard deviation of the relative errors is 0.046 for the binding activity of α(2-6)-linked sugar chains to H1N1, and 0.084 for H5N1. Therefore, the standard deviation of the binding activity between H5N1 and α(2-6)-linked sugar chains is reduced to 0.046. Highly concentrated clusters could have higher binding activity with smaller fluctuations.
the scaffolding provided by the BSA molecules, the clustered sugar chains on BSA with fluctuation can work effectively [17]. Therefore, we achieved effective use of sugar chains and high binding activity with fewer free sugar chains. In general, when the density of viruses is low under actual working conditions, the high clustering and effective use of sugar chains seems to be important for the binding molecules. With this in mind, SGP-BSA appears to be a potential candidate for use in initial reactions of influenza virus sensors.

IV. CONCLUSION

We have discussed several clustered sugar chains that engage in binding to influenza viruses. α(2-6)-linked sugar chains have higher binding activity to human influenza viruses compared to α(2-3)-linked sugar chains. Single sugar chains of SGP display conformational variability due to the presence of multiple amino groups as binding sites. As a result, SGP shows large fluctuations in binding activity. On the other hand, clustered sugar chains such as sialoglycopolymer and SGP-BSA show small fluctuations in binding activity. The standard deviation of the relative errors could be reduced to 0.071 for sialoglycopolymer and 0.046 for SGP-BSA, while that for SGP was 0.114. Thus, we have succeeded in reducing the fluctuations in binding activity by clustering. For SGP-BSA, the high clustering using scaffold molecules could enhance the effective use of sugar chains to yield high binding activity with fewer free sugar chains.

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Statistical theory of mixed-valence selectivity in biological ion channels

W.A.T. Gibby∗, M.L. Barabash ∗, C. Guardiani∗, D.G. Luchinsky∗† and P.V.E. McClintock∗
∗Department of Physics, Lancaster University, Lancaster, UK, LA1 4YB
Email: w.gibby@lancaster.ac.uk
†SGT Inc., Ames Research Center, Moffett field, CA, 94035, USA
Email: d.luchinsky@lancaster.ac.uk

Abstract—In this article we apply our multi-component statistical and linear response theory to the process of mixed-valence ionic transport in a biological sodium ion channel. We analyse the free energy spectra, and statistical fluctuations (which are proportional to the conductivity) to investigate conditions and optimal parameters required for selectivity.

I. INTRODUCTION

Understanding, predicting, and controlling how ions permeate nanoscale pores is increasingly important in the fields of nano- and biophysics. Ionic transport governs a large and growing number of applications including, but not limited to, DNA sensing [1], water treatment [2] and natural gas purification [3]. In general the pores are designed to fulfill two key criteria: (i) their ability to select amongst ionic species; and (ii) ability to conduct ions at a high rate. Fortunately, biological ion channels which are abundant in nature, have been perfectly optimised by evolution to perform these tasks. Ion channels are holes through proteins, formed from thousands of atoms, and even minor mutations in structure can greatly influence their function and can be associated with disease [4]. Although it is clear that structure is a direct determinant of function, the mechanism through which this occurs has remained unclear and represents a major scientific challenge that we now address.

We will consider mixed sodium Na⁺ and calcium Ca²⁺ transport and selectivity through the selectivity filter of a typical biological Na⁺ channel with 2 binding sites. In doing so, we will apply our multi-species equilibrium statistical theory [5]–[8], calculating the linear response conductivity from the fluctuations, using generalised Einstein relations. The development of theory in this area has already led to several important theoretical insights. These include the creation of a continuous statistical theory and its relationship with molecular dynamics [9], the analysis of selectivity through density functional theory [10], [11], and multi-species transport in narrow charged nanopores [12]–[15].

II. THEORETICAL APPROACH

A. Defining the system

We consider two bulk reservoirs with aqueous solutions containing ionic species i ∈ 1, ⋅⋅⋅, S, diffusively and thermally coupled via a pore of volume Vc (see Fig. 1 taken from [7]). Under standard conditions the thermal de Broglie wavelength is smaller than system dimensions, and thus the system is classical and described by canonical statistics. The pore itself can be considered as a one-dimensional lattice with M binding sites, and individual site occupancy is characterised by the number n_im. We further assume that the steric properties of the site prohibit multiple ion occupancy and so n_im ∈ 0, 1 for all i ∈ 1, ⋅⋅⋅, S, m ∈ 1, ⋅⋅⋅, M sites. As a result the total occupancy which is given by the double sum \( \sum_i \sum_m n_{im} \) represents a state of the system. If the sites are assumed to be identical then each state \( \{n_j\} \) is defined by the total number of ions of each species in the pore and its total set is given by \( \{n_j\} \).

Within the bulk the electrochemical potential can be calculated using Widom’s insertion [18]. It contains contributions from the natural logarithm of the mole fraction \( x_i^b \), the electric potential \( \phi^b \) and the excess chemical potential \( \mu_i^b \). This latter term provides the non-polar (surface tension and excluded volume) and electrostatic (Born and Debye-Hückel) contributions.
to the solvation free energy [11], [19], [20]. Meanwhile we can establish two scales of interaction in the confined pore. It is known that pores repleted with charge (including total pore charge $Q_f = n_f q$ where $q$ is the proton charge) must reproduce strong electrostatic correlations $E$ [13], dependent on its geometry and total enclosed charge. However on the scale of the binding sites, the ions must also interact locally through short-range interactions $\mu_{ii}$ including solvation and bonding [21].

### B. Energy spectrum

The total Gibbs free energy of each state can be expressed by writing the total energy and separating the degrees of freedom from the bulk and pore phases. Thus it reads as,

$$G\{\{n_i\}; n_f\} = E\{\{n_i\}; n_f\} - \sum_i n_i \Delta \mu_i$$

$$+ \sum_i kT \ln (n_i) + kT \ln \left( M - \sum_i n_i \right)$$

where: $k$ and $T$ are the Boltzmann constant and system temperature respectively and $\Delta \mu_i$ is the summation of the difference in excess chemical potential between bulk and pore $\Delta \mu_i = \mu_i^b - \mu_i^p$ plus the natural logarithm of the mole fraction. The final term represents the entropy of mixing for both the ions $n_i$ and empty sites ($n_w = M - \sum n_i$). We note that these energetic contributions are in exact agreement with the density functionals that are introduced to investigate ionic transport between charged surfaces and the related double layer effect see e.g. [22].

Here, we shall consider the form ($\mathcal{E} = U_c(n_f + \sum_i z_i n_i)^2$), taken from [13], were $U_c$ represents the capacitative charging energy, which is dependent on pore geometry and it takes the value of $\sim 6kT$ in Na$^+$ channels. It is of interest to note that this interaction is identical to that used in electron transport through quantum dots [23].

In (A) and (B) of Fig. 2 we display the free energy profiles for the full configuration of states. We consider $\Delta \mu_{Ca} = 5kT$ and $\Delta \mu_{Na} = 4kT$ and $-20kT$ for (A) and (B) respectively. Solid blue and red lines represent pure Ca$^{++}$ and Na$^+$ states, whereas the mixed species states are presented in green. Each curve is parabolic in nature due to its quadratic dependence on $n_f$, and minimises when the total charge is neutralised i.e. equal to zero. Due to the difference in valence both $\mathcal{E}$ and $\Delta \mu_i$ govern the selectivity because these parameters translate the curves vertically and horizontally. To observe this let us consider the differences between (A) and (B). When $\Delta \mu_{Na}$ is positive and comparable to $\Delta \mu_{Ca}$, the Ca$^{++}$ levels are vertical translated when $n_f$ is small due to the large electrostatic barrier; and hence unfavoured. Consequently the pure Na$^+$ crossings occur at a lower energy (highlighted for the 0-1 transition by the purple circle at $n_f \sim -0.1$). As this parameter is made strongly negative it imposes a blocking barrier for Na$^+$ which ensures that the pure Ca$^{++}$ crossings now occur at a lower energy and shifted value of $n_f$.

### C. Statistical fluctuations and conductivity

The statistical fluctuations can be computed from the grand potential ($\Omega = -kT \ln Z$) which reads as follows,

$$\Omega = -kT \ln \left( \sum_{\{n_i\}} \frac{1}{n_{i(1)}! \prod_i 1/n_i!} \exp \left( \sum_i n_i \tilde{\mu}_i - \mathcal{E}(\{n_i\}) \right) \right)$$

(2)

which includes the average number of ions in the pore of species $i$ at constant system temperature and volume $V$,

$$\langle n_i \rangle \equiv -\frac{\partial \Omega}{\partial \tilde{\mu}_i^{(T,V)}}$$

(3)

together with the variance and covariance [20], [24],

$$\langle (\Delta n_i)^2 \rangle = \frac{\partial^2 \Omega}{\partial \tilde{\mu}_i^2}, \quad \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle = \frac{\partial \langle n_i \rangle}{\partial \tilde{\mu}_j^{(T,V)}}$$

(4)

respectively. Here, $\eta^c$ is the chemical potential in the pore which, at equilibrium, is equal to its bulk counterpart.

In multicomponent mixtures, non-equilibrium thermodynamics states that the Fickian diffusivity $D_{ij}$ is related to the flux density $j_i$ [15] via,

$$j_i = -\sum_{j \neq i} \frac{S}{1-q} \left( D_{ij} \nabla c_i + \sum_{j \neq i} D_{ij} \nabla c_j \right) - \sum_{j=1}^{S} \sigma_j \nabla \phi$$

(5)

Here, $c$ is the concentration in the pore ($c_i = \langle n_i \rangle / V$). Cross diffusivities $D_{ij}$ represent the ability for one species type to conduct in the presence of the concentration gradient for the other ion, and are equal to zero in independent solutions. We have shown in earlier work [5], [6], and it follows closely from [25], that the conductivity $\sigma$ can be calculated under linear response conditions. If we assume that the kinetic properties of the ions are comparable such that $D_{ii} \sim D_{jj} \sim D_{ij}$, then the total conductivity through the pore at linear response...
Fig. 3. Occupancy staircase for Na\(^+\) (blue) and Ca\(^{++}\) (grey) vs. \(n_f\) and \(\Delta\tilde{\mu}_{Na}\) with \(\Delta\tilde{\mu}_{Ca} = 5kT\). Plateaus correspond to stability with the pore being locked into one state, meanwhile steps correspond to energies at which the pore can reside in multiple states.

\[
\sigma = \sum_{i=1}^{S} \sigma_i, \text{ is proportional to the variance in total particle number i.e.}
\]

\[
\sigma \propto \left( \sum_i S \frac{\partial \langle n_i \rangle}{\partial \eta_i} + \sum_{j \neq i} S \frac{\partial \langle n_j \rangle}{\partial \eta_i} \right).
\]

(6)

III. Analysis

To investigate the selective properties of the pore we should also analyse both the mean number of ions (occupancy) of Na\(^+\) (blue) and Ca\(^{++}\) (grey) in Fig. 3, and the variance of total particle number in Fig. 4 (which is proportional to the conductivity Eqn. (6)), vs. \(\Delta\tilde{\mu}_{Na}\) and \(n_f\), with \(\Delta\tilde{\mu}_{Ca} = 5kT\).

Occupancy forms a staircase, where plateaus correspond to integer values of ions in the pore. As \(n_f\) becomes more negative the total number of ions residing in the pore increases because of the increased electrostatic attraction to the pore. As we have already seen the selectivity is defined by both \(E\) and \(\Delta\tilde{\mu}\). To observe this let us consider \(n_f = -2\) because the electrostatic interaction can then be neutralised with either 1 Ca\(^{++}\) ion or 2 Na\(^+\) ions. Consequently when \(\Delta\tilde{\mu}_{Na} \lesssim \Delta\tilde{\mu}_{Ca}/2\) the pore favours Ca\(^{++}\). Non-selectivity occurs when each of the species states are energetically indistinguishable and so the vertical steps for each species converge. An example of this occurs when \(\Delta\tilde{\mu}_{Na} \sim -4kT\) and \(n_f \sim 0.8\).

As shown in Fig. 4, the variance in particle number forms a complicated structure with two sets of peaks (\(T_1\) and \(T_2\)) corresponding to 0-1 and 1-2 ion transitions, that are separated by a region of negligible conductivity. Peaks form at the midpoints of the vertical occupancy steps where there is a change in total number of ions in the pore, i.e. a step in the total occupancy. This result is intuitively clear, because these points correspond to degeneracies in the system, when adding or removing an ion does not cost any energy. Away from these points there is a corresponding energy barrier impeding conduction. As a result we can establish two conditions on that must be satisfied for optimal conductivity: (i) the pore must be degenerate i.e. \(\Delta G \sim 0\) and (ii) both states must correspond to the favoured occupancy’s of the pore i.e. lowest free energy (see purple circles in Fig. 2). Since this occurs between neighbouring states it also corresponds directly to knock-on conduction. Collectively these phenomena are known as ionic Coulomb blockade a classical analogy [26]–[28] of the process observed in electron transport [23].

The structure of the peaks differs because \(T_2\) allows for conduction involving mixed states. As a result, it can produce Na\(^+\) conduction between the pure (1Na\(^+\) and 2Na\(^+\)) and mixed (1Ca\(^{++}\) and 1Ca\(^{++}\)+1Na\(^+\)) states and pure Ca\(^{++}\) conduction. Mixed state conduction between these states has been qualitatively observed in simulations of Na\(^+\) channels.

In [29], the system was initiated with both Na\(^+\) and Ca\(^{++}\) present in the pore, and Na\(^+\) was shown to bypass the resident Ca\(^{++}\) ion, in a similar manner to [30]. Finally we must note that the branch parallel to \(n_f\) appears to suggest an anomalous conduction event. This branch occurs when \(-1.4 \lesssim n_f \lesssim -2.1\) and \(\Delta\tilde{\mu}_{Na} \sim -2.8kT\), and corresponds to the degeneracy between the 1Ca\(^{++}\) and 2Na\(^+\) states. As a result conduction does not occur via the knock-on mechanism. It’s importance and implications will be further explored in future work.

These properties can be combined to generate a phase diagram, further helping to elucidate the picture, Fig. 5. The coloured blocks correspond to the occupancy and are labelled, meanwhile the purple regions denote instability because the pore can reside in multiple states of different numbers of particles. As a result there is a quasi-phase-boundary separating different locations of non-zero conductivity. The blue and orange lines correspond to the midpoints of the Na\(^+\) and Ca\(^{++}\) occupancy step, and the black line corresponds to the maximal conductivity. It is clear that the pore switches from Na\(^+\) to Ca\(^{++}\) conduction when \(\Delta\tilde{\mu}_{Na} \lesssim -6kT\), and that mixed state conduction occurs when the mixed state is the
Fig. 5. Phase diagram of the pore. Coloured blocks refer to its occupancy, with purple denoting locations where it is made from a superposition of $n^−$ and $n^+$, and blue and orange lines correspond to the midpoint of the Na$^+$ and Ca$^{2+}$ occupancy steps, and the black lines display the maximal conductivity.

favoured 2 ion state which occurs when $-2.1 \lesssim n_f \lesssim -2.7$.

IV. Conclusion

In conclusion, we have investigated the mixed-valence transport properties of narrow ion channels, through analysis of a multi-component statistical and linear response theory. In particular we have focused on Na$^+$ vs. Ca$^{2+}$ selectivity in a typical biological Na$^+$ channel. Strong selectivity is observed in both the variance in total particle number and the mean number of ions in the pore. This is determined by both the electrostatic interaction because the ions have a differing valence, and also the local interactions. We can identify two conditions required for optimal selective transport: (i) the pore must be degenerate i.e. $\Delta G \sim 0$ and (ii) both states must correspond to the energetically favoured occupancy of the pore i.e. lowest free energy. Therefore, it is possible to predict the pore changes required to shift the selectivity and/or the conduction mechanism.

In future we will further analyse the far-from-equilibrium regime through the development of a master equation model. Finally we expect that the proposed formalism should also be applicable to other artificial nano-pores or biological channels.

References

Interplay between channel and shot noise in subthreshold voltage fluctuations of neural membranes

B. G. Vasallo, J. Mateos and T. González
Department of Applied Physics, University of Salamanca, Plaza de la Merced s/n, 37008 Salamanca, Spain
e-mail: bgvasallo@usal.es

Abstract—Subthreshold voltage fluctuations of neural membranes are studied by means of a stochastic model based on the Monte Carlo technique. Gating channels for sodium and potassium cations and leakage channels are considered following the Hodgkin-Huxley equations. Ion channel noise is included by means of Langevin sources and ion shot noise is considered by using the Gillespie’s method in terms of the probabilities for different ions to cross the cell membrane. Both noise sources are found to play a role around spiking threshold conditions.

Keywords—Stochastic methods; neural membranes; ion channel noise; ion shot noise

I. INTRODUCTION

In order to contribute to the development of bioinspired electronic devices mimicking the brain functionality and further assist the knowledge of the human brain [1-5], the study of the noise behavior in biological membranes is compulsory [6,7]. The time evolution of electrical quantities in neural membranes, essential to explain the physical properties of neurons and axons [8,9], is typically modelled by means of the phenomenological Hodgkin-Huxley (HH) model [10]. This model accounts for the equilibrium conditions and the so-called action potential, which is a spike appearing in the voltage $V_m$ between the inside and the outside regions of cell membrane activated by sufficient external current $I_{appr}$.

As nonlinear systems that evolve from a threshold, the analysis of their subthreshold regime, and, in particular, the role played by noise, is extremely interesting [11,12]. This is the main goal of this work. In our approach, ion channels are taken into account as a whole by means of a continuous model [13-16]. The presence of ion channel noise, i.e., the noise related to the random opening and closing of the ion gates through the membrane, is taken into account by means of the Langevin generalization for the HH equations [13]. Ion shot noise, i.e., the noise associated to the random passage of ions across the cell membrane, is included by using the Gillespie’s method, from the probabilities for ions to cross the membrane [14-16]. The main purpose of this work is to analyze the interplay between channel and shot noise under excitation conditions around the threshold for the onset of spiking.

The noise of the system is studied in terms of the spectral density of the membrane voltage fluctuations $S_{V_m}(f)$. The subthreshold voltage behavior is analyzed for large membrane patches, when channel and shot noise are insufficient for the onset of spikes in the absence of external excitation. The limit of subthreshold conditions will be reached by applying increasing values of $I_{appr}$. Even if ion shot noise has been typically considered as negligible as compared to other electrical sources of neural noise [6,13-17], according to our results the signature of shot noise becomes visible in the limits of subthreshold conditions by virtue of the interplay with channel noise.

II. PHYSICAL MODEL

The neuron membrane can be considered as an insulator separating the intracellular and the extracellular spaces, which are electrolytes mainly containing sodium and potassium cations, Na$^+$ and K$^+$, and chloride anions, Cl$^-$. Ion leakage and voltage-gated channels connect the inside and the outside regions through the membrane. According to the HH model [10], the electrical evolution of the cell membrane is described in time-domain in terms of the membrane voltage $V_m(t)$, linked to the amount of charge at each side of it, and therefore to the ion currents crossing the membrane and the so-called gating variables, $m(t), h(t)$ and $n(t)$, which determine the opening or the closing of potassium and sodium channels. In our model, the equations are solved in time domain by means of the standard Euler algorithm with a time step $\Delta t \approx 2 \mu s$ [13-16]. At every $\Delta t$, a new value of $V_m$ is evaluated as

$$C_m \frac{dV_m(t)}{dt} = I_{appr} - \sum_{ion} I_{ion}$$

where $C_m$ is the membrane capacitance per unit surface and $I_{appr}$ is an external current density (here considered as noiseless), which can initiate a voltage spike. $I_{ion}$, with the subindex ion = Na, K or leak, is the current density due to sodium cations, potassium cations or ion leakage, respectively.

In the HH model, the ion currents $I_{ion}^{HH}$ are described in terms of channel conductances, which in the case of Na and K depend on the membrane voltage $g_{ion}(V_m)$ (gated channels). $g_{Na}$ depends on the dimensionless gating parameters $m(t)$ and $h(t)$, which are interpreted, respectively, as the fraction of activation or inactivation molecules in the open state; $g_K$ depends on the dimensionless gating parameter $n(t)$, interpreted as the proportion of activation molecules in the open state [10].

In our model, ion channel noise is introduced by means of Langevin sources in the equations governing the time evolution of the gating parameters $\gamma(t)$, with $\gamma(t)$ = $m(t), h(t)$ or $n(t)$. In the simulation, $\gamma(t)$ is updated every time step $\Delta t$ following the HH model as [13]
\[ \dot{y}(t) = \alpha_{y}(V_{m}) (1 - y) - \beta_{y}(V_{m}) y + \xi_{y}(t) , \]  

where \( \alpha_{y} \) and \( \beta_{y} \) are the voltage dependent transition rates [10], and \( \xi_{y}(t) \) is the source of channel noise. Eq. (2) is the Langevin generalization of the deterministic HH equations taking into account independent Gaussian white-noise sources of vanishing mean, with autocorrelation function [13]

\[ \langle \xi_{y}(t)\xi_{y}(t') \rangle = \frac{2}{\rho_{ion}} \alpha_{y}\beta_{y} \delta(t - t') , \]

where \( \gamma = m, h \) when \( \text{ion} = \text{Na} \) and \( \gamma = n \) when \( \text{ion} = \text{K} \). \( \rho_{\text{ion}} \) represents the ion channel densities, assumed as homogeneous, and \( S \) the area of the membrane patch.

Ion shot noise is included by means of a stochastic model in which the Monte Carlo technique is employed for the determination of the time of passage of ions through the membrane following the HH equations [14-16]. The probabilities for the different ions to cross the cell membrane are considered to be independent of each other, and Gillespie method [21,22] is used to account for their stochastic transmembrane kinetics. The probability per unit time that a given ion crosses the membrane is calculated as [16]

\[ P_{\text{ion}} = \frac{|I_{\text{ion}}|^2}{e} , \]

where \( e \) is the elementary charge. Assuming the crossing of ions through the membrane as a memoryless process, the time between crossing events is calculated following Poissonian statistics. The particular type of ion crossing the membrane is randomly determined according to the respective probabilities. The values of \( I_{\text{ion}} \), calculated from the number of ions actually crossing the membrane during \( \Delta t \), are then used in Eq. (1) to calculate the total ion current density \( \sum_{\text{ion}} I_{\text{ion}} \). Thus, \( I_{\text{ion}} \) and \( V_{m} \) contain the influence of the fluctuations associated to the random passage of ions through the cell membrane. All the details of this model and the values of the involved parameters can be found in [16]. When ion shot noise is not included in the calculations, \( I_{\text{ion}} \) are evaluated as \( I_{\text{ion}}^{\text{shot}} \).

The noise behavior of the system is studied in terms of the spectral density of the membrane voltage fluctuations \( S_{\text{Vm}}(f) \), calculated from the Fourier transform of a \( V_{m} \) sequence of 300 s.

In order to illustrate the influence of the different noise sources on the \( V_{m} \) fluctuations with respect to the deterministic case (in the absence of any noise source), \( S_{\text{Vm}}(f) \) for an external excitation \( I_{\text{app}}=0.08 \text{ Am}^{-2} \) is presented in Fig. 1 in the deterministic case and when channel noise or shot noise is considered (separately) in the simulation. This value of \( I_{\text{app}} \) leads to the appearance of spike trains in \( V_{m} \) and, as a consequence, \( S_{\text{Vm}}(f) \) presents peaks at a characteristic spiking frequency around 60 Hz [13,15] and its harmonics. In the deterministic case, the \( \delta \)-like peaks evidence the coherence inherent to a regular spiking. Under this excitation condition, shot noise very slightly modifies \( S_{\text{Vm}}(f) \) with respect to the deterministic case and the coherence is essentially preserved. In contrast, the presence of channel noise, much more intense, noticeably deteriorates the coherence, so that peaks in \( S_{\text{Vm}}(f) \) are less pronounced and cover a wider frequency range around the characteristic frequency and its harmonics.

### III. Results

Firstly, \( S_{\text{Vm}}(f) \) is analyzed for a cell membrane of \( S=1000 \text{ \mu m}^2 \), for which channel and shot noise are insufficient for the onset of spikes in the absence of external excitation. Fig. 2 presents \( S_{\text{Vm}}(f) \) when (a) ion channel noise and (b) ion shot noise are considered separately, for \( I_{\text{app}} \) ranging from 0 to 0.08 \text{ Am}^{-2}, covering the subthreshold regime and the onset of oscillations. For \( I_{\text{app}}=0 \text{ Am}^{-2} \) and 0.04 \text{ Am}^{-2}, channel noise [Fig. 2(a)] is not strong enough for the onset of voltage spikes, as shown in Fig. 3, in which the mean value of \( V_{m} \) and the number of oscillations is calculated.
of spikes occurring in a simulation time of 300 s in the presence and the absence of channel noise is plotted as a function of \( I_{\text{app}} \). For 0.04 Am\(^{-2}\), \( S_{\text{rad}}(f) \) shows a peak around 80-90 Hz, evidencing a rhythm in the \( V_m(f) \) fluctuations, related to the internal dynamics of the system [23]. This frequency behavior of the \( V_m \) fluctuations originated by channel noise has been also reported in [11,12]. When \( I_{\text{app}}=0.05-0.06 \) Am\(^{-2}\), still insufficient for the onset of spike trains but close to their appearance, \( S_{\text{rad}}(f) \) exhibits a remarkable increase [23], typical at the onset of instabilities, as observed in other physical systems like electronic devices [24]. For increasing values of \( I_{\text{app}} \) the mean value of the membrane voltage increases [12], taking similar values to those found in the deterministic case [Fig. 3]. The onset of voltage spikes takes place for lower values of \( I_{\text{app}} \) when channel noise is considered, since the noise assists their appearance below threshold (~0.065 Am\(^{-2}\) in the deterministic case). In contrast, above threshold conditions the noise suppresses a significant amount of spikes with respect to the deterministic case. For \( I_{\text{app}}>0.07 \) Am\(^{-2}\), when a full train of voltage spikes is achieved, \( S_{\text{rad}}(f) \) presents peaks around the characteristic spiking frequency and its harmonics.

In the case of considering only ion shot noise [Fig. 2(b)], \( S_{\text{rad}}(f) \) exhibits a maximum at around 80-90 Hz for the lower values of \( I_{\text{app}} \), which reveals the existence of the mentioned intrinsic rhythmic oscillations in \( V_m \) even for large values of \( S \), for which shot noise is extremely weak [15]. Indeed, the values of \( S_{\text{rad}}(f) \) are around four orders of magnitude lower than when channel noise is considered. For \( I_{\text{app}}=0.08 \) Am\(^{-2}\), once the spike train is triggered, \( S_{\text{rad}}(f) \) is very similar to that found in the deterministic case [Fig. 1].

To illustrate the interplay between channel and shot noise in subthreshold voltage fluctuations of neural membranes, Fig. 4 shows the frequency behavior of \( S_{\text{rad}}(f) \) when a cell membrane of \( S=1000 \) \( \mu \text{m}^2 \) is excited with different values of \( I_{\text{app}} \), both when channel noise is the only noise source considered in the system and in the case in which also shot noise is taken into account. In the absence of excitation, both cases coincide, thus evidencing a negligible influence of shot noise. For \( I_{\text{app}}=0.07 \) A/m\(^2\), high enough for generation of spikes, the peaks in \( S_{\text{rad}}(f) \) denote their presence and both cases again essentially overlap. However, when \( I_{\text{app}}=0.04 \) A/m\(^2\), still insufficient for the onset of spikes but close to their appearance, the spectral density exhibits a significant increase when ion shot noise is included in the simulation with respect to case in which channel noise is the only source considered. Remarkably, in these subthreshold conditions, shot noise plays a role in the noise behavior by the interplay with channel noise.

A similar conclusion can be achieved when, instead of applying an external current in large membrane patches, the emergence of the noise influence is achieved, in the absence of excitation, by reducing the membrane area \( S \). Fig. 5 presents the frequency behavior of \( S_{\text{rad}}(f) \) when a cell membrane is reduced from 1000 \( \mu \text{m}^2 \) to 80 \( \mu \text{m}^2 \), when channel noise is the only noise source considered in the system and in the case in which also shot noise is taken into account. In the limit of the onset of spiking due to channel noise, for around 120 \( \mu \text{m}^2 \), shot noise also displays a visible influence.

### IV. CONCLUSIONS

Subthreshold voltage fluctuations of neural membranes modeled by means of the HH model have been studied in terms of \( S_{\text{rad}}(f) \) when considering ion channel noise and ion shot noise sources in the system. In the model, ion channel noise is included by means of Langevin sources and ion shot noise is considered in terms of the probabilities for different ions to cross the cell.
membrane. The threshold for spiking activity has been reached by applying increasing values of $I_{\text{app}}$ in a large membrane patch. Around such threshold conditions, the signature of shot noise is significantly visible when combined with channel noise, while its influence is otherwise negligible as typically predicted. A similar conclusion can be achieved if the limit in the subthreshold regime is obtained by reducing the membrane patch in the absence of external excitation.

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**I/f DNA Hydrogen-Bond Energy Noise**

Nazarii Boichuk  
Bioelectronics (ICS-8),  
Forschungszentrum Jülich,  
52425 Jülich, Germany  
n.boichuk@fz-juelich.de

Yuiri Kutovyi  
Bioelectronics (ICS-8),  
Forschungszentrum Jülich,  
52425 Jülich, Germany  
y.kutovyi@fz-juelich.de

Nicolas Lobato-Dauzier  
LIMMS/CNRS-IIS,  
The University of Tokyo,  
Tokyo 153–8505, Japan  
lobato@iis.u-tokyo.ac.jp

Anthony Genot  
(LIMMS/CNRS-IIS),  
The University of Tokyo,  
Tokyo 153–8505, Japan  
genot@iis.u-tokyo.ac.jp

Teruo Fujii  
Institute of Industrial Science (IIS),  
The University of Tokyo,  
Tokyo 153–8505, Japan  
tfujii@iis.u-tokyo.ac.jp

Nicolas Clément  
LIMMS/CNRS-IIS,  
The University of Tokyo,  
Tokyo 153–8505, Japan  
nclément@iis.u-tokyo.ac.jp

Svetlana Vitusevich  
Bioelectronics (ICS-8),  
Forschungszentrum Jülich,  
52425 Jülich, Germany  
s.vitusevich@fz-juelich.de

Andreas Offenhäusser  
Bioelectronics (ICS-8),  
Forschungszentrum Jülich,  
52425 Jülich, Germany  
a.offenhaeusser@fz-juelich.de

**Abstract**—The power spectrum of the hydrogen-bond energy fluctuation of DNA was investigated by molecular dynamics simulation. A 1/f frequency response is found, even for simple configurations of DNA composed only of thymine bases and with the presence of an equal number of complementary strands. This suggests that various relaxation processes are involved as a general trend. The noise amplitude strongly depends on the temperature. These results indicate that experimental measurements of the hydrogen-bond energy noise may provide additional degrees of freedom for a wide range of studies from DNA nanotechnologies to biosensors.

**Keywords**—DNA, hydrogen-bond energy, 1/f noise, biosensors, DNA nanotechnology

I. INTRODUCTION

Hydrogen-bond (hb) interactions are ubiquitous in nature, and their dynamics is expected to play a key role in chemical reactions and biomolecular recognition [1, 2]. Recent experimental setups allow the direct observation of single-molecule hydrogen-bond dynamics with single-molecule resolution for solvent/surface interaction open up new opportunities for the study of hb dynamics [2]. In particular, it has recently been shown that liquid water yields 1/f noise in hb energy, whereas simple liquids such as liquid argon exhibit a near white spectrum [3]. This observation was related to network rearrangement dynamics. In DNA systems, hb interactions are typically studied from temperature ramps in order to assess a melting temperature $T_m$ and are measured by fluorescence. The direct investigation of hb dynamics is extremely difficult at the molecular level as the noise in these experimental systems is typically too high to directly observe hb energy fluctuations, but the application of fluorescence correlation microscopy is a very promising approach for such studies [4]. Recent progress in molecular dynamics (MD) simulations means that experimental results are satisfactorily reproduced, in particular for DNA [5, 6]. Nonetheless, hydrogen-bonding dynamics was not revealed in these studies. By means of MD simulations, we investigate the dynamics of hb energy, and we explore the potential for using DNA hb fluctuations as an additional signal for studies in DNA nanotechnology and biosensors. Used methods, including MD, are described in Sec. II. The simulation results and the discussion are presented in Sec. III and conclusions are put forward in Sec. IV.

II. SIMULATION METHOD

Our work was carried out by means of an oxDNA code where the coarse-grained DNA model was introduced [7]. For simplicity, we examined an archetype system for DNA with a nucleic acid sequence of only 10 thymine bases ($T_{10}$) and with the presence of an equal number of complementary strands ($A_{10}$) (Fig. 1). 1, 10, and 100 double-stranded DNAs (dsDNA) were randomly confined in cubic boxes whose size corresponded to 133 μM dsDNA. The selected concentration is rather high when compared to typical experimental conditions, but the aim was to preserve the computational time for hb energy dynamics monitoring over a sufficiently long period to study low-frequency noise. A periodic boundary condition was considered to keep the concentration constant. Additionally, the system was studied at salt concentrations equivalent to 0.1 M of NaCl, which is typical experimentally, and which corresponds to a Debye screening length of approximately 1 nm.

Fig. 1. Representation of the investigated dsDNA. Red and white spheres show backbone repulsion sites, and cyan ellipsoids indicate bases.

The computation was performed in two steps. During the first step, dsDNAs were inserted into the box at a random position and the molecules were relaxed by Monte Carlo simulation. The trajectory and hb energy calculations were subsequently computed for the system using molecular dynamics simulation with a Langevin thermostat for at least $10^5$ steps. Note that hb energy, $E_{hb}$, corresponds to the sum of all hb interaction energies.
The power spectral density of hb energy was estimated as
\[
S_{Ehb}(f) = \frac{1}{\Delta t} \left| \int_{t_{\text{min}}}^{t_{\text{max}}} E_{\text{hb}}(t) e^{2\pi if t} dt \right|^2
\]
(1)

where \(E_{\text{hb}}(t)\) is the total hydrogen bond energy of the system at time \(t\). To obtain quality curves, we introduced an averaging procedure for each spectrum.

In order to investigate the effect of temperature variations, we calculated the noise spectra for different temperatures from 10 °C to 80 °C.

III. SIMULATION RESULTS AND DISCUSSION

We first evaluated an hb energy averaged over time, \(<E_{\text{hb}}(t)\>\), as a function of temperature to confirm that the system is in equilibrium. Typical time dependences of \(E_{\text{hb}}(t)\) calculated for 1, 10 and 100 molecules are shown in Fig. 2(a-c) when \(T = 10 °C\). As can be seen, the value of \(E_{\text{hb}}\) fluctuates around \(E_{\text{hb}}\). The dependences of \(<E_{\text{hb}}(t)\>\) on temperature for 1, 10 and 100 molecules are shown in Fig. 2(d-f). As shown in the figure, \(<E_{\text{hb}}(t)\>\) displays a sigmoid shape. Such behavior of \(<E_{\text{hb}}(t)\>\) can be obtained analytically by considering a two-state approximation for the system. The process of denaturation as a forward bimolecular reaction and the reverse reaction of hybridization is given by

\[
AB \leftrightarrow A + B
\]
(2)

where AB denotes the ordered state of ten-base dsDNA, and A and B stand for single strands, A10 and T10, in the random coil state. Note that no intermediate states are considered in this case. The equilibrium constant for the reaction is given by the law of mass action (square brackets symbolize the concentration):

\[
K = [A][B]/[AB]
\]
(3)

Therefore, by considering the concentration of A10 strands to be equal to the concentration of T10 strands ([A] = [B]) and taking into account the conservation of mass \([A] = [AB] + [A_{\text{eq}}]\), with \([A_{\text{eq}}]\) the initial concentration of dsDNAs, the equation for the fraction of paired strands, \(x = [AB]/[A_{\text{eq}}]\), can be derived from (3):

\[
x = 1 + \frac{K}{2[A_{\text{eq}}]} - \sqrt{1 + \frac{K}{2[A_{\text{eq}}]}} = 1
\]
(4)

It should be noted that only the 0 \(\leq x \leq 1\) root of the solution was used to satisfy the physical meaning of the fraction of paired strands.

Importantly, there is a relationship between the Gibb's free energy change, \(\Delta G\), and the equilibrium constant at temperature, \(T\), \(K = \exp(-\Delta G/RT)\), where \(R\) is the gas constant and \(C\) is a matching coefficient. The results of fitting with the relation \(E_{\text{hb}}(T) = E_{\text{max}} \cdot x(T)\) are shown by dashed lines in Fig. 2(d-f). From the calculated fitting, the free energy change is in the order of 400-550 kJ/mol (Table I.). \(E_{\text{max}}\) represents the total energy of the fully hybridized dsDNA. Considering a typical enthalpy for the A-T bond around \(\Delta H = 0.287\ eV\) per bond as a value for comparison [8], \(E_{\text{max}}\) extracted from fittings using the sigmoid function acquires the values that are of the same order as \(\Delta H\) for 1 (2.87 eV), 10 (28.7 eV) and 100 dsDNAs (287 eV), which proves the correctness of the simulations.

The melting temperature, \(T_{\text{m}}\), of the duplex formation is defined as the temperature at which half of the available strands are in the double-stranded state and half of the strands are in the random coil state. From the curves in Fig. 2(d-f), we can extract the melting temperature by solving (4) when \(x(T_{\text{m}}) = 1/2\). The melting temperature calculated for different numbers of molecules is in the range of 51.5-53 °C, which is as expected for relatively large dsDNA concentrations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1 dsDNA</th>
<th>10 dsDNAs</th>
<th>100 dsDNAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{\text{max}}), eV</td>
<td>1.63</td>
<td>16.22</td>
<td>163.14</td>
</tr>
<tr>
<td>(T_{\text{m}}), °C</td>
<td>52.7</td>
<td>51.7</td>
<td>52.8</td>
</tr>
<tr>
<td>(\Delta G), kJ/mol</td>
<td>470</td>
<td>398</td>
<td>550</td>
</tr>
</tbody>
</table>

The power spectrum of the hb energy fluctuation \(S_{\text{hb}}(f)\) for 100 dsDNAs is shown in Fig. 3a. We find a 1/f \(^\gamma\) frequency response. Moreover, the amplitude of \(S_{\text{hb}}\) strongly depends on the temperature (Fig. 3b). More quantitatively, a maximum is observed at \(T = T_{\text{m}}\). The hb energy dependence (simulated data from Fig. 2) is also depicted for comparison. Thus, the following power proportionality between noise at 3 MHz and the energy level can be obtained: \(S_{\text{hb}}(3\ \text{MHz}) \propto E_{\text{hb}}\). The power density spectra of 1, 10 and 100 dsDNAs molecules are shown in Fig. 4a. It should be emphasized that...
Fig. 3. (a) The power spectra of hb energy in the 100 dsDNAs system. The system was investigated at different temperatures. Averaging was implemented for 100 spectral curves for each temperature. A dashed line indicates $1/f$ behavior. The line with an arrow at the end shows the temperature increase. (b) Temperature dependence of noise level at different frequencies for 100 dsDNAs system compared with hb energy. Data was taken from Fig. 3a at 300 kHz, 1 MHz, and 3 MHz.

Fig. 4. Power spectra density for different numbers of molecules calculated at 10 °C (a) and 55 °C (c). Spectra normalized per number of molecules are shown in (b) and (d).

similar $1/f$ behavior in the frequency range 0.1 MHz - 33 MHz was observed for different numbers of molecules. More precisely, at low temperature, when all molecules are hybridized, the noise level is directly proportional to the number of molecules (Fig. 4b). This is also the case when $T = T_m$ (Fig. 4c-d), with a slight deviation observed when the number of molecules is small.

These results suggest that $\pi-\pi$ intermolecular interactions [9] between neighboring bases at the single-molecule level may play a greater role than the statistical hybridization fluctuations relative to the number of molecules.

Another interesting feature is that the slope ($\gamma$) of the flicker noise ($1/f^\gamma$) shows a strong dependence on temperature, as depicted in Fig. 5. In a temperature range from 10 °C to $T_m \approx 52 °C$, parameter $\gamma$ yields a linear dependence on temperature with a slope of 0.008 °C$^{-1}$. However, when the system reaches melting point, the slope of gamma becomes almost three times as high as at the lower temperatures. This leads to the suggestion that $T_m$ is a crossover stage for noise behavior, which requires further investigation.

Fig. 5. Dependence of gamma parameter of flicker noise for the 10 dsDNAs system on temperature. Cross-over point corresponds to the melting temperature.

IV. CONCLUSIONS

We analyzed the power spectra of the hydrogen bond energy fluctuation in DNA by coarse-grained molecular dynamics and found near $1/f$ noise behavior. We suggest that these results may be related to multiple relaxation processes induced by the fluctuation of $\pi-\pi$ molecular interactions as well as possible partial hybridization and a statistical distribution of these relaxation processes in the case of multiple molecules. The temperature dependence of the noise also demonstrates that noise could be a powerful approach providing supplementary information in addition to the ensemble and average measurements depicted by the usual sigmoid response. For example, it appears that the melting temperature is a crossover for the $1/f^\gamma$ noise slope. We suggest that further investigation of DNA sequence, length, concentration and ionic strength may provide useful information for future experiments. An analytical model for hb noise is under development.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the Seed Money funds for supporting the establishment of a new international collaboration as part of the RTS Biosensor project (January 15, 2018 – March 31, 2019). Y. Kutovyi greatly appreciates the research grant from the German Academic Exchange Service (DAAD). N. Clément acknowledges the interdisciplinary research funds from CNRS for the NANOCAP and BIOSTAT projects.
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The dynamics of quasiparticles in a toy model of the KcsA biological ion channel


*Department of Physics
Lancaster University, Lancaster, UK, LA1 4YB
Email: m.barabash@lancaster.ac.uk
†SGT Inc., Ames Research Center, Moffet Field, CA, 94035, USA
‡School of Engineering University of Warwick, Coventry, CV4 7AL, UK

Abstract—We study the highly-concerted motion of ions in a narrow biological ion channel (KcsA) by considering the notion of a quasiparticle, with specific focus on the transition process. Namely, we show that the ion entering or exiting the channel is correlated with the position of the quasiparticle. This result is of importance in the rate theories of ion conduction through narrow channels and artificial nanopores.

Index Terms—Ion channel; highly-correlated ionic motion; quasiparticle; effective potential; transition process; Brownian dynamics; statistical physics

I. INTRODUCTION

The healthy evolution of a biological cell severely depends on the functioning of ion channels – proteins allowing for the passive transport of ions across the otherwise impermeable cellular membrane [1]. Due to its pharmacological importance, this topic has attracted the enormous attention both theoretically and experimentally. The underlying question – how to connect the channel’s function to its structure, – still uncovers new unstudied facets and, consequently, provides challenges and reason for further investigation.

A major challenge is the theoretical description of conductivity and selectivity of a given channel. The permeation of ions through the channel’s pore can be described by a number of methods, depending on the required level of details. Molecular dynamics, arguably the most realistic approach, considers all atoms and ions separately, but due to the pairwise nature of interactions a computation requires a lengthy simulation run. Brownian dynamics avoids this caveat at the cost of implicit inclusion of ion-water collisions via the noise terms. Continuous methods, e.g. the classical Poisson-Nernst-Planck theory [2], represent ions as a continuum, but due to the absence of ion-ion interactions it fails to properly describe correlated ionic motion inside the channel.

Recently, the concept of the quasiparticle (QP) was introduced to describe the highly correlated motion of ions in a narrow ion channel [3], [4]. A quasiparticle represents the center of mass of the ions in the channel. It is the motion of the QP, or equivalently, that of the ions as a whole, that leads to the electric current through the channel – the main experimental observable.

In the previous work [5], we have connected the QP’s potential landscape and the MD-generated potential of the mean force. Here, we focus on the transitions between channel’s occupancy states using Brownian dynamics (BD) simulations. In many cases, noninteracting ions in the mean field are considered [6], [7] thus omitting the correlations between ions [8]. The latter is of primary importance for the rate theories [9], [10] where generally the entrance / exit probabilities still remain an unsolved problem. Using BD simulations, we provide important insight into the motion of the QP and its connection to the motion of bulk ions close to the channel mouth.

II. BROWNIAN DYNAMICS

KcsA is a prokaryotic potassium channel [1]. It became one of the first channels whose crystal structure became known [11]. The amino acid sequence of its selectivity filter appears highly conserved amongst potassium channels, and as a result it became one of the most heavily studied channel.

Here, we describe the motion of ions in the KcsA channel using a toy model [3]. The BD domain is shown in Fig. 1. The central narrow tube represents the pore, and the two
large surrounding cylinders are the bulk reservoirs of radius and length 2nm. We consider a KCl solution with the ions highlighted by the coloured spheres. The pore axis spans along the z coordinate from -2nm to 2nm. We apply a harmonic radial potential \((k = 10k_B T / \AA^2)\) to ensure the one-dimensional motion of ions within the channel. To prevent ions exiting the domain through a cylindrical boundary, a short-range harmonic repulsive force is applied to approaching ions.

The motion of the ions is governed by the Langevin equation [12]

\[
m \frac{dv_i}{dt} = -\gamma m v_i + F_i + \sqrt{2m\gamma k_B T} \xi(t),
\]

where \(m\) is ion's mass, \(\gamma\) is its friction coefficient which is coupled to its diffusivity \(D\) by the Einstein relation \(D = k_B T / m\gamma, v_i\) is the ion’s velocity, \(k_B\) is Boltzmann’s constant, \(T\) represents the absolute temperature, and \(\xi\) is three-dimensional white noise of unit intensity \((\langle \xi \rangle = 0, \langle \xi(t) \xi(t') \rangle = \delta(t - t')\). A third-order algorithm [12] was used to numerically solve Eq. (1) with a timestep \(\Delta t = 0.2\) ps.

The forces \(F_i = -\frac{\partial V(r)}{\partial r}\) acting on the ion located at \(r\) arise from the potential

\[
V(r) = U(r) + \sum_{m} V(|r - r_m|),
\]

where the sums run over all other ions in the system. \(U\) stands for the potential of the interaction between the ion and the pore [3]

\[
U = -U_0 e^{-(z/a)^2},
\]

and \(U_0 = 10.5 k_B T, a = 9\) Å. We note that this is a toy model reflecting only the overall potential landscape along the pore. For instance, the presence of negatively charged amino acids in the selectivity filter of the channel leads to a potential well. A more detailed description should include the binding sites, and is available via the potential of the mean force (PMF) for the individual ions in the channel [13].

\(V\) represents the screened Coulomb potential with a short-range repulsive term

\[
V = \epsilon_0 e^{-r/d} + F_0 r_0 \left( \frac{r_0}{r} \right)^9.
\]

Here \(r_m\) represents ion’s valence, \(d = 2.8\) Å is a screening constant [3], \(\epsilon_0\) and \(\epsilon\) are the dielectric permittivities of vacuum and the pore, respectively, \(F_0 = 2 \times 10^{-19}\) N and \(r_0 = 2.8\) Å. The last term represents the soft-wall [14] repulsive potential acting at short distances.

The presence of the strong repulsive term in Eq. (4) prevents ions from approaching each other too closely. Occasionally the ion-ion distance becomes small \(\leq 2.8\) Å and consequently this term provides a strong force that results in a large displacement of the ions, known as long jumps exceptions [15]. We apply an adaptive time step method to overcome these unphysical artifacts, as described in [5] in more details. The fraction of corrected steps was less than one per thousand steps.

To maintain a constant number of ions in the simulation, the standard periodic boundary conditions were used [16].

Thus an ion that leaving the domain through the left (right) domain side will reappear on the rightmost (leftmost) position of the domain. These are the cheapest method for equilibrium simulations in the absence of an electrochemical gradient (cf. [17]).

We consider 0.5mM KCl solutions in both bulks with no applied voltage. This allows us to consider the equilibrium ionic transitions in and out of the pore.

A. Quasiparticles

The strong correlation of ionic motion in the pore allows us to introduce the quasiparticle (QP) – the center of mass of \(N\) ions located at \(z_m\) in the pore, – according to Ref. [3]

\[
q_N = \frac{\sum_{m} z_m}{N}.
\]

The Langevin equation (1) can be further simplified [5] to describe the diffusive motion of the QP in the pore

\[
\dot{q}_N = \frac{D_N}{k_B T} \frac{\partial U_{eff}^q}{\partial q_N} + \sqrt{2D_N \xi(t)}.
\]

In this equation, the index “\(N\)” reflects the fact that the QPs consist of different number of ions \(N\) (\(N = 1,2,3\)), and thus different physical properties. The effective potential \(U_{eff}^q\), faced by the QP, can be derived from the PMF [5]. Likewise, the transport diffusivity \(D_N^q\) differs from the product \(N \cdot D\) due to the ion-ion interactions [18].

III. Results and Discussion

Figure 2 shows the distribution of 1, 2, and 3 ions inside the pore. For 1 ion, this is given by the Boltzmann distribution \(P \sim e^{-U(q)/k_B T}\) as in this case the ion and quasiparticle coincide. When two ions occupy the pore the ion-ion repulsion creates a two-headed distribution at approximately \(\pm 0.5 nm\). With three occupying ions, the outermost ion gains the freedom to reach the edge of the pore and eventually leave it.

The probabilities of each occupancy state are shown by the inset. The pore primarily accommodates 1 - 3 ions, with a negligible chance of a 4-th ion entering. This occupancy...
distribution in fact represents the probability to find each individual QP inside the pore. Switching from one QP to another suggests an ion leaving or entering the channel, and thus produces electric current – the main experimental observable. For a better understanding let’s consider the individual ions’ trajectories and these of the QPs.

A typical trajectory of the ions and the QPs are shown in Fig. 3. Individual ions move inside the channel revealing a high level of correlation [5]. An ion (green trace) approaches the channel from the bulk and enters at $t \approx 550\text{ns}$. This immediately transforms the QP from $q_2$ (black line) into $q_3$ (orange line) with an immediate jump to a new position. After that, the quasiparticle diffuses until another jump at a later moment $t \approx 555\text{ns}$ when the outermost ion leaves the channel. The QP $q_3$ transforms into $q_2$ with a new location again. Overall, this process, known as the knock-on conduction, corresponds to the diffusion of the QP within the channel and switching its type in a jump-like manner.

![Fig. 3. Dynamics of ions and the QP. An ion (green trace) approaches the channel from the bulk and enters at $t \approx 550\text{ns}$. This immediately transforms the QP from $q_2$ (black line) into $q_3$ (orange line) with an immediate jump to a new position. At a later moment $t \approx 555\text{ns}$ when the outermost ion leaves the channel. The QP $q_3$ transforms into $q_2$ with a new location again. Overall, this process is known as knock-on conduction. Dashed lines demarcate the edges of the channel.](image1)

The transition processes describing either an ion entering the pore from the bulk, or exiting from the pore, need further clarification. With that in mind, we introduce a cylinder of radius and height 5Å at each end of the channel, representing the channel’s mouths. Thus the dynamics of the entering and leaving processes can be monitored. The inset of Fig. 4 shows the distribution of residence times (identically, of the cis trajectories [19]), showing how long on average an ion stays in the mouth.

However, these times should also depend on the location of the QP. To provide some preliminary insight into this question, we fix a pair of ions at a 2nm separation inside the channel and analyze the corresponding changes in the distribution near the pore’s edges. This mimics the ionic configuration at a specific position of the QP analogous to Fig. 2. For clarity, we consider three values of $q_2 = \{-0.5, 0, 0.5\}$ nm. Eventually, the number density of $K^+$ ions in the mouths and the channel is measured.

One can see that once the QP is localised closer to the left (right) edge of the channel, ions do not enter the channel from that side, but penetrate from the “free” right (left) mouths of the channel. If the QP is at the centre of the pore then it provides an equal probability of entrance by an ion from either mouth. The increased distribution in the channel or near the mouths suggests an increased probability of entry, and therefore higher probability of permeation.

These findings can be understood in light of the screened Coulomb interaction Eq. (4) between the ions. Thus, the bulk ions can be influenced by ions in the channel at a finite distance ($\sim d$ in our simulations). This interaction between bulk and channel ions defines which transition occurs. If the QP approaches the edge of the pore it prevents an ion entering from the bulk on this side but allows an ion to enter on the opposite side. When an ion enters it pushes the remaining ions such that the outermost ion leaves the channel (knock-on conduction). This is an example of a trans trajectory [19], [20].

These transitions are vital to understanding the channel’s permeation mechanism. One often assumes noninteracting ions moving in a mean field [6], [7], [10], based on the unconditional probability densities [8], [20]. However, this approach evidently ignores the correlations between ions emerging from their interaction. A more sophisticated analysis requires the application of the conditional probability densities [8] that are closely related to the pair-distribution functions [8]. This improved description of the transition process under arbitrary experimental conditions – voltage, concentrations, and chemical composition [2] – can thus pave the way to a more adequate

![Fig. 4. Distribution of the mobile ions in the channel and the mouths when a pair of ions in the pore is kept immobile. Thus the QP’s location $q_2$ is fixed. Inset: distribution of the residence times in the mouth.](image2)
definition of the transition rates in the kinetic [9], [10], [21] theories.

IV. CONCLUSIONS
We have considered the motion of the quasiparticle (QP) in a narrow ion channel with a specific emphasis on the transition process. We show that the motion of the QP and the bulk ions in the mouths is correlated. Namely, the presence of the QP near one of the channel’s edges suggests a higher exit probability from that edge and a higher entering probability on the opposite side. This applies to kinetic and rate theories where the transition process is the key to successive description of conduction and selectivity.

In further work we plan to consider a more realistic description of the channel. First, it is desirable to estimate the forces from the potentials of the mean force (PMF) obtained from Molecular Dynamics (MD). This is proven to be successful in the translocation of DNA through nanopores [22]. Secondly, it’s important to extend the analysis for a system far from equilibrium. Finally we plan to run a full scale MD simulation to verify the predictions of this paper.

It is also believed that the concept of quasiparticles can turn out useful in constructing a functionalised nanotubes with prescribed properties. The latter appears at the front end of the research towards efficient water desalination atomically thin membranes [23].

ACKNOWLEDGEMENTS
We are grateful to Robert S. Eisenberg, Aneta Stefanovska, and the late Igor Kh. Kaufman for helpful discussions.

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The Origin and the Measurement of Phase Noise in Oscillators

Enrico Rubiola
FEMTO-ST Institute, Besancon, France, and INRiM, Torino, Italy
Home page http://rubiola.org

The scientific and technological interest for lowest noise oscillators and lasers is ever growing because phase noise relates to the time fluctuation. In this lecture, we go through the origin of phase noise in oscillators and in resonators, and the measurement techniques.

Letting $V_0[1 + \alpha(t)] \cos[2\pi f t + \varphi(t)]$ the clock signal, $\alpha(t)$ is the random fractional-amplitude fluctuation, and $\varphi(t)$ is the random phase. Such noise is properly described in terms of the power spectral $S_\alpha(f)$ and $S_\varphi(f)$, while industries prefer $L(f) = (1/2) S_\varphi(f)$. The Allan deviation $\sigma_y(\tau)$ is also often used, for slower fluctuations and for timekeeping.

The Leeson effect is a system-oriented heuristic approach which explains how the noise of the electronics turns into the AM and PM noise of the oscillator, and how the AM noise – often neglected – impacts on PM noise. Being broader than just a model, the Leeson effect applies also to lasers and to microwave-photonic oscillators. The analysis relies on a low-frequency space, where cyclostationary noise of parametric nature is easily represented as additive noise. The resonator is seen a low-pass filter, or a delay line in the case of lasers and photonic oscillators. The same framework helps to understand the Pound and the PDH frequency control used to stabilize lasers and microwave oscillators to a reference cavity, ending in a stability of $10^{-15} \ldots 10^{-16}$.

However, the Leeson effect fails in a number of practical cases, where the frequency flicker ($1/f^3$ PM noise) is set by the flicker ($1/f$) fluctuation of the natural frequency. Flicker noise in the resonators is understood only in some cases. A theory starts from the fact that mechanical dissipation in low-loss condensed matter is of structural nature, as opposed to viscous nature. The dissipation shows up as a hysteresis cycle which keeps its aspect ratio from microscopic scale to microscopic scale. Combining structural dissipation with the fluctuation dissipation theorem predicts successfully the stability of Fabry-Perot etalons made of ULE glass or monocrystalline Si, of the order of $10^{-15} \ldots 10^{-16}$. Unfortunately, the same theory fails to explain the stability of quartz resonators ($10^{-12} \ldots 10^{-13}$). A heuristic theory, suggesting that flicker is proportional to the reciprocal of the volume, applies to the resonator’s frequency fluctuations and to the phase fluctuations of two-port devices.

Virtually all modern phase-noise analyzers exploit two separate channels, each consisting of a detector and a frequency reference, that measure simultaneously the oscillator under test. Under the hypothesis that the two channels are independent, the cross spectrum converges to the oscillator’s $S_\varphi(f)$, or $S_\alpha(f)$. The large memory and computing power of nowadays digital electronics give the false impression of high rejection of the single-channel noise, while the actual limit is set by the systematic (B-Type) uncertainty, generally not documented. Gross errors and nonsensical results are around the corner.
Noise spectroscopy-based gas identifying methods to improve the selectivity of MOX gas sensors.

Thierry Contaret
Aix Marseille Univ, Univ Toulon,
CNRS, IM2NP
Marseille, France
thierry.contaret@im2np.fr

Nicolas Morati
Aix Marseille Univ, Univ Toulon,
CNRS, IM2NP
Marseille, France
nicolas.morati@im2np.fr

Tomas Fiorido
Aix Marseille Univ, Univ Toulon,
CNRS, IM2NP
Marseille, France
tomas.fiorido@im2np.fr

Sami Gomri
METS Unit
National School of Ingenieurs of Sfax
Sfax, Tunisia
samigomri14@gmail.com

Jean-Luc Seguin
Aix Marseille Univ, Univ Toulon,
CNRS, IM2NP
Marseille, France
jean-luc.seguin@im2np.fr

Nicolas Morati
Aix Marseille Univ, Univ Toulon,
CNRS, IM2NP
Marseille, France
nicolas.morati@im2np.fr

Marc Bendahan
Aix Marseille Univ, Univ Toulon,
CNRS, IM2NP
Marseille, France
marc.bendahan@im2np.fr

Abstract—Air quality monitoring and analysis have become a major issue in recent years. Metal oxide gas sensors are very sensitive due to high variability of its resistivity in presence of gas. However, they are not selective. It is not possible to determine the both gas nature and concentration using sensors resistance variation. Noise spectroscopy is one of solution to improve the selectivity. In this paper, we apply the recent developed noise spectroscopy-based gas identifying methods in order to show the possibility of each method to discriminate the nature of different gases. Noise measurements have been carried out on metal-oxide gas micro-sensors with tungsten trioxide (WO3) sensing layer, under several NO2, O3 and CO concentrations in dry air. The obtained results have demonstrated that a selective sensing of the studied gases is possible using a single MOX gas micro-sensor.

Keywords—noise spectroscopy, adsorption-desorption noise, selectivity, metal oxide gas sensors

I. INTRODUCTION

Nowadays, there is a growing need for low-cost, low-power miniaturized gas sensors mainly due to the connectivity of sensing devices on global network (internet of things) enabling immediate sharing of information in a variety of fields such as portable and connected devices for domestic and industrial air quality control, security and defence, food quality control … In this context, a great deal of research and development work is being carried out to design small and cheap gas sensors with high sensitivity, selectivity and stability, with respect to a given application. Metal oxide (MOX) gas sensors are readily available and widely used in portable and low cost gas monitoring devices because of their high sensitivity, stability, and attractive life time. However, this type of gas sensor suffers an inherent lack of selectivity, because the gas detection mechanism is rather unspecific and more or less any type of reducing or oxidizing gas is detected.

Due to this poor selectivity, MOX gas sensors are more often assembled into a multi sensor array that forms the core of an electronic nose. Electronic nose is a complex system used in the identification of gas mixtures. It consists of a multi sensor array, an information-processing unit, software with digital pattern-recognition algorithms and reference-library databases [1-2]. The sensor array is composed of different sensors chosen to respond to a wide range of chemical classes. The output of individual sensors are collectively assembled and integrated to generate a distinct digital response pattern. Identification and classification of an analyte mixture is accomplished through recognition of this unique chemical signature (electronic fingerprint) of collective sensor responses. The challenge in miniaturizing devices and lowering power consumption is to minimize the number of sensors required for a given application. To do this, we must try to increase the amount of information provided by one sensor using advanced measurements, like temperature modulation or fluctuation enhanced sensing (FES). [3,4].

The FES principle uses the fluctuations of the gas sensors’ response as an information source. This experimental technique is based on noise spectroscopy: the measurement and the analysis of the power spectral density (PSD) of the fluctuations measured at the terminals of sensors in the presence of one or more gases. Measuring these fluctuations caused by adsorption-desorption and diffusion noise provides enhanced selectivity and sensitivity. Several studies have shown that noise spectroscopy is a relevant signal-processing tool able to extract selective informations on multiple gases with a single sensor [4-6].

II. GASES IDENTIFICATION METHODS

In MOX gas micro sensors, noise depends strongly on the oxygen stoichiometry and oxygen motion. The adsorption-desorption process of oxygen atoms, the presence of defects and grain boundaries in metal-oxide cause fluctuations of the oxygen density, and, thus fluctuations of the sensing layer electrical conductance. In a gaseous environment, the sensing layer conductance fluctuations due to free carrier’s number and mobility fluctuation are related to concentration and distribution fluctuations of adsorbed chemical species.

A. Noise current spectral decomposition

In earlier work [7], we presented a model of adsorption-desorption (A-D) noise in MOX gas sensors, developing the idea that the fluctuation of the gas sensor resistance is, among others noise sources, due to the fluctuation of the density of gas molecules on the surface of the sensing film. The modeling was developed by taking into account the polycrystalline structure of the sensing layer and the effect of the adsorbed molecule’s density fluctuation on the grain boundary barrier height.

If the gas sensor resistance $R_{sensor}$ is biased by a voltage $V_0$ and the measured noise is a current fluctuation, the PSD of the fluctuations of the total terminal current across the gas sensor resistance writes [7]:
\[ S_{a\_\_\_}(f) = \frac{V^2}{R_{\text{sensor}}} \sum_{i=1}^{N} \frac{1}{f_i} \left( 1 + \frac{f}{f_i} \right) + \frac{4kT}{R_{\text{sensor}}} \]  

(1)

The first term in (1) is a sum of lorentzians having a cut-off frequency \( f_i \) and a low frequency magnitude \( S_i \). Their expressions are given in [7], and they depend on the nature of the detected gas, and on the grain size. \( g \) is the number of most prevalent grain sizes involved in the sensing layer, \( T \) is the temperature and \( k \) is the Boltzman constant.

**B. First derivative of noise current spectral density**

In our recent work [8], we calculated the theoretical expression of the first derivative of the PSD of the gas sensor noise, and showed that it admitted a minimum which depends on the nature of the detected gas.

Using "(1)" , the expression of the first derivative of the PSD of the gas sensor noise can be written as follows:

\[ \frac{dS_{a\_\_\_}(f)}{df} = -\frac{V^2}{R_{\text{sensor}}} \sum_{i=1}^{N} S_i \left[ f \left( 1 + \frac{f}{f_i} \right)^2 \right] \]  

(2)

First derivative of each single Lorentzian has a minimum at the frequency \( f_i / \sqrt{3} \) that depends on the nature of the detected gas. We have demonstrated that the complete expression of the first derivative of the PSD of the gas sensor noise had a minimum at a frequency between \( f_i / \sqrt{3} \) and \( f_{cgf} / \sqrt{3} \) [8].

**C. Product of the noise current spectral density by the frequency**

Another gas identification method is based on the product \( f \cdot (S_{a\_\_\_}(f) - S_{in}(f)) \) where \( f \) is the frequency, \( S_{a\_\_\_}(f) \) is the PSD of gas sensor noise current as (1) and \( S_{in}(f) = \frac{4kT}{R_{\text{sensor}}} \). This product writes as follows:

\[ f \cdot (S_{a\_\_\_}(f) - S_{in}(f)) = f \sum_{i=1}^{N} S_i \left( 1 + \frac{f}{f_i} \right) \]  

(3)

with \( f^2 = \frac{V^2}{R_{\text{sensor}}} \)  

(4)

The product of frequency by the PSD of the gas sensing layer resistance fluctuations often has a maximum which is characteristic of the gas and, that is the combination of maxima of each term [9].

**III. EXPERIMENTAL**

The experimental protocol used in the present work is the same one used in our recent paper [9]. Indeed, the noise measurements have been performed on WO3 based gas sensors. For more details concerning the experimental setup (see figure 1) and the preparation of the WO3 based sensing layer, one can see the descriptions given in [9].

<table>
<thead>
<tr>
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<th>Concentrations</th>
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<tr>
<td></td>
<td>C1</td>
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<tr>
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</tr>
<tr>
<td>NO₂</td>
<td>1 ppm</td>
</tr>
<tr>
<td>O₃</td>
<td>80ppb</td>
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**IV. RESULTS**

The developed gas identification methods are based on precise spectral decomposition of measured noise responses. To extract useful information from the measured noise across the sensor in the presence of gas, we take into account the characteristics of low noise amplifier by:

\[ S_{\text{sensor}} = \frac{S_{\text{measured}}}{(A_{\text{LNA}}(f))} \]  

(6)

where \( S_{\text{sensor}} \) is the intrinsic current spectral density of sensor, and \( S_{\text{measured}} \) is the measured noise voltage spectral density. \( A_{\text{LNA}}(f) \) is the measured frequency response of the gain of the SR570 low noise current amplifier. Fig. 2 gives an example of current spectral density of the sensor under 10ppm of CO (curve1). All measured spectra clearly show Lorentzian components according to adsorption-desorption noise theory. Indeed, the noise generated due to the adsorption of a gas on an adsorption site with a given adsorption energy has a Lorentzian spectrum. For the three studied gases, we observe that two Lorentzians dominate the low frequency spectrum. Over 1 kHz we observe white noise mainly composed of the thermal noise due to sensitive layer resistance and the thermal noise of the amplifier due to the...
feedback resistor. Details of spectral decomposition are also plotted in Fig. 2. Measured noise spectra is higher by one or more orders of magnitude compared to amplifier noise level (curve 6). The extracted noise parameters of spectral decomposition of all measured spectra are reported in Tab. 2. For all studied gas concentrations, the thermal noise is below $10^{-26}$ $A^2/Hz$ indicating a resistance of the sensitive layer varying around 1M$\Omega$. The evolution of thermal noise is consistent with the evolution of the resistance of the sensitive layer in the presence of each of the three studied gases.

![Fig. 2. Spectral decomposition of a noise response under CO (10 ppm): measurements (curve 1), total noise (curve 2), adsorption-desorption noise modeled by two lorentzians (curve 3 and 4), white noise proportional to the resistance of the sensor (curve 5); extracted noise model of low-noise current preamplifier (curve 6).](image)

**TABLE II.** VALUES OF THE LORENTZIANS CUT-OFF FREQUENCY $f_{C1}$ AND LOW FREQUENCY MAGNITUDE $S1$ FOR THE VARIOUS GASES CONCENTRATION USED IN THE MEASUREMENT PROCESS.

<table>
<thead>
<tr>
<th>Gas / Concentration</th>
<th>$S1$ $(A^2/Hz)$</th>
<th>$f_{C1}$ $(Hz)$</th>
<th>$S2$ $(A^2/Hz)$</th>
<th>$f_{C2}$ $(Hz)$</th>
<th>$S_{Th}$ $(A^2/Hz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O$_3$ / C1</td>
<td>$7 \times 10^{-23}$</td>
<td>0.1</td>
<td>$1 \times 10^{-23}$</td>
<td>5</td>
<td>$3 \times 10^{-26}$</td>
</tr>
<tr>
<td>O$_3$ / C2</td>
<td>$8 \times 10^{-23}$</td>
<td>0.1</td>
<td>$1 \times 10^{-23}$</td>
<td>5</td>
<td>$2.5 \times 10^{-26}$</td>
</tr>
<tr>
<td>O$_3$ / C3</td>
<td>$1 \times 10^{-20}$</td>
<td>0.1</td>
<td>$4 \times 10^{-25}$</td>
<td>5</td>
<td>$1.5 \times 10^{-26}$</td>
</tr>
<tr>
<td>O$_3$ / C4</td>
<td>$1.2 \times 10^{-20}$</td>
<td>0.1</td>
<td>$5 \times 10^{-25}$</td>
<td>5</td>
<td>$1.5 \times 10^{-26}$</td>
</tr>
<tr>
<td>NO$_2$ / C1</td>
<td>$3 \times 10^{-23}$</td>
<td>0.2</td>
<td>$7 \times 10^{-25}$</td>
<td>5</td>
<td>$5 \times 10^{-26}$</td>
</tr>
<tr>
<td>NO$_2$ / C2</td>
<td>$2 \times 10^{-23}$</td>
<td>0.2</td>
<td>$9 \times 10^{-25}$</td>
<td>5</td>
<td>$4.5 \times 10^{-26}$</td>
</tr>
<tr>
<td>NO$_2$ / C3</td>
<td>$1.5 \times 10^{-23}$</td>
<td>0.2</td>
<td>$7 \times 10^{-25}$</td>
<td>5</td>
<td>$4 \times 10^{-26}$</td>
</tr>
<tr>
<td>NO$_2$ / C4</td>
<td>$1 \times 10^{-23}$</td>
<td>0.2</td>
<td>$7 \times 10^{-25}$</td>
<td>5</td>
<td>$3.5 \times 10^{-26}$</td>
</tr>
<tr>
<td>CO / C1</td>
<td>$1.2 \times 10^{-22}$</td>
<td>0.2</td>
<td>$7 \times 10^{-25}$</td>
<td>10</td>
<td>$2.5 \times 10^{-26}$</td>
</tr>
<tr>
<td>CO / C2</td>
<td>$1.3 \times 10^{-22}$</td>
<td>0.2</td>
<td>$3 \times 10^{-25}$</td>
<td>10</td>
<td>$2.8 \times 10^{-26}$</td>
</tr>
<tr>
<td>CO / C3</td>
<td>$1.5 \times 10^{-22}$</td>
<td>0.2</td>
<td>$3 \times 10^{-25}$</td>
<td>10</td>
<td>$3 \times 10^{-26}$</td>
</tr>
<tr>
<td>CO / C4</td>
<td>$2 \times 10^{-22}$</td>
<td>0.2</td>
<td>$3 \times 10^{-25}$</td>
<td>10</td>
<td>$3.2 \times 10^{-26}$</td>
</tr>
</tbody>
</table>

The first derivative of noise current spectral density of the measured gas sensor noise is calculated using (2) and extracted parameters of TABLE II. In Fig. 3 we present the plots of the first derivative of the PSD of the gas sensor noise response under four concentrations of nitrogen dioxide. We obtain similar curves for ozone and carbon monoxide. For the three gases, we clearly observe that the first derivative of the measured gas sensor noise spectrum presents a negative minimum according to the developed theory in [8]. These negative minimums are reported in TABLE III and have specific value ranges for each gas.

![Fig. 3. Plots of the first derivative of noise current spectral density of the four concentrations of nitrogen dioxide.](image)

**TABLE III.** MINIMUM OF THE FIRST DERAVATIVE OF THE PSD OF THE GAS SENSOR NOISE $(A^2/Hz^2)$

<table>
<thead>
<tr>
<th>Gas</th>
<th>Concentration</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>O$_3$</td>
<td>$-4 \times 10^{-26}$</td>
<td>$-5.2 \times 10^{-26}$</td>
<td>$-6.2 \times 10^{-26}$</td>
<td>$-7.8 \times 10^{-26}$</td>
<td></td>
</tr>
<tr>
<td>NO$_2$</td>
<td>$-9.8 \times 10^{-26}$</td>
<td>$-6.5 \times 10^{-26}$</td>
<td>$-5 \times 10^{-26}$</td>
<td>$-3.2 \times 10^{-26}$</td>
<td></td>
</tr>
<tr>
<td>CO</td>
<td>$-3.9 \times 10^{-26}$</td>
<td>$-4.2 \times 10^{-26}$</td>
<td>$-4.9 \times 10^{-26}$</td>
<td>$-6.5 \times 10^{-26}$</td>
<td></td>
</tr>
</tbody>
</table>

In Fig. 4, we plot the product $f \cdot \left( S_{\text{sensor}}(f) - S_{Th}(f) \right)$ in the case of ozone, nitrogen dioxide and carbon monoxide using the extracted parameters in TABLE II. Over 100 Hz, we observe a $1/f$ slope as predicted by (3) for all concentrations of studied gases. On the other hand, at low frequencies, the behavior is different for each gas. For NO$_2$, the curves of the product of noise current spectral density by the frequency show two maximums of approximately equal values while the maximum at lower frequency is higher for CO. The curves for ozone have a particular behavior because of the higher sensitivity of tungsten trioxide to this gas. This high sensitivity to ozone is characterized by a higher low frequency noise level compared to other gases. The values in TABLE II show that the first Lorentzian for ozone is predominant due to a desorption-adsorption process on a preferred adsorption site. So, the quantity $f \cdot \left( S_{\text{sensor}}(f) - S_{Th}(f) \right)$, has a maximum which is characteristic of the nature of the detected gas. Detecting this maximum is a sensitive method to identify a gas. This result confirms that the choice of this parameter seems interesting for the identification of the detected gas compared to other parameters such as the average slope of the
product \( f \cdot S_{\max}(f) \) \([10]\) or the characteristic frequency of the maximum of this same product \([11]\).

Fig. 4. Plots of the quantity \( f \cdot (S_{\max}(f) - S_{n}(f)) \) for four concentrations of ozone (O\textsubscript{3}), nitrogen dioxide (NO\textsubscript{2}) and carbon monoxide (CO) detected by metal-oxide gas microsensor with tungsten trioxide (WO\textsubscript{3}) sensing layer.

The ability to discriminate several gases using our different noise spectroscopy-based gas identifying methods has been evaluated by the principal component analysis (PCA). PCA is a commonly used unsupervised and robust pattern recognition approach for analysis of multivariable data. It is a statistical procedure that enables to convert a set of observations of possibly correlated variables into a new set of values called principal components. In PCA, the score plots show the relations between these analyzed variables (different concentrations of the three gases in our studies). PCA has been performed using the extracted spectral parameters of the spectral decomposition method (TABLE II), the first derivative method's (TABLE III) and the method of the product of the frequency by the noise current spectral density (maximum of the product). The scores plot of the first two principal components is given in Fig. 5 and shows that a clear discrimination is possible between the three studied gases.

V. CONCLUSION

In this paper, we have presented the recent developed gases identification methods based on noise spectroscopy. These methods are based on precise spectral decomposition of measured noise responses of micro sensor under gases. For each method, we observe possibilities to discriminate the nature of detected gas. In particular, the minimum of the first derivative of noise current spectral density is different for each studied gases. Similarly, we have observed that the maximum of the product of noise current spectral density by the frequency is characteristic of the nature of the detected gas. Finally, the PCA multivariable analysis method has been applied to all extracted spectral data. So, we have shown that it is possible to identify the nature of detected gas using noise spectroscopy-based methods.

REFERENCES

Bulk induced 1/f noise in topological insulators

Saurav Islam,1 Semonti Bhattacharyya,1 Hariharan Nhalil,1 Anthony Richardella,2 Abhinav Kandala,2 Suja Elizabeth,1 Nitin Samarth,2 and Arindam Ghosh1

1Department of Physics, Indian Institute of Science, Bangalore: 560012
2Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802-6300, USA.

Email: isaurav@iisc.ac.in

We present a detailed study of 1/f noise in both mesoscopic and large area, epitaxially grown thin films of topological insulators. Our investigation of gate-voltage dependence of noise has revealed that, whereas Hooge type mobility fluctuation noise is the dominant mechanism of 1/f noise in thick (height ~100 nm) mesoscopic samples, in case of thin (height ~10 nm) samples, the noise is governed by universal conductance fluctuations.[1,2] In case of epitaxially grown samples, the noise in the surface transport regime is governed by correlated mobility-number density fluctuation noise, caused by the bulk defects with a density of $D_b = 3.2 \times 10^{17}$ cm$^{-2}$eV$^{-1}$.[3] In the presence of magnetic field, the 1/f noise follows a parabolic dependence which is qualitatively similar to mobility and charge density fluctuation noise in nondegenerate semiconductors. The temperature dependence of 1/f noise displays sharp peaks at characteristic temperatures depending on the material, is both mesoscopic and epitaxially grown films, which can be attributed to generation-recombination processes in the impurity band in the bulk band gap. The Hooge parameter, which is a figure of merit, however, can be very small, comparable to some of the other low noise systems, implying that they can be used in low noise interconnects. Our studies reveal that in TIs, irrespective of the thickness and substrate, the intrinsic bulk defects are the dominant source of noise, contrary to other two-dimensional materials, where the noise originates at the contacts or at the channel-substrate interface.

References:

Trapping Investigation of the GaN HEMT Devices Using the Low Frequency Noise Characterization

Mohamed Bouslama
Xlim Laboratory
University of Limoges
France, Brive-la-Gaillarde
mohamed.bouslama@unilim.fr

Jean-Christophe Nallatamby
Xlim Laboratory
University of Limoges
France, Brive-la-Gaillarde
jean-christophe.nallatamby@unilim.fr

Michel Prigent
Xlim Laboratory
University of Limoges
France, Brive-la-Gaillarde
michel.prigent@unilim.fr

Abstract—This paper, proposes the characterization of the signature of traps existing in the new AlGaN/GaN HEMT of 0.15 µm ultra-short gate length and 8x50 µm gate width through the output and the input Low Frequency (LF) noise measurement technique. These measurements were performed for varying chuck temperatures (Tchuck) ranging between 25 °C and 125 °C and for the same biasing condition by measuring the output or input noise spectral density. The output drain noise spectral density characteristics demonstrate the existence of an acceptor-like traps. The peak value of those traps shifts towards higher frequencies as the temperature increases. The activation energy $E_a$ around 0.51 eV and the cross section $\sigma$ around 5x10$^{-15}$ cm$^2$ were extracted using Arrhenius equation. Furthermore, the input gate noise spectral density characteristics demonstrate the presence of another type of traps. The peak of this traps does not show the frequency shift as the temperature increases. The leakage current measured before and after LF measurements for $V_{GS} = -6$ V, -7 V and for $V_{DS}$ varying from 0 V to 10 V remains lower than 40 µA/mm.

Keywords—Arrhenius plot, LF noise, leakage current, traps.

I. INTRODUCTION

The AlGaN/GaN high-electron-mobility transistor (HEMT) is a focus of interest in efforts to fabricate high power and high-frequency microwave circuits due to the remarkable properties of the GaN material. This is possible thanks to the high electron mobility, the high breakdown voltage and the high thermal conductivity. However, trapping effects remain the biggest problem for GaN community because they significantly limit the dynamic performances of the device and also detriment the device reliability due to the presence of deep level states into the heterostructure, called traps. Several measurement characterization techniques can help researchers to identify and localize these defects. The so-called “traps” can be located in different region and/or interfaces. The origin of these traps is not correlated with the physical phenomenon occurring in the device. For GaN HEMTs, 1/f noise and Generation-Recombination (G-R) noise are considered to be the most important noise sources [1]. The 1/f noise could be originated from material due to microscopic degrees of freedom interacting with quantum variables of nano-devices [2]. The G-R noise originates from traps that randomly capture and emit charge carriers, thereby causing fluctuations in the total number of charge carriers available for current transport in the channel [3].

Therefore, to determine the signature of these traps (activation energy ($E_a$), cross section ($\sigma$)) in the devices, we propose in this study a Low Frequency (LF) Noise characterization based on the measurement of the output drain noise and the input gate noise spectral densities using the FFT HP89410A vector signal analyzer [4].

The noise measurement has been performed at different chuck temperatures (Tchuck) ranging between 25°C and 125°C and this allows to extract the activation energy ($E_a$) and capture cross section ($\sigma$) of the traps located in the device. The paper is organized as follows. Section II describes very briefly the structure of the transistor. Section III describes the setup noise characterization for HEMT device under test (DUT). In Section IV, we present the measured LF drain noise characteristics (output noise power spectral density) and the LF gate noise characteristics (input noise power spectral density) of AlGaN/GaN HEMT of 0.15µm ultra-short gate length and 8x50 µm gate width and we discuss the extracted trap parameters. We also verify the leakage current in the end of these measurements to ensure their reliability. Finally, Section VI concludes the paper.

II. DEVICE DESCRIPTION

The structure of the transistor used in this work is based on AlGaN/GaN HEMTs grown on SiC substrate with a gate length of 0.15 µm and a slanted T-shape profile. A source terminated field-plate is processed to improve the breakdown voltage and the transistor RF gain by a reduction of feed-back capacitance. Experimental details on the thickness of each layers of the device cannot be given due to confidential reasons. This technology offers to frequency bands up to 35GHz covering the need for 5G in particular.

III. CHARACTERIZATION SETUP

A. Noise Work bench

The LF noise of the semiconductor devices can be carried out as equivalent input voltage and current noise sources or in terms of input gate short-circuit current and output drain short-circuit current noise sources as shown in Fig.1. The transistor is modeled as a noiseless transistor with noise current sources at both the ports and their corresponding input
and output noise spectral densities are denoted as $S_{\text{in}}$ and $S_{\text{out}}$, respectively.

![Noise model of Transistor](image1)

**Fig. 1.** Noise model of Transistor

In order to measure the noise current sources, different setups have been proposed in the literature [5].

The Low Frequency noise bench measurement [1], [4] used in this work is shown in Fig. 2. The noise voltages generated by the voltage amplifiers are measured using the HP89410A vector signal analyzer. The FFT vector signal analyzer allows the measure of the signal power either in frequency or the time domain. The noise floor of the measurement test-bench is $3 \times 10^{-27}$ A$^2$/Hz at 1 KHz and $7 \times 10^{-27}$ A$^2$/Hz at 100 KHz, respectively. On-wafer LF noise measurements have been performed by including the thermal chuck in the measurement bench and the associated thermal calibration has been performed. The device is biased under deep class-AB operation mode ($V_{DS} = 10$ V and $I_{DS} = 50$ mA/mm). The bias tees are connected in order to avoid the unnecessary oscillations in the RF band. By characterizing the bias tees connected at the input and output terminals of the transistor, the measured power noise spectral densities $S_{\text{in}}$ and $S_{\text{out}}$ are related to the short circuit noise power spectral densities $S_{\text{in}}$ and $S_{\text{out}}$ of the transistor. In order to avoid the influence of one noise on another, a large value of capacitance (30-mF) is connected at the In or Out node, while measuring the noise at the other terminal. Further details about the noise measurement can be found in [1].

### IV. RESULTS AND DISCUSSIONS

#### A. Output Power Spectral Density

The output noise voltage spectral density $S_{\text{out}}$ was measured using the noise test-bench shown in Fig. 2 and the corresponding noise current spectral density obtained is shown in Fig. 3. This device is biased under deep class AB operating conditions: $V_{DS} = 10$ V and $I_{DS} = 20$ mA.

It can be noticed that the 1/f noise dominates at low frequencies whereas the GR noise dominates at higher frequencies.

![Complete setup for noise characterization](image2)

**Fig. 2.** The complete setup for noise characterization

The peaks observed correspond to the existence of traps in the device structure. The traps cut-off frequencies have been extracted at various temperatures and then, by using the Arrhenius equation [6], the trap activation energy ($E_a$) and capture cross sections ($\sigma_c$) have been determined. A correct extrapolation of the time constant of the de-trapping processes needs to evaluate the self-heating effects of the device. Therefore, the method proposed in [7] is used to extract the thermal resistance $R_{\text{TH}}$ of the devices using on-wafer pulsed I(V) measurements. The thermal resistance of the transistor ($R_{\text{TH}}=13\,\text{°C}\,\text{mm/W}$) has been taken into account while extracting the trapping physical parameters. The corresponding extracted Arrhenius plot is shown in Fig. 5. The apparent activation energy determined is 0.51 eV and the
corresponding capture cross section determined is $5 \times 10^{-15}$ cm$^2$.

Fig. 5. Extracted Arrhenius plot using LF noise measurement under the biasing conditions of $V_{DS} = 10$ V and $I_{DS} = 50$ mA/mm.

To achieve complementary information on the trapping phenomena, measurements are carried out for $I_{DS} = 60$ mA while maintaining the drain voltage $V_{DS} = 10$ V. The corresponding output noise current spectral density is shown in Fig. 6. In the same manner as previously described, the corresponding activation ($E_a$) and cross section ($\sigma_n$) obtained using the Arrhenius plot are 0.53 eV and 1.5x$10^{-14}$ cm$^2$.

Fig. 6. Output noise power spectral density vs. frequency for the AlGaN/GaN device measured for various Tchuck ranging between 25 °C and 125 °C and under the biasing conditions of $V_{DS} = 10$ V and $I_{DS} = 150$ mA/mm.

Fig. 7 investigates the effect of the measured output noise spectral density multiplied by frequency for the drain current $I_{DS} = 60$ mA for the temperature ranges between 25°C and 125°C. It can be noticed that the cut-off frequency of the detrapping processes depend on the choice of the bias conditions. In our study, the time constant decreases with increasing the drain current. Nevertheless, the values of the activation energy and cross section are very close to the ones obtained for $V_{DS} = 10$ V and $I_{DS} = 20$ mA.

Fig. 8 exhibits the summary of the deep levels investigated by several research groups for the case of iron doped GaN buffer HEMT devices. On the same plot, we report the curves corresponding to activation energies extracted using low frequency noise measurements. The physical origin of these deep levels could be related to the Fe-doped GaN buffer. Moreover, the extracted activation energies of these deep levels are in good agreement with the already reported value of Fe-doped GaN HEMT devices [4], [8], [9], [10].

C. Input Power Spectral Density

The input noise voltage spectral density $S_{nI}$ was measured using the same biasing condition. The corresponding noise current spectral density obtained is shown in Fig. 9.
To distinguish GR noise from other measured noise sources, the measured input noise spectral density is multiplied by frequency and the corresponding obtained plot for different temperature is shown in Fig. 10. These data show an absence of temperature dependence. So, the extraction of the signature of traps cannot be highlighted. In order to understand, to localize and identify these traps, two-dimensional (2D) TCAD physics-based device simulations must be performed.

![Image](https://example.com/image.png)

**Fig. 10.** Input noise power spectral density multiplied by frequency for the AlGaN/GaN device measured for various Tchuck ranging between 25°C and 125 °C and under the biasing conditions of $V_{DS} = 10$ V and $I_{DS} = 50$ mA/mm.

V. LEAKAGE CURRENT VERIFICATION

To ensure the validity of our measurements, we present in Fig.11 the drain current before and after LF noise measurements which is in a range of $\mu$A/mm. So, for a $V_{GS} = -7$ V and -6 V, we measured the dc drain current ($I_{DS}$) using the Keysight B1500A analyzer. We can notice that no leakage current was detected (less than 40 $\mu$A/mm), in addition, the current decrease after LF noise measurements.

![Image](https://example.com/image.png)

**Fig. 11.** Measurement of the drain current vs drain voltage for the AlGaN/GaN transistor at $V_{GS} = -6$ V and -7 V before and after LF noise measurements.

VI. CONCLUSION

In this work, we have investigated the LF drain and gate noise characteristics of the new AlGaN/GaN HEMTs Transistor of 0.15 $\mu$m ultra-short gate length. The measured drain noise characteristics confirm the existence of traps in the device. The activation energy of traps determined could be related to the iron doping existing in the GaN buffer region of the device. However, the measured gate noise characteristics confirm also the existence of another type of traps in the structure. Further investigations using TCAD physical simulations are strongly required to identify the type of those traps existing in the device and also their corresponding physical location. No leakage current was identified before and after measurements.

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REFERENCES

1/f Noise Generated with the Branching Process Model

Takayuki Kobayashi
Department of Physics, Shiga University of Medical Science,
Otsu, Shiga, 520-21, Japan
E-mail: jokyoji@gmail.com

Abstract--It was demonstrated that the branching process model is useful to generate a series having a 1/f spectrum in a wide range of frequency more than seven decades.

Keywords-- 1/f noise, branching process, computer simulation

I. INTRODUCTION

In the stochastic process such as the Markov process, an event in the process is strongly influenced by the event happened just before. The time series composed of these events shows usually a 1/f²-like spectrum. In the present work, a medium, in which many particles exist and each single particle may be divided into several particles and be absorbed, is considered, where the particle number at a moment is decided stochastically by the number just before. If these particles are observed by a detector, the detecting events may show another kind of stochastic behavior. This is the basic idea of this work and is illustrated in Fig. 1.

The particle numbers at time $t_1$ and $t_2$ in Fig. 1 are 7 and 10, respectively. A detection may correlated with another detection through the branching paths as given by $a$, $b$ and $c$ in the figure. The detection $d$ has no correlation with $a$, $b$ and $c$, because it is on another branching chain different from that of $a$, $b$ and $c$. The length of the path between the detections has statistical correlation with the physical time interval. The time interval, for example, between $a$ and $b$ is approximately equivalent to that between $b$ and $c$, but the correlation between $b$ and $c$ may be far weaker than that between $a$ and $b$, because the path between $b$ and $c$ is longer than that between $a$ and $b$.

Fig. 1. Chain of the branching process. The square, circle and black spot represent a particle immigrated randomly in the medium, absorption and detection of a particle, respectively. The path of a particle is shown by a full line.

II. DETECTION PROBABILITY

We suppose a medium in which a particle may be subjected to absorption, branching multiplicative reaction, and detection with the probabilities $\lambda_a$, $\lambda_m$ and $\lambda_d$, respectively. The statistics of the particles and detections in the medium can be obtained in close forms. Their detailed mathematical expressions are given in my previous works [1] and [2] by using the branching process model. In the present work, we
focused only on the case that exactly two particles are produced by a branching reaction, a detection of a particle has no influence on the particle number and the absorption rate is equivalent to the branching rate, i.e., $\lambda_a = \lambda_m$. The case that a particle is absorbed by detection and the case of $\lambda_a \neq \lambda_m$ are discussed in [1], [2], [3] and [4].

We consider the probability $P_k(m,n,t)$ that $m$ detection counts have been recorded during the time interval $(0, t)$ and $n$ particles are found in the medium at time $t > 0$ after $k$ particles exist at $t = 0$. It is rather complicated to obtain this probability for every single value of $m>0$, but the probability $P_k(0,n,t)$ can be described closely as

$$P_k(0,n,t) = K_k^{(0,n)} = \sum_{i=0}^{n} p(0,l,t) \cdot K_{k-1}^{(0,n-l)},$$  \hspace{1cm} (1)

where $K_0^{(0,i)} = \delta_{i,0}$  \hspace{1cm} (2)

and

$$p(0,l,t) = \begin{cases} \eta_0 \xi_0 V & \text{for } l = 0 \\ \eta_0 \xi_0 \eta_0(e^{-\eta_0 t})/(\eta_0 - \xi_0) e^{-\eta_0 t} & \text{for } l = 1 \\ V \cdot p(0,l-1,t) & \text{for } l \geq 2 \end{cases},$$  \hspace{1cm} (3)

Here

$$V = \frac{1-e^{-\eta_0 t}}{\eta_0 \xi_0 e^{-\eta_0 t}},$$  \hspace{1cm} (4)

$$\theta_0 = \lambda_a \sqrt{\varepsilon(\varepsilon + 4)},$$

$$\eta_0 = \frac{(1+\varepsilon + \sqrt{\varepsilon(\varepsilon + 4)})}{2},$$

$$\xi_0 = \frac{(1+\varepsilon - \sqrt{\varepsilon(\varepsilon + 4)})}{2},$$

and

$$\varepsilon = \frac{\lambda_a}{\lambda_m}.$$

The probability that we get some detections during the time interval $(0, t)$ and $n$ particles are found in the medium at $t > 0$ after we had $k$ particles at $t = 0$ is given by

$$\sum_{m=n}^{\infty} P_k(m,n,t) = P_k(n,t) - P_k(0,n,t),$$  \hspace{1cm} (7)

where $P_k(n,t)$ is the probability that $n$ particles are found in the medium at time $t > 0$ after we had $k$ particles at $t = 0$. When the probability (7) is much smaller than $P_k(0,n,t)$, the probability recording more than two counts can be negligible and the following relation holds approximately,

$$P_k(1,n,t) \approx P_k(1,n,t) - P_k(0,n,t).$$  \hspace{1cm} (8)

The probability $P_k(n,t)$ is given in [1], [2] and [3] as

$$P_k(n,t) = K_k^n = \sum_{l=0}^{n} p(l,t) \cdot K_{k-1}^{n-l},$$  \hspace{1cm} (9)

where

$$K_0^{l} = \delta_{l,0},$$  \hspace{1cm} (10)

$$p(l,t) = \begin{cases} W & \text{for } l = 0 \\ \left(\frac{W}{\lambda_a t}\right)^{l-1} & \text{for } l = 1 \\ W \cdot p(l-1,t) & \text{for } l \geq 2 \end{cases},$$  \hspace{1cm} (11)

and

$$W = \frac{\lambda_a t}{\lambda_a t + 1}. $$  \hspace{1cm} (12)

III. COMPUTER SIMULATIONS

Whether a particle detection has occurred or not in a very short time interval was decided successively by using the Monte Carlo method with the probabilities described in (1), (8) and (9), and the series formed by the time intervals between two successive detections (count series) was generated in the case of $\varepsilon = 1$. The obtained series was analyzed using the fast Fourier transformation technique (FFT) and the power spectral density (PSD) was calculated. One of the results is shown in Fig. 2, where the frequency is related to the detection counts not to time. The PSD behaves like a $1/f$ distribution for six decades or more of frequency when the series size is shorter than 2097152. The PSD behavior starts to deviate from the $1/f$ line in a low-frequency range when the series size is over 4194304. The condition $\varepsilon = 1$ means that each particle is detected once, on average, before absorbed
in the medium. When $\varepsilon < 1$, the PSD deviates from $1/f$ line at a shorter series size [3]. On the other hand, when $\varepsilon > 1$, the PSD is very similar to the case of $\varepsilon = 1$ [3].

**IV. RESULTS AND DISCUSSIONS**

There are at least two limitations on performing the computer simulations. The existing particle number should be avoided to be zero, because no branching will arise from no particle situation. A very large number of particles takes unreasonably long time to process on a computer, and so it should be set a limit on the number of particles. In the present simulations the upper limit of the number of particles (maximum particle number) was set to 1000. The effect of the limitation of the particle number on the PDS was examined, the results of which is shown in Fig. 3, where the count series were generated under the same condition with the case in Fig. 2 but the upper limits were set to 200, 400 and 700. As can be seen in Fig. 3, the frequency range with the $1/f$ behavior of the spectrum is so sensitive to the limit, and is increasing steadily with the maximum particle number. It indicates clearly that the frequency range with the $1/f$ behavior can be extended more if the upper limit is set to larger than 1000. It is not sure that this trend in Fig. 3 continues on and on endlessly, but if the limit is set up to 2000 or more, the PSD may behave like $1/f$ for about eight decades of frequency.

In order to see the above expectation, two count series with the upper limits set to 2000 and 3000 were generated in the case of $\varepsilon = 1$. Their obtained results are shown in Figs. 4 and 5, respectively. In Fig. 4, the PSD of the series size of 33554432 behaves like a $1/f$ distribution, but it turns off from the $1/f$ line in a low-frequency range at a longer series size. In Fig. 5, when the series size is 67108864, the PSD has a $1/f$ distribution over seven decades of frequency, but at a longer size of the series it is not clear whether the PSD deviates from the $1/f$ line or not. It was difficult to generate a fully long series in this case because of limited computing time, and so the statistical precision of the FFT results in Fig. 5 is insufficient. It can be said, however, that the frequency range having the $1/f$ behavior spreads out wider compared with the case that the upper limit is
2000.

V. CONCLUSION

The branching process model has been applied to discuss the $1/f$ problem, and the $1/f$ behavior has been demonstrated in the spectrum of the obtained series in a wide range, as wide as seven decades, of frequency. This frequency range is expected to expand up to eight decades or more by simulating on a higher-speed computer. It is expected, from the present results, that the $1/f$ behavior may be realized by observing a part of the familiar $1/f^2$ phenomena.

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Thomson scattering in inhomogeneous plasma:
The Role of the Fluctuation-Dissipation Theorem

Viacheslav V. Belyi
Theoretical Department
IZMIRAN, Russian Academy of Sciences
Troitsk, Moscow, Russia
email address: slava.belyi(at)gmail.com

Abstract—A self-consistent kinetic theory of Thomson scattering of an electromagnetic field by a non-uniform plasma is derived. We show that not only the imaginary part, but also the time and space derivatives of the real part of the dielectric susceptibility determine the amplitude and the width of the Thomson scattering spectral lines. As a result of inhomogeneity, these properties become asymmetric with respect to inversion of the sign of the frequency. Our theory provides a novel and unique method of a remote probing and measurement of electron density gradients in plasma; this is based on the demonstrated asymmetry of the Thomson scattering lines.

Index Terms—non-equilibrium fluctuations, FDT, Thomson scattering

I. INTRODUCTION

When an electromagnetic wave propagates in a plasma, its interaction with fluctuation oscillations of the plasma may result in scattering of the wave, which can be accompanied by a change in its frequency and wave vector. The intensity of scattered waves depends on both the intensity of the incident wave and the level of plasma fluctuations. Since the spectrum of plasma fluctuations exhibits sharp maxima at proper plasma frequencies, the spectrum of scattered waves will also exhibit sharp maxima at frequencies differing from the frequency of the incident wave by the according frequencies of the plasma fluctuations. The shift, width and shape of spectral lines carry information on such parameters of the plasma as its density, temperature, mean velocity, ion composition etc. A method of remote probing of a plasma, termed Thomson scattering, is a powerful plasma diagnostic tool that is widely employed in measurements of plasma parameters over a fairly broad range of plasma densities from the ionosphere to strongly coupled plasma. In such measurements the plasma must be transparent to the probe electromagnetic radiation. This may be microwave [1], laser [2] or X-ray radiation [3].

The differential Thomson scattering cross section, within an elementary solid angle $d\theta$ and for a frequency interval $d\omega'$ is described by the expression [4]:

$$
\frac{d\Xi}{d\theta} = \frac{1}{4\pi} \left( \frac{e^2}{m_e c^2} \right) \frac{\omega'^2}{\omega^2} \sqrt{\frac{\epsilon(\omega')}{\epsilon(\omega)}} (1+\cos^2 \theta) (\delta n_e \delta n_e)_{k\omega} d\omega d\omega',
$$

where $k = k' - k_0$, $\omega = \omega' - \omega_0$; $k_0$, $k'$, $\omega_0$, $\omega'$ are the wave vectors and the frequencies of the incident and scattered electromagnetic fields, $\theta$ - is the scattering angle between $k'$ and $k_0$. Thus, the problem reduces to finding the spectral characteristics of electron density fluctuations $(\delta n_e \delta n_e)_{k\omega} = S(k, \omega)$ - the dynamic electron structure factor (structure factor). The theory of equilibrium and nonequilibrium plasma fluctuations was successfully developed in the second half of the past century [5]–[8]. In accordance with the Poisson equation, the structure factor in a spatially homogeneous system is directly linked to the electrostatic field fluctuations. In thermodynamic equilibrium, the electrostatic field fluctuations satisfy the famous Callen-Welton Fluctuation-Dissipation Theorem (FDT) [9], linking their intensity to the imaginary part of the response function and to the temperature T. A first comprehensive exposition of the state-of-the-art and application of the FDT and response functions to plasma is presented by Golden and Kalman [10]. In the case of plasma physics, if the electric field $\delta E$ is considered to be the fluctuation variable $x$, then the response function $\alpha(\omega)$ is inversely proportional to the dielectric permittivity of the plasma $\epsilon(\omega, k)$. In this case the Callen-Welton formula assumes the form:

$$
(\delta E \delta E)_{k\omega} = \frac{8\pi T \Im \epsilon(k, \omega)}{\omega |\epsilon(k, \omega)|^2}.
$$

The matter becomes more delicate even in the local equilibrium case. We have indeed shown [11], that in the collisional regime the Callen-Welton formula should be revised. There then appear new terms explicitly displaying dissipative non-equilibrium contributions and containing the interparticle collision frequency, the differences in the temperatures and the velocities, and also functions of the real parts of the dielectric susceptibilities. Taking into account this additional correlation for non-isothermal plasma may result in corrections to the measured temperatures amounting to tens of percents.

Eq. (2) refers to the steady state, for a space uniform system. However, it is not evident that the plasma parameters can be kept constant in both space and time. Inhomogeneities in space and time of these quantities will certainly also contribute to the fluctuations. Hence it is challenging to formulate the generalization of the FDT for inhomogeneous plasma and reformulate accordingly the results for the Thomson scattering.

The FDT for a local equilibrium state was proved by Balescu [12]. The parameters of a system in a local equilib-
rium state can be changed adiabatically on a scale greater than the particle mean free path. Inhomogeneity and nonstationarity of plasma fluctuations are manifested via a non-local dependence upon time [13] and coordinates [14]. The FDT for a non-local plasma was given in our paper [15]. A generalization of the Callen-Welton formula for systems with slowly varying parameters is presented in [16].

In the present paper, applying the Klimontovich-Langevin approach [17] and the time-space multiscale technique, we show that not only the imaginary part but also the derivatives of the real part of the dielectric susceptibility determine the amplitude and width of spectral lines of the electrostatic field fluctuations and of the structure factor, as well. As a result of the inhomogeneity, these properties become asymmetric with respect to inversion of the sign of the frequency. In the kinetic regime the dynamic electron structure factor is more sensitive to space gradients than the spectral function of the electrostatic field fluctuations. Note that for simple fluids and gases a general theory of hydrodynamic fluctuations for nonequilibrium stationary inhomogeneous states has been developed in [18], [19]. In particular, it has been found that there exists an asymmetry of the spectrum for Brillouin scattering from a fluid in a shear flow or in a temperature gradient. The situation for the plasma problem we are considering is, however, quite different.

II. Results

To treat the problem, a kinetic approach is required, especially when the wavelength of the fluctuations is larger than the Debye wavelength. To derive nonlocal expressions for the spectral function of the electrostatic field fluctuation and for the structure factor we adopt the Klimontovich-Langevin approach to describe kinetic fluctuations [17]. A kinetic equation for the fluctuation $\delta f_a$ of the one-particle distribution function (DF) with respect to the reference state $f_a$ is considered. In the general case the reference state is a nonequilibrium DF which varies in space and time both on the kinetic scale (free path $l_\text{ci}$ and interparticle collision time $\nu_\text{ci}^{-1}$) and, also, on the larger hydrodynamic scales. These scales are much larger than the characteristic fluctuation time $\omega^{-1}$. In the nonequilibrium case we can, therefore, introduce a small parameter $\mu = \nu_\text{ci}/\omega$, which allows us to describe fluctuations on the basis of a multiple space and time scale analysis. Obviously, the fluctuations vary on both the "fast" $(r,t)$ and the "slow" $(\mu r, \mu t)$ time and space scales: $\delta f_a(x,t) = \delta f_a(x,t,\mu t, \mu r)$ and $f_a(x,t) = f_a(p,\mu t, \mu r)$. Here $x$ stands for the phase-space coordinates $(r,p)$. The Langevin kinetic equation for $\delta f_a$ has the form [17]

$$\hat{L}_{\text{ext}}(\delta f_a(x,t) - \delta f_a^S(x,t)) = -e_a \delta E(r,t) \frac{\partial f_a(x,t)}{\partial p} - \delta I_\text{a},$$

(3)

where $e_a$ is the charge of the particle of species $a$, $\delta E$ is the electrostatic field fluctuation, and the operator $\hat{L}_{\text{ext}}$ is defined by

$$\hat{L}_{\text{ext}} = \frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial r} + \hat{G}_a(x,t); \hat{G}_a(x,t) = e_a E \frac{\partial}{\partial p} - \hat{I}_\text{a},$$

(4)

and $\delta I_\text{a}$ is the linearized collision operator. A model collision operators for plasmas is presented in [20]. The Langevin source $\delta f_a^S$ in Eq. (3) is determined by the following equation [17]:

$$\hat{L}_{\text{ext}} \delta f_a(x,t) - \delta f_a^S(x',t') = e_a \delta(t-t') \delta(x-x') f_b(x',t').$$

(5)

The solution of Eq. (3) has the form

$$\delta f_a(x,t) = \delta f_a^S(x,t) - \sum_b \int d\chi \int_{-\infty}^{t} dt' G_{ab}(x,t,x',t') e_b \delta E(r',t') \frac{\partial f_b(x',t')}{\partial p}.$$

(6)

where the Green function $G_{ab}(x,t,x',t')$ of the operator $\hat{L}_{\text{ext}}$ is determined by

$$\hat{L}_{\text{ext}} G_{ab}(x,t,x',t') = \delta_{ab} \delta(x-x') \delta(t-t'),$$

(7)

with the causality condition:

$$G_{ab}(x,t,x',t') = 0,$$

(8)

Thus, $\delta f_a(x,t) = \delta f_a^S(x,t)$ and $G_{ab}(x,t,x',t')$ are connected by the relation:

$$\delta f_a(x,t) = G_{ab}(x,t,x',t') f_b(x',t'),$$

(9)

For stationary and spatially uniform systems the DF $f_a$ and the operator $\Gamma_\text{a}$ do not depend on time and space. In this case, the dependence on time and space of the Green function $G_{ab}(x,t,x',t')$ is manifest only through the difference $t-t'$ and $r-r'$. However, when $f_a(p,\mu t, \mu r)$ and $\Gamma_\text{a}(p,\mu t, \mu r)$ are slowly varying quantities in time and space, and when nonlocal effects are considered, the time and space dependence of $G_{ab}(x,t,x',t')$ is more subtle:

$$G_{ab}(x,t,x',t') = G_{ab}(p, p', r-r', t-t', \mu r', \mu t').$$

(10)

For the homogeneous case this non-trivial result was obtained for the first time in our previous work [13]. This result was extended to inhomogeneous systems [14]. Here we want to stress that the nonlocal effects appear due to the slow time and space dependencies $\mu r'$ and $\mu t'$.

In the first order expansion with respect to $\mu$ from Eq. (6) and the Poisson equation

$$\frac{\partial E(r,t)}{\partial t} = -\frac{\partial}{\partial r} \sum_b e_b \int \frac{1}{|r-r'|} \delta f_b(x',t) dx',$$

(11)

follows the generalized Fluctuation-Dissipation relation [15]

$$\langle \delta E(\delta E) \rangle_{k,\omega} =$$

$$= \sum_a \frac{\delta \pi T_a m(1+i\mu \frac{\partial}{\partial \omega} - \frac{\partial}{\partial t} - i\mu \frac{\partial}{\partial \omega} + \frac{\partial}{\partial \mu} \chi_a(k,\omega,\mu,\mu t)}{\omega_a \left(1+i\mu \frac{\partial}{\partial \omega} - \frac{\partial}{\partial t} - i\mu \frac{\partial}{\partial \omega} + \frac{\partial}{\partial \mu} \chi(k,\omega) \right)^2}$$

(12)
where \( \omega_a = \omega - kV_a \), and \( \chi_a(k, \omega, \mu_r, \mu_t) \) is the susceptibility for a local-equilibrium plasma.

The effective dielectric function \( \varepsilon(\omega, k) \) in the denominator of Eq. (12) defines the spectral properties of the electrostatic field fluctuations and imaginary part of the susceptibility \( \chi(k, \omega) \) determines the half-width of the spectral line \( \delta \varepsilon(\varepsilon E)_{k, \omega} \) near the resonance:

\[
\gamma = (\Im \chi + \mu \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu} \Re \chi - \mu \frac{\partial}{\partial \mu} \frac{\partial}{\partial k} \Re \chi) / \frac{\partial}{\partial \omega} \Re \chi. \tag{13}
\]

In Eq. (13) there appear additional first-order terms of the small parameter \( \mu \). It is important to note that the imaginary part of the dielectric susceptibility is now replaced by the real part, which in the plasma resonance may be greater than the imaginary part by the same factor \( \mu^{-1} \). Therefore, the second and third terms in Eq. (13) in the kinetic regime have an effect comparable to that of the first term. Second-order corrections in the expansion in \( \mu \) only appear in the imaginary part of the susceptibility, and they can be reasonably neglected. It is therefore sufficient to retain the first-order corrections to resolve the problem. The half-width of the spectral lines Eq. (13) is affected by new nonlinear terms. They are not related to Joule dissipation and appear because of an additional phase shift between the induction vector and the electric field. This phase shift results from the finite time needed to set the polarization in the plasma with dispersion [21]. Such a phase shift in the plasma with space dispersion appears due to the medium inhomogeneity. For the case when the system parameters are homogeneous in space but vary in time, the correction to the half-width of the spectral lines in Eq. (13) is still symmetric with respect to the change in sign of \( \omega \). However, when the plasma parameters are space dependent this symmetry is lost. The real part of the susceptibility \( \chi(k, \omega) \) in Eq. (13) is an even function of \( \omega \). This property implies that the contribution of the space derivative to the expression for the width of the spectral lines is odd function of \( \omega \). Moreover, this term gives rise to an anisotropy in \( k \) space.

For the spatially homogeneous case there is no difference between the spectral properties of the longitudinal electric field and of the electron density, because they are related by the Poisson equation. This statement is no longer valid when an inhomogeneous plasma is considered. Indeed the longitudinal electric field is linked to the particle density by the nonlocal relation (11). In the latter case, an analysis similar to that made above can also be performed for the particle density.

\[
S^a(k, \omega) = (\delta n_e(\delta n_e)_{k, \omega}) = \frac{2n_e k^2}{\omega_i k_D^2} \left[ 1 + \frac{\chi_a(k, \omega)}{\sigma(k, \omega)} \right]^2 \Im \chi_a(k, \omega) + \left( \frac{\chi_a(k, \omega)}{\sigma(k, \omega)} \right)^2 \left( \frac{T_i}{T_e} \frac{2 \pi n_e k^2}{\omega_i k_D^2} \right) \Im \chi_a(k, \omega), \tag{14}
\]

where \( k_D \) is the inverse Debye length,

\[
\sigma(k, \omega) = 1 + \sum_a \chi_a(k, \omega), \tag{15}
\]

\[
\bar{\chi}_a(k, \omega) = (1 + i \mu \frac{\partial}{\partial \omega} \frac{\partial}{\partial \mu} - i \mu \frac{\partial}{\partial \mu} \frac{\partial}{\partial k} \chi_a(k, \omega, \mu_r, \mu_t)). \tag{16}
\]

The inhomogeneous corrections \( \frac{1}{k} \frac{\partial}{\partial k} k_j \frac{\partial}{\partial \mu} \Re \chi_a \) in Eq. (15) are not the same as in Eq. (13) \( \frac{1}{k} \frac{\partial}{\partial \mu} \frac{\partial}{\partial \mu} \Re \chi_a \). The origin of this difference is that the Green functions for the electrostatic field fluctuations and density particle fluctuations are not the same in an inhomogeneous situation. As above, we can expand \( \bar{\varepsilon}(\omega, k) \) near the plasma resonance \( \omega = \omega_L \). Thus, for the Langmuir line,

\[
S^r(k, \omega) = \frac{\gamma}{(\omega - \omega_L sgn \omega)^2 + \gamma^2 \omega^2 k_D^2 \partial Re \chi / \partial \omega |_{\omega = \omega_L}^2, \tag{17}
\]

where

\[
\gamma = [\Im e - \mu \frac{\partial^2 \Re e}{\partial \mu \partial \omega} - \mu \frac{\partial}{\partial \mu} \frac{\partial}{\partial k} k_j \frac{\partial}{\partial k} \Re e] / \frac{\partial \Re e}{\partial \omega} |_{\omega = \omega_L sgn \omega \omega_L}, \tag{18}
\]

is the half-width of the structure factor. An estimate for the plasma mode is then:

\[
\gamma = [\nu_e + \frac{2n_e}{n} \omega_L + \frac{\omega_L}{nk^2} \frac{k \delta n}{\partial k} (1 + 9k^2/k_D^2) sgn \omega] / 2. \tag{19}
\]

This from equation we see that the inhomogeneous correction in Eq. (19) is greater than the one in Eq. (13) by the factor \((1 + k^2/k_D^2)/2 \). For the same inhomogeneity; i.e., the same gradient of the density, we plot \( S^r(k, \omega) \) together with \( \delta \varepsilon(\varepsilon E)_{k, \omega} \) as functions of frequency (Fig. 1).

![Fig. 1. The electron structure factor \( S^r(k, \omega) \) (solid line) and the spectral function of electrostatic field fluctuations \( \delta \varepsilon(\varepsilon E)_{k, \omega} \) (dashed line) as a function of frequency. \( k_D/k = 3 \); \( k \frac{\delta n}{\partial n} = \nu_e k_D/27 e \gamma \).](image)

This figure shows that the asymmetry of the spectral lines is present both for \( S^r(k, \omega) \) and \( \delta \varepsilon(\varepsilon E)_{k, \omega} \). However, this effect is more pronounced in \( S^r(k, \omega) \) than in \( \delta \varepsilon(\varepsilon E)_{k, \omega} \). Such asymmetry has been indeed detected in inhomogeneous plasma. The asymmetry of lines \( S^r(k, \omega) \) can be used as a new diagnostic tool to measure local gradients in the plasma by Thomson scattering.

The Langmuir line (17) takes the Lorentz form and the amplitude of the spectral line \( A \) is inversely proportional to its width

\[
A = \frac{\pi n_e k^2}{18 k_D}. \tag{20}
\]
The amplitude of the Langmuir line is seen to be more sensitive to the electron density gradient, than the line width. For example, in the case of a density gradient equal to \( \partial n/n = \omega_{ei}/9v T \) and \( k_D = k \), the red line width decreases by 67 percents, while at the same time the amplitude becomes 3 times larger (see Fig 1). From Eqs. (19) and (20) quite a simple formula for calculation of the electron density gradient from the Thomson scattering spectrum follows [24], [25]:

\[
\frac{\partial n}{n \partial r} = \frac{\nu_{ei} A^R - A^B}{A^R + A^B} \frac{k_D}{k^2 + 9} \left( \frac{\gamma_R + \gamma_B}{\omega_L} \right) \left( A^R + A^B \frac{k_D^2}{k^2 + 9} \right)
\]

(21)

where \( A^R, A^B \) and \( \gamma_R, \gamma_B \) are the amplitudes and the half-widths of the red and blue Langmuir satellites, respectively (Fig. 1).

Thus, intensity and width measurements of the red and blue lines of the spectrum allow to determine the scalar product of the electron density gradient and the scattering vector at a given point. To determine the vector \( \partial n/n \partial r \) it is sufficient to measure the radiation scattered in three directions simultaneously.

From (21) it follows that the Knudsen number \( Kn = \frac{\nu_{ei} k_D}{\omega_L} \) is equal to

\[
Kn = \frac{A^R - A^B}{A^R + A^B} \left( \frac{k_D}{k^2 + 9} \right) \cos \beta
\]

(22)

where the amplitudes \( A^R \) and \( A^B \) are in arbitrary units, \( \beta \) is the angle between the electron density gradient and the scattering vector \( k \).

III. CONCLUSION

A first-principle kinetic theory of Thomson scattering in a non-uniform plasma is constructed, which agrees with the basic FDT and provides quantitatively correct results. Moreover, our theory provides a novel and unique method for remote probing and measurement of electron density gradients in plasma; this is based on the demonstrated asymmetry of the Thomson scattering lines in an inhomogeneous plasma. Such asymmetry has been indeed detected in spectroscopic studies of inhomogeneous plasma flows in magnetic traps [22], [23]. This method may be important for numerous technological applications, e.g. for the tokamak [26], for x-ray Thomson scattering on inhomogeneous targets [27] etc.

REFERENCES

Quantum 1/f Noise --a Decoherence Phenomenon

Peter H. Handel
Department of Physics and Astronomy and Center for Nanoscience, University of Missouri--St. Louis (UMSL), St. Louis, USA, handel@umsl.edu

Abstract—Our quantum theory of fundamental 1/f noise anticipated since 1975, and used, the theory of decoherence, which was developed two decades later. The Quantum Theory of the conventional and coherent quantum 1/f effects started precisely from the mixture of states selected by the interaction with the environment, with random phases, states later called “einselected states.” The two quantum 1/f effects are thus both “infrared divergence” and “decoherence” phenomena that include the interaction with the rest of the world. This misunderstanding, initiated by van Kampen and two others, delayed progress 40 years.

Keywords—1/f noise, decoherence, quantum 1/f effects, quantum 1/f noise, einselection, infrared divergence, phase noise

I. INTRODUCTION

The Quantum Theory of 1/f Noise is a new aspect of quantum mechanics, introduced [1] 1975 as an infrared divergence phenomenon in both of its forms. It is also a decoherence phenomenon in both, the conventional and coherent Quantum 1/f Effects (Q1/fE). Indeed, in the 1980’s, the Quantum Theory of 1/f Noise was misunderstood by some for many reasons, but finally for only one reason: In general, the bremsstrahlung energy loss components emerging from any scattering, or from other processes, are known not to interfere with the main wave function. They are orthogonal, with phases differing by 90 degrees in a certain representation. Therefore, some said that my theory, based on such interference, is not valid. But the experiment always verified our Quantum 1/f (Q1/f) formulas in all domains, universally.

Their mistake was ignoring the universal phenomenon of decoherence. Indeed, soon after the superposition state with 90 degrees phase differences is created, decoherence scrambles the phases, randomizing them, as we always assumed in our theory of the conventional and coherent quantum 1/f effects, which we introduced. In general, in times of the order of 10^8 s the superposition of states interacts with the environment and is replaced by a mixture of “einselected” states [2]-[4]. The latter are states that are much more stable in interaction with the environment. This is also why the Schrödinger cat is never found half dead and half alive, and why we don’t have good quantum computers!

This may have been understood now by the scientific community, particularly after a 2013 paper [5], and there are no other objections left to our theory and to its simple, practical, universal engineering formulas. These are applicable to all high-tech applications, for materials, devices and systems, existing in any domain in time and space, including also sensors or high stability resonators, oscillators and clocks. It’s as basic as time and space, as it also happens in the macroscopic world with gravitons as infraquanta, instead of photons. It shapes all existence in time and space it is the universal way of existence itself. It is caused, in final analysis, by the interaction of our system with the rest of the world.

Alternatively, as we show below, the same result can be obtained in the Quantum Information Theory (QIT) approach, by considering this environment under our observation threshold, i.e., by simply ignoring it and possible negative entropy states left in it. The latter not only fail to provide us with the expected choice in a potential measurement, but have instead a pre-selected outcome.

II. DECOHERENCE

As pointed out by us [6] since 1982, the phases of the bremsstrahlung energy loss components of the outgoing wave function in a scattering process are randomized, as implied by the density matrix formalism used in that paper in anticipation of later results of the theory of Decoherence that was developed later, in the eighties.

Decoherence is the process of continuous, measurement-like, interaction of the quantum system with its environment. It quickly replaces exquisite, often complex (coherent) superpositions of states resulting from interaction processes like scattering, with an (incoherent) mixture of simpler states, that are much more stable in interaction with the environment, and have thus a more objective existence, because one can find them in repeated measurements over times much larger than the decoherence time. These states are called the “einselected pointer states” [2]-[4], [7], and the process in which the environment selects them as the relatively stable descendants of the initial coherent state is known as “einselection.” For scattering of an electron or other charged particle on a center of force or arbitrary potential, this mixture of einselected outgoing Heisenberg states is

\[
\phi(x) = \frac{C}{\sqrt{\pi}} e^{iKx} [1 + 2\sum b(k,l)a^*_{k,l} e^{i\gamma_{k,l}}].
\]

Here \(C\) is an amplitude factor, \(K\) the electron wave vector magnitude, \(m\) its mass, \(x\) the distance from the scattering center, \(b(k,l)=|b(k,l)|e^{i\gamma_{k,l}}\) the bremsstrahlung amplitude for photons of wave vector \(k\) and polarization \(l\), with a random phase \(\gamma_{k,l}\) while \(a^*_{k,l}\) is the corresponding photon creation operator, allowing the photon state to be created from the vacuum if Eq. (1) is inserted into Eq. (9) below. The momentum magnitude loss \(q\hbar = mck/K=2\pi mfK\) is necessary for energy conservation in the Bremsstrahlung process with hf being the bremsstrahlung energy loss. This yields the quantum 1/f noise as known [1], [8]-[11].

All this was found decades after the discovery of the quantum 1/f noise in the Fall of 1974 [1], [8]-[11], anticipated the decoherence theory by decades, introducing for the first time the einselected states ad hoc, with different name, through physical insight, as the true mixture of states expected to emerge from the scattering of a charged particle like an electron, with interaction with the electromagnetic field, including thus the presence of bremsstrahlung and virtual photons of arbitrarily low frequencies. This author gave these anticipated de facto einselected states, the name of “stochastic Schrödinger field,” introducing random phases.
of the bremsstrahlung energy loss components resulting in practice from the outgoing state in any scattering experiment involving an electron or any other charged particle in interaction with the environment.

Aldert van der Ziel had a deep abiding respect for the fundamental 1/f noise. He had struggled his whole life to understand the physical nature and origin of this universal phenomenon. As soon as he realized the conceptual simplicity and universality of the quantum 1/f (Q1/f) derivation, as a property of the new notions of physical cross sections and process rates, he started a good cooperation with this author and with a dozen of graduate students, to verify the Q1/f formulas in all situations in which 1/f fluctuations are observed. In his final invited review paper [12], Aldert wrote: Our project cannot check the validity or invalidity of Handel’s derivation of his predictions for $a_{0n}$. This is the domain of the theoreticians. They have every right to criticize the derivation and replace it by a better one. In the latter case, they should see to it that their prediction for $a_{0n}$ agrees with Handel’s prediction for $a_{0n}$ when the latter has been verified experimentally. Up to now this has not been done by them. It is difficult for some scientists to understand how a theory that is in their opinion incorrect can give correct predictions. It must be emphasized that only experiment can decide whether a conclusion is correct or incorrect. In our situation experiments decided that the predictions were right, and I see no way to avoid this conclusion. We now understand the wisdom here better than ever ....

At the end of the Abstract (No. 145 in Appendix 3) of the paper presented by C.M. Van Vliet at the Xth (ICNF) Int. Conf. on Noise in Physical Systems in Budapest, Aug. 21-25, 1989, C.M. Van Vliet wrote: “In retrospect we believe that the ’non-conventional’ approach in Handel’s papers, based on a ’stochastic Schrödinger field’ is hereby mainly vindicated; in fact Handel has added a novel aspect of quantum mechanics, which stands out by its simplicity, as compared to the very lengthy perturbation expansions used in the diagrammatic as well as the present quantum field theoretical approaches.” This was the first time that the concept of a “new aspect of quantum mechanics” was introduced. On the solid basis of the theory of decoherence, we see today how its later developments, already included in the conventional and coherent Q1/fE, were also understood and anticipated by Van Vliet and Van der Ziel.

III. QUANTUM INFORMATION THEORY APPROACH

Let’s simplify our world and assume only one electromagnetic mode of the universe would be present, with frequency $\omega$ and wave vector $k$. Consider the field mode in its ground state and a pair of 2 incoming identical charged particles with the same well-defined wave vector being scattered by some potential. Both the initial (incoming) and the final (scattered) state represent a pure state. The initial state is $|++\rangle_{0}$, where we reserved the first two arguments of the ket for the two electrons and the last (-) for the field. Due to the interaction with the field, the final state is $|>| = |++\rangle + \gamma|+->\rangle/\sqrt{2} + |+-\rangle/\sqrt{2} (\sqrt{1+\gamma^{2}})$, \( \text{where } \gamma = \text{the emission amplitude of a bremsstrahlung photon, i.e., for the excitation of the field oscillator from its ground state (-) to its first excited state (+).} \]

The presence of an energy loss with (-) and the persistence in the same energy state with (+). The third argument of the kets always labels the field oscillator, as mentioned.

The corresponding density operator of the pure state obtained is $\rho = |+\rangle<+|$. Its von Neumann entropy $S/k = \sigma = -\text{Tr}(\rho \log \rho)$ is zero.

A. Paradoxical Entropy Increase in 1/f noise

Ignoring the field oscillator, i.e., taking the trace of over the field oscillator label, we obtain a classically correlated system, a mixture of 2 pure states, described by the density operator

$$[|++><++| + |-+><-+|]/(1+\gamma^{2}).$$

The second term gives the Q1/fE at $F = \omega/2\pi$, as we show below. The corresponding entropy is

$$\text{log}(1+\gamma^{2}) - (\gamma^{2}\log\gamma^{2})/(1+\gamma^{2}) > 0 \text{ for } 0<\gamma^{2}<1.$$

The entropy of the system thus appears to have increased although according to quantum mechanics it cannot increase. Indeed, according to quantum mechanics, time evolution of a state occurs through a unitary transformation. The latter, however, is known to leave $\sigma = -\text{Tr}(\rho \log \rho)$ invariant.

B. Quantum Information Theory

QIT [29] solves this paradox. When two systems A and B with quantum von Neumann entropy $\sigma(A) = -\text{Tr}_{A}(\rho_{A}\log\rho_{A})$ and $\sigma(B) = -\text{Tr}_{B}(\rho_{B}\log\rho_{B})$ form a composite system AB of entropy $\sigma(AB) = -\text{Tr}_{AB}(\rho_{AB}\log\rho_{AB})$, we can prove that we have to write

$$\sigma(AB) = \sigma(A) + \sigma(B) = \sigma(A) + \sigma(AB),$$

in perfect analogy with classical entropies or information. We have introduced $\sigma(AB)$ as the von Neumann entropy of A conditional on B, or the entropy of A when we know B:

$$\sigma(A|B) = -\text{Tr}_{AB}(\rho_{AB}\log\rho_{AB}).$$

Here we have introduced the conditional density matrix $\rho_{AB} = \rho_{AB}(1_{A}\otimes\rho_{B})^{-1}$, the quantum analog similar to the classical conditional probability, where $\otimes$ is the tensor product in the joint Hilbert space and $\rho_{AB} = \text{Tr}_{B}(\rho_{AB})$ is the marginal density matrix obtained by taking a partial trace over the variables associated with A.

C. Negative entropy

The conditional entropy is usually negative in quantum entangled states. Finally, we introduce the quantum mutual entropy which represents the shared entropy, corresponding to the mutual information between A and B:

$$\sigma(A:B) = -\text{Tr}_{AB}(\rho_{AB}\log\rho_{AB}) = \sigma(A) + \sigma(B) - \sigma(AB),$$

where $\rho_{AB} = \rho_{AB}(1_{A}\otimes\rho_{B})^{-1}$.

Taking all logarithms in the base of 2, the entropies will be expressed in bits. Considering our paradoxical case discussed above for simplicity with $\gamma^{2}=1$, we obtain the following entropy diagram (Fig. 1) of our quantum mechanically entangled triplet of three systems comprising the charged particles A and B, as well as the field oscillator C.
IV. Simplified Derivation of the Conventional Quantum 1/\(f\) Effect

We start with the expression of the Heisenberg representation state \(|f>\) of \(N\) identical bosons of mass \(M\) emerging at an angle \(\theta\) from some scattering process with undetermined bremsstrahlung energy losses reflected in their one-particle waves \(\psi_i(\xi_i)|f> \Rightarrow (N!)^{1/2}\Pi^j\xi_j^3\psi_i(\xi_i)\psi_i^*(\xi_i)|0> = \Pi^j\xi_j^3\phi_i(\xi_i)|0>, \)

where \(\psi_i(\xi_i)\) is the field operator creating a boson with position vector \(\xi_i\) and \(|0>\) is the vacuum state, while \(|f0>\) is the vacuum field state with \(N\) bosons of position vectors \(\xi_i\) with \(i = 1...N\). All products and sums in this paper run from 1 to \(N\), unless otherwise stated.

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

\[ O(x_1,x_2) = \psi^*(x_1)\psi^*(x_2)\psi(x_2)\psi(x_1), \]

known as the operator of the pair correlation. Using the commutation properties of the boson field operators, we first calculate the matrix element

\[ N!\langle S(0)|f0> = \sum \Sigma_{mn} \delta(\eta-n\xi-2)\delta(\xi_n-\xi_1)\delta(\xi_m-x_2)\Sigma_{(i,j)}\Pi_{ij}\delta(\eta_j-\xi_j). \]

Here the prime excludes \(\mu=v\) and \(m=n\) in the summations and excludes \(i=m, i=n, j=\mu\) and \(j=v\) in the product. The summation \(\Sigma_{(i,j)}\) runs over all permutations of the remaining \(N-2\) values of \(i\) and \(j\). On this basis we now calculate the complete matrix element

\[ \langle f|O|f> = \langle f|S(0)|f0> = \sum \Sigma_{mn} \delta(\eta_n-x_1)\psi(\eta_n-x_1)\psi(\xi_n-x_2)\Sigma_{(i,j)}\Pi_{ij}\delta(\eta_j-\xi_j). \]

Here \(C\) is an amplitude factor, \(K\) the boson wave vector magnitude, \(b(k,l) = |b(k,l)|e^{i\gamma(k)}\) the bremsstrahlung amplitude for photons of wave vector \(k\) and polarization \(l\), with a random phase \(\gamma(k)\), while \(a_k\) is the corresponding photon creation operator, allowing the photon state to be created from the vacuum if Eq. (13) is inserted into Eq. (9). The momentum magnitude loss \(q=Mck/hK=Me/hK\) is necessary for energy conservation in the Bremsstrahlung process. Substituting Eq. (13) into Eq. (12), we obtain

\[ \langle f|O|f> = \langle f|S(0)|f0> + 2(N-1)\Sigma_{k|b(k,l)|^2[1+\cosq(x_1-x_2)]}, \]

where \(\Sigma_{k|b(k,l)|^2[1+\cosq(x_1-x_2)]}\) is the bremsstrahlung coefficient \(\gamma\) on \(f\), and from the summation over all field oscillators, as we see below.
where we neglected a small term of higher order in $b(k,l)$. To perform the angular part of the summation in Eq. (14), we calculate the current expectation value of the state in Eq. (13), and compare it to the well known cross section without and with bremsstrahlung

$$j = (hK/Mx^2)(1 + \Sigma_k)b(k,l)^2 = J_0(1 + \alpha Adf f),$$

where the quantum fluctuations have disappeared, where $\alpha = e^2/hc$ is the fine structure constant, $\alpha = (2\alpha/3\pi)(\Delta \mu/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent, and the $1/f$ dependence of the bremsstrahlung part displays the well-known infrared catastrophe, i.e., the emission of a logarithmically divergent number of photons in the low frequency limit. Here $\Delta \mu$ is the velocity change $h(K-K_0)/M$ of the scattered boson, and $f = c/k/2\pi$ the photon frequency. Eq. (14) gives

$$<\langle f|O|f\rangle> = \left[\frac{e^2}{\alpha} \sum_{N=1}^{N-1}\left[1 + \cos(q_{X1}X2)\right]\right],$$

which is the pair correlation function, or density autocorrelation function along the scattered beam with $df/df = dq/\Delta$. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency $f$. According to the Wiener-Khintchine theorem, we obtain the spectral density of fractional scattered particle density $p$, (current $j$, or cross section $\sigma$) fluctuations in frequency $f$ or wave number $q$ by dividing the coefficient of the cosine by the constant term $N(N-1)$:

$$\rho^2S_j(f) = \rho^2S_\sigma(f) = 2\alpha A/N \quad \text{[or]} \quad 2\alpha A/N(N-1) \quad \text{[for fermions]},$$

where $N$ is the number of particles or current carriers used to define the current $j$ whose fluctuations we are studying. Quantum $1/f$ noise is thus a fundamental $1/N$ effect.

The exact value of the exponent of $\Gamma$ in Eq. (17) can be determined by including the contributions from all real and virtual multiphoton processes of any order, and turns out to be $\alpha A - 1$, rather than $-1$, which is important only philosophically, since $\alpha A < 1$. The spectral integral is thus convergent.

For fermions we repeat the calculation replacing in the derivation of Eq. (11) the commutators of field operators by anticommutators, which finally yields in the same way

$$\rho^2S_j(f) = \rho^2S_\sigma(f) = 2\alpha A/\gamma(N-1),$$

which causes no difficulties, since $N \geq 2$ for particle correlations to be defined, and which is practically the same as Eq. (17), since usually $N = 1$. Eqs. (17) and (18) suggest a new notion of physical cross sections and process rates which contain $1/f$ noise, and express a fundamental law of physics, important in most high-tech applications [11] - [21].

V. DISCUSSION

We introduce the degree $D$ of quantum $1/f$ decoherence and will focus on the determination of the decoherence time scales. We determine $D$ from comparison of the calculated and measured $1/f$ noise, or phase noise levels close to carrier. These $D$ values can be determined in a wider class of materials, devices and systems, at various temperatures, in order to use the $D$ values for a new, independent method of investigating special materials, devices and systems, e.g., materials with lower dimensionality, for their transport properties and reliability, including the study of low-temperature behavior of $D$. A focus of future research is the application of these results to the improve quantum computers, by opening a new way of studying the decoherence.

In materials, devices and systems, $D$ is usually close to 1, or 100% for QED quantum $1/f$ noise. However, decoherence happens only to about 10% in beta radioactive decay [14], i.e., $D = 0.1$ in this case. It is also practically important to find how partial decoherence can reduce the observed $Q_1/f$ noise in devices.

Low frequency noise of GaSb layers on GaAs substrate

Lukasz Ciura
Department of Electronics
Fundamentals
Rzeszow University of Technology
Rzeszow, Poland
lciura@prz.edu.pl

Andrzej Kolek
Department of Electronics
Fundamentals
Rzeszow University of Technology
Rzeszow, Poland
akoleknd@prz.edu.pl

Iwona Sankowska
Institute of Electron Technology
Warsaw, Poland
isanko@ite.waw.pl

Agata Jasik
Institute of Electron Technology
Warsaw, Poland
ajasik@ite.waw.pl

Krzysztof Czuba
Institute of Electron Technology
Warsaw, Poland
kczuba@ite.waw.pl

Abstract—The low frequency noise of GaSb layer was studied with four-point probe method. Such measurements supported by numerical calculations allow for the identification of the non-resistance low frequency noise introduced by metal/interface (contact), as well as the resistance noise from near-contact area and the noise related to the entire layer can be evaluated. The low frequency noise related to contacts for Te-doped GaSb sample is significantly larger than for Be-doped samples with the comparable doping level. No low frequency noise from contacts and the layer was detected for highly Be-doped GaSb.

Keywords—GaSb, low frequency noise, contact noise, four-point probe method

I. INTRODUCTION

Among the III-V compounds, gallium antimonide (GaSb) is one of the most important for optoelectronic devices. It can be used as the basic component, e.g., InAs/GaSb superlattice or as the substrate. There are many various ternary and quaternary III-V compounds as well as superlattices lattice-matched to GaSb, which makes it an attractive substrate for growing both infrared (IR) emitters and detectors. However, growing IR detector on GaAs substrate instead of on GaSb has a few advantages [1] [2], e.g., better thermal conductivity, lower cost, and much lower absorption coefficient in infrared spectral range, which makes the conversion into immersion lens, and so increase the detectivity, possible. On the other hand, lattice mismatch exists between epitaxial GaSb layer and GaAs substrate, so the growing technique should be optimized to balance the strain and reduce density of dislocations [3]. It is possible provided the necessary feedback is obtained on the quality of the grown GaSb-on-GaAs layer. Among many available methods, measurement of the low-frequency (lf) noise is very interesting as it is extremely sensitive to the quality of both materials and devices [4]. Nevertheless, this is non-trivial technique, as a result the lf noise parameters were not included in Dutta et al. [5] comprehensive review covering physics and technology of gallium antimonide. The main difficulty in application of the lf noise technique in characterizing properties of GaSb layer is the extraction of the lf noise originating from this layer from the total measured noise. This is a challenge due to the high contact-related noise contribution, which can be much higher than the lf noise of the layer. It is not so unexpected because contact metal-semiconductor interface contains various defects, e.g.,

<table>
<thead>
<tr>
<th>Label</th>
<th>Carrier Concentration at 300 K [cm⁻³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D581, D585, D631, D580, D584</td>
<td>1×10¹⁷ (p), 7×10¹⁷ (p), 5×10¹⁷ (p), 2×10¹⁷ (n), 7×10¹⁷ (n)</td>
</tr>
</tbody>
</table>

This paper explores the idea of using low frequency noise as a tool for evaluation of various GaSb layers deposited on GaAs substrate. It is demonstrated that the contact/layer related noises can be evaluated, to some extent, with measurements and numerical calculations for the four point-probe samples.

II. EXPERIMENT

A. Samples

The GaSb layers were grown on the semi-insulating GaAs substrates using the interfacial misfit array growth method. The growth temperature was identical for all examined the layers. The thickness of GaSb layers was in the range of 1-1.3 μm. The samples differed both in doping type and concentration: D581, D585, and D631 are p-doped with beryllium (Be), while D580, D584 are n-doped with tellurium (Te). For electrical measurements the square-shape samples with four 1 mm in diameter In₉₅Zn₅ contacts located near the corners were prepared. The following procedure used. A 5×5 mm samples were degreased in 2-propanol and then placed in an oven with laminar N₂ or N₂+HCl flow. In₉₅Zn₅ fragments were etched in 1:1 HCl:H₂O solution for 60 s before being placed at the corners of the samples. Next, the annealing in N₂+HCl flow was started. After the temperature reached contact melting point hydrochloric acid vapor flow was turned off. Subsequently, the samples were further annealed in 300°C for 10 minutes. After that, the heater was turned off and the samples were allowed to cool. In general, this procedure forms ohmic contacts, which allow for Hall measurements. In Table I the carrier concentrations in GaSb layers obtained from this measurement are provided.

TABLE I. MEASURED CARRIER CONCENTRATION OF GASB LAYERS.
B. Four-point probe method

The low frequency noise of devices under test (DUT) was measured with four-point probe method. The setup for this measurement is presented in Fig. 1. The bias current was provided to the DUT current terminals (CT) from the quasi-current source formed by external dc voltage $U_i$ and the large-value resistance $R_0$ ($R_S$ was about fifty times greater than DUT resistance $R_{DUT}$). Power supply noise was suppressed by the filter sections. The resistance $R_{DUT}$ was known, because the voltage across the DUT, $U_{DUT}$, and bias current $I$ were continuously monitored. The voltage noise signal was acquired from the voltage terminals (VT) $V_1$-$V_2$ and amplified by two differential amplifiers, W1, W2. The cross-correlation technique \[7\] was then used to eliminate the (uncorrelated) equivalent input voltage noise of the amplifiers. In practice, the 30 dB lower signal could be detected (in acceptable averaging time) in comparison to the measurements with only one amplifier.

\[ S_{VT} = \frac{\sum (i_a j_b) S_{ij}}{I^2} \]  

(1)

In this equation, the lumped network model of DUT was adopted after ref. \[8\]. Namely, $i_a$ and $j_b$ are the currents in $\alpha$-branch of the original or adjoint networks, and $S_{ij}$ is the psd of the resistance fluctuations in this branch (local noise). The adjoint network is identical to the original network except for the current $I$ being provided through VT and the voltage noise being measured at CT. It stems from Eq. (1) that $S_{ij}$ is proportional to the bias current squared $I^2$. The bias independent quantity can be defined as:

\[ \frac{S_{VT}}{I^2} = S_a = \frac{\sum (i_a j_b)^2 S_{ij}}{I^2} \]  

(2)

The coefficient $(i_a j_b)^2/I^4$ in equation (2) can be interpreted as the amplification factor of the local noise $S_a$. Equation (2) was used to calculate the noise amplification factor for the geometry of examined samples with four finite size contacts near the sample corners. In the numerical calculations the DUT was modeled as two dimensional homogeneous ($r_a = r$) structure and node-voltage method was used to find local currents $i_a j_b$. In Fig. 3, the distribution of the coefficient $(i_a j_b)^2/I^4$ is shown for two different arrangements of the current and voltage terminals. In the first case (Fig. 3a), the current is provided diagonally to terminals AD, and the noise is measured at the same terminals. Such configuration is denoted as ADCTVT. As can be seen, in this case the noise amplification factor is significantly non-uniform and high in the near-contact areas. Consequently, the noise measured in this configuration is related almost exclusively to these areas. In the second case, the current is provided in the same way (AD terminals) but the voltage noise is measured at different pair (BC) – configuration ADCTBCVT. Results for this case are presented in Fig. 3b. The noise amplification factor is distributed more regularly. Consequently, the noise measured in this configuration is related to almost the entire sample, however, contributions from near-contact areas are still substantial.

\[ (i_a j_b)^2/I^4 \leq 0.1 \max [(i_a j_b)^2/I^4] \]

The four contacts of the DUT provide the possibility to measure $1/f$ noise in several configurations. For the arbitrarily placed current and the voltage terminals the spatially uncorrelated resistivity fluctuations produce fluctuations (resistance noise) at voltage terminals with psd given \[8\]

\[ S_{VT} = \sum (i_a j_b)^2 S_{ij} \]

(3)

Fig. 1. The setup for voltage noise cross-correlation measurements with four-point probe configuration.

Fig. 2. The measured power spectral densities of voltage fluctuations for sample D580 biased with constant current $I = 160$ $\mu$A at several temperatures.

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For resistance noise, i.e., the noise originated from resistance fluctuations, the reciprocity principle forces that power spectral densities measured in two reciprocal configurations, e.g., AD<sub>CT</sub>BC<sub>VT</sub> and BC<sub>CT</sub>AD<sub>VT</sub> should be equal. The principle is valid even when the sample is inhomogeneous, and contains local noise sources with different physical origin [9]. Breaking of this rule indicates that the measurement results are related to non-resistance noise of the contacts rather than to resistance noise of the semiconductor layer. The former can be explained e.g., by the depleted region of metal/semiconductor interface.

When the principle rule is obeyed the four-point probe allows for further evaluation of the noise sources homogeneity. For homogeneous distribution, i.e., for $S_{S_o} = S_s$, the frequency dependence of psds $S_p(f)$ measured in different configurations should remain unchanged, and their magnitudes should remain in a constant relation. For example for configurations AD<sub>CT</sub>BC<sub>VT</sub> ($S_{S_{AD}}$), BC<sub>CT</sub>AD<sub>VT</sub> ($S_{S_{BC}}$) and AD<sub>CT</sub>BC<sub>VT</sub> ($S_{S_{AD_{BC}}}$) the calculated ratio of psd magnitudes is $S_{S_{AD_{BC}}} = S_{S_{BC}}/S_{S_{AD}}$ ($\approx 2.9$). In other words, the noise measured on current contacts in reciprocal configurations should be equal. Any deviation from this relation means that the distribution of noise sources in the sample is inhomogeneous. In particular, larger values of this ratio indicate that noise sources located near the contacts are more prominent.

### III. RESULTS

Samples were tested for obeying the reciprocity rule by the measurements in complementary AD<sub>CT</sub>BC<sub>VT</sub>/BC<sub>CT</sub>AD<sub>VT</sub> configurations. Apart from these, noise was also measured on certain current contacts i.e. in XY<sub>CT</sub>VT configuration (XY = AD, AB, AC, BC, ...). Out of five DUTs only for samples D584, D585 reciprocity rule held. For sample D584 the ratio $S_{S_{AD}}/S_{S_{AD_{BC}}} \equiv S_{S_{BC}}/S_{S_{AD_{BC}}} \approx 1$, whereas for sample D585 $S_{S_{AD}}/S_{S_{AD_{BC}}} \approx 3.5$ and $S_{S_{BC}}/S_{S_{AD_{BC}}} \approx 10$. These results are consistent with the concept that the distribution of local noise sources in these samples is inhomogeneous. In particular, in sample D585 the noise of near-contact regions.

DUTs D580, D581 did not obey the reciprocity rule and moreover noise measured in XY<sub>CT</sub>VT configurations (i.e. on current contacts) depended significantly on the terminals (XY) selected for measurements. Thus, non-resistance noise of the contacts dominates in these samples.

#### A. Contact-related low-frequency noise

Regardless of its origin (resistance/non-resistance), the contact-related if noise can be measured in XY<sub>CT</sub>VT configuration. In such configuration the noise can be related to the sample resistance and \textit{relative contact noise} can be defined as: $C_f = S_p/i^2 = S_p/R^2$. In Fig. 4, the coefficient $C_f$ determined at 300 K for different samples and CT-configurations is shown as a function of free carrier concentration (doping) in GaSb layer. The highest p-doped layer D631 did not exhibit if noise: the value $7 \times 10^{-19}$ Hz<sup>-1</sup> attributed to this sample in Fig. 4 is the amplifier noise limit. For other devices, the values of $C_f$ are scattered depending on the terminals used during the measurements. Nevertheless, a rough dependence on carrier concentration can be observed: relative noise decreases as carrier density increases.

Another observation that can be derived from Fig.4 is that for a similar carrier concentration the p-type layer (sample D585) exhibits about two orders of magnitude lower contact noise than n-type layer (sample D584). As shown in Fig. 5, in which the temperature dependence of the coefficient $C_f$ is presented, this relation holds in a wide range of temperatures. The curves for beryllium p-doped and tellurium n-doped samples exhibit different behaviors: for p-type layers (D585, D581) the contact noise weakly depends on the temperature as opposed to n-type layers (D584, D580), for which $C_f$ changes by ~2 orders of magnitude with a complex temperature dependence.

The magnitude of the coefficient $C_f$ found in this study can be compared with the only other results found in the literature. Rolland et. al. [10] studied if noise of Te-Au contact to n-type GaSb and Au or AuZn contacts to p-type GaSb. Both GaSb layers were highly doped to $N \approx 10^{18}$ cm<sup>-3</sup>. The general conclusions that can be drawn from theirs measurements are consistent with ours: both Au or AuZn-alloy contacts to p-doped GaSb exhibit no if noise after annealing in pure hydrogen atmosphere. The Te-Au contact to n-type GaSb exhibits 1/f noise with the magnitude $C_f = 3.3 \times 10^{-17}$ Hz<sup>-1</sup> much lower than our estimates, but consistent with the conclusion that contacts to n-type layers are more noisy than to p-type GaSb layers. (It is also noteworthy, that lower noise is expected for higher doping density). In addition, theirs Au-Te based contacts are much better than ours In-based contacts.
Fig. 5. Relative contact low frequency noise $C_f$ versus reciprocal of temperature for samples with different doping density/type.

B. Low frequency noise of GaSb layer

The estimation of layer-related component of $C_f$ noise is possible only with four-point probe measurements with separated current and voltage terminals for samples, which obey reciprocity rule (D584 and D585). As shown in Fig. 3b, in diagonal AD$_2$BC$_4$T or BC$_2$AD$_4$T configurations the quantity $S_V$ is determined by the entire sample not only by near-contact areas. Thus, the layer-related component is more prominent in this configurations. Then, as the measured noise can be attributed to the entire layer the normalization of $S_V$ by layer sheet resistance $R_{sheet}$ is justified. In Table II, the values of $S_V$ at 1 Hz measured at 300 K and normalized quantity, $L_{eff} = S_V/(R_{sheet})^2$ are gathered. The relative low frequency noise of the Be-doped layer is significantly smaller than that of Te-doped layer. As no $C_f$ noise was observed for sample D631 (Be-doped to $10^{18}$ cm$^{-3}$), the conclusion is that the relative noise of the layers ($L_{eff}$) for p-doped samples decreases with the increasing doping concentration.

In ref. [11] N. Hooge et al. provided the value of parameter $\alpha_S = 3 \times 10^6$ for p-type GaSb material. Then, the estimate of relative noise of homogeneous sample of volume $V$ doped $N_i$ is: $L_{eff} = \alpha_S/V(N_i)^2 \approx 10^{-16}$/Hz, if the values of $V$ and $N_i$ for sample D585 are used. This value can be viewed as the lower limit for measured $C_f$ noise. Our estimate is much larger due to current crowding effects, which make the effective volume much lower than $V$.

TABLE II.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$S_V (\Omega^2/Hz)$</th>
<th>$R_{sheet} (\Omega)$</th>
<th>$L_{eff} (1/Hz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D584 (n- GaSb)</td>
<td>$1.1 \times 10^7$</td>
<td>35</td>
<td>$9 \times 10^{-12}$</td>
</tr>
<tr>
<td>D585 (p- GaSb)</td>
<td>$2.3 \times 10^{18}$</td>
<td>155</td>
<td>$9.7 \times 10^{-12}$</td>
</tr>
</tbody>
</table>

IV. Conclusions

The resistance or non-resistance low frequency noise related to the contacts and the semiconductor layer can be qualitatively investigated by four point-probe method. Such measurements can be performed with the different terminal (contacts) configurations, which amplifies the noise coming from different regions of the sample/layer. The contact (metal/semiconductor interface) can be source of significant low frequency non-resistance noise. The relative contact $C_f$ noise (both resistance and non-resistance) is inversely proportional to layer doping concentration. For comparable doping densities the $C_f$ contact noise for n-type Te-doped GaSb layers are significantly larger than for p-type Be-doped layers. The $C_f$ noise related to the layer with similar doping level but different doping type is higher for n-type Te-doped layer. Nevertheless, both layers exhibited inhomogeneous distribution of the noise sources. The overall low frequency noise performance of highly Be-doped (>10$^{18}$ cm$^{-3}$) GaSb is very good: both contact and layer $C_f$ noise were not measurable.

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Using the velocity auto-correlation function to characterize functional ”noise” in bio-molecules

M. Sidore∗, Y. Meriguet∗, A. Kudashova∗, M. Lechelon†, M. Gori†, M. Pettini†, J. Torres∗, L. Varani∗

∗Institute of Electronics and Systems (CNRS UMR 5214), University of Montpellier, 860 rue St. Priest, 34095 Montpellier, France
†Center of Theoretical Physics (CNRS UMR 7332), Aix-Marseille University, Marseille, France

Abstract—Characterization of current noise in simulated electrical systems relies on the Fourier transform of the auto-correlation of electrons velocity. A similar characterization can also be done in the simulation of biological systems, which relies on a statistical mechanics description through the calculation of the trajectories of the atoms over time. In such a system, the quantity calculated in the same way as electrical noise can be defining, whether it has functional significance or can be compared to experiments in the Mega to TeraHertz range. To exemplify this, we carried out the simulation of a model protein, the Bovine Serum Albumin, and outline the necessary precautions needed to obtain biologically relevant information.

I. INTRODUCTION

Characterization of noise in simulated electrical systems often relies on the Fourier transform of the auto-correlation function of the velocity, or the fluctuations of the velocity, of electrons.

A similar approach can also be used in a biological context, with macromolecules such as proteins or nucleic acids, albeit the ”noise” calculated through the auto-correlation function of the velocity of the particles (here, atoms), translates to the vibrational spectrum of the molecule [1], [2]. Using this approach through the simulation of a protein, which relies on a statistical mechanics description through the calculation of the trajectories of the atoms over time, we can uncover dynamical sub-millisecond movements that are often required for an adequate apprehension of biological function [3]–[6].

In the simulation of biomolecules using molecular dynamics, the position and velocity of each atom is recorded every \( \Delta t \). This allows us to characterize what we can then call molecular ”noise”, that is in fact how the molecules vibrate. When calculating the Fourier transform of the auto-correlation function of the velocities of the atoms of a bio-molecule, we gain access to its vibrational density of states which can then directly be compared to experimental spectra such as the THz spectra of the same bio-molecule [2]. Through this approach, we can couple simulation and experiment to gain insights into this molecule’s signature molecular vibrations, which can be important from a biological and physical point of view.

To exemplify the vibrations we can uncover through this method, we have simulated a system containing the model protein Bovine Serum Albumin (BSA) and obtained its vibrational density of states. BSA is a 583 amino-acid protein (Figure 1) involved in the maintain of oncotic pressure in blood vessels that also acts as a carrier for steroids, fatty acids and hormones. Its main use in biochemistry is as a standard in many experiments.

Fig. 1. Cartoon representation of the Bovine Serum Albumin (PDB ID: 3V03). The alpha helices are in purple, the turns are in cyan, the coils are in white and the \( \pi \)-helices are in blue.

After outlining the conditions which need to be met to simulate a system that is physical enough to obtain relevant vibrations, we compare the obtained spectra with previously published data.

II. METHODS

Construction of the system

The all atom coordinates of the Bovine Serum Albumin were taken from the Protein Data Bank (PDB ID: 3V03 [7]). BSA is a model protein extensively used in biophysics which is an ideal candidate to be simulated given its lack of cofactors and can moreover be compared to experiments. Missing atoms were built with a homemade script which used the MODELLER [8] software, available at https://framagit.org/msidore/ieee_vdos. The system was then built using the GROMACS tools. During this procedure, a solvation shell of 20 Å was used to ensure that the protein wouldn’t see itself through periodic boundary conditions; indeed, THz experiments have
uncovered solvation shells in the nanometer scale [2]. The resulting system has a size of 13 nm$^3$. The system was then ionized with 0.15 M Na$^+$ and Cl$^-$. The final system has 75106 water molecules, 230 Na$^+$ and 214 Cl$^-$. The CHARMM36 forcefield [9], [10] coupled to the TIP3P water model was used. This rigid 3 points water model was chosen because it is sufficient to reproduce long range solvation sphere perturbations [2].

A control system, containing only a 3 Å solvation sphere in a vacuum, has also been built as a control and similarly equilibrated and simulated.

**Simulation parameters**

All Molecular Dynamics simulations were performed using the GROMACS 2018 simulation package [11]–[15]. A two-step minimization comprised of a steepest gradient step and a conjugate gradient step up to an energy tolerance of 500 kJ/mol/nm. The Van der Waals and Coulomb cutoffs were set at all times to 1.2 nm. The system was then carefully equilibrated.

The first step was a 50 ps NVT run during which the C$\alpha$ of the protein were position restrained, with a 1 ps $\Delta t$ and a velocity rescale [16] thermostat set at 300 K with a 0.2 ps time constant. The second step was a 50 ps NPT run with the same parameters plus a Berendsen barostat [17] set at 1 bar with a 2.0 ps time constant.

After these first steps of equilibration, 40 ps of NPT were carried out with constraints set to none and $\Delta t$ set to 0.5 fs. While this parameter is usually set to all bonds, which is reasonable in most situations [18], constraining the bonds could negatively affect the vibrational density of states by removing small degrees of freedom. A subsequent 160 ps NPT run was then used to remove the position restraints. A return to the NVT ensemble was then carried out for 400 ps with a Nose-Hoover thermostat [19] set at 300 K with a 0.2 ps time constant.

The production run was then carried out in NVT for 1 ns using Particle Mesh Ewald [20] for long-range electrostatics with a PME order of 4 and a grid spacing of 0.12 nm. During the production run, the coordinates and velocities were saved every 8 steps (4 ps) to ensure sufficient sampling to obtain the vibrational density of states.

**Analysis**

The VMD software [21] was used to render the protein and the assignment of the secondary structure was predicted with the STRIDE software.

Graphical output has been produced with the Grace software.

The vibrational density of states is estimated from the Fourier transform of the autocorrelation function of atomic velocities:

$$VDOS(f) = \int \frac{\langle \vec{v}(0) \cdot \vec{v}(t) \rangle}{\langle \vec{v}(0) \cdot \vec{v}(0) \rangle} \exp(\text{i}2\pi ft) dt$$

The analyses were performed using the GROMACS post-processing tools *dos* and *vacf* on the protein atoms. The vibrational density of states characterizes the oscillations of the molecule in the frequency domain because the motion of the atoms is of an oscillatory nature (atomic velocities self-correlate in a periodic manner). More precisely, all vibrational bands of covalently bonded atoms (such as bond stretching and bond bending) are shown in the vibrational density of states spectrum. For these analyses, water molecules were discarded from the trajectory because the saving of the velocities every 4 fs led to storage issues.

**III. Results**

The aim here is to produce and outline how the autocorrelation of velocities, which can be conceptualized as "noise", can also be used within biological molecules to obtain signature vibrations. Figure 2 shows the calculated velocity autocorrelation function according to equation 1.

![Fig. 2. The velocity autocorrelation function using the velocities of the atoms of the protein in Arbitrary Units (AU) as a function of time. The velocities of the atoms self-correlate to 0 on the scale of a few picoseconds.](image)

The velocity autocorrelation function exhibits a fast oscillating behaviour superimposed to an exponential decay and vanishes on a time scale of few picoseconds. The Fourier transform of this autocorrelation function then turns the periodicities into frequency bands and Figure 3 shows the whole normalized vibrational density of states.

The general shape of the density of states is rather complicated showing several resonances in a wide frequency domain. The frequency peaks found at frequencies higher than 50 THz correspond to the fastest motions in our system. Most importantly, their highest amplitudes are of the same level as the peaks below 50 THz, which indicate that the system contains enough water molecules. Indeed, the 90 THz band has a high amplitude if the protein doesn’t have enough water molecules for a complete solvation (Data not shown) and it has been shown that enough water is needed to allow its full range of vibrations.

However, since we are in a solvated environment and water absorbs THz radiations heavily, only the frequencies in the
low-frequency THz range can be compared with available experiments obtained using THz spectroscopy techniques. Figure 4 shows a zoom of the vibrational density of states in this range.

In this frequency range, we can then compare with experiments. In solution [22], BSA displays a monotonous increase between 0 and 2 THz, which is in accordance with our calculations (Figure 4). After 2 THz, the frequency slowly decreases and reaches a plateau, which is not found in experiments but can be explained by the effects of water on the measurements.

IV. DISCUSSION AND CONCLUSION

In this paper, we have outlined the relevance of the autocorrelation of the velocities of particles in biology to investigate molecular "noise", translated to specific molecular vibrations. In this protocol, the preparation of the system takes the most considerations, from a careful equilibration to the absence of bond constraints and the size of the solvation sphere.

Indeed, the solvation sphere is measured and hypothesized to be several nanometers wide. In our simulations, this matter is entangled with the choice of the water model: can a rigid 3 point water model reproduce a perturbation far enough from the protein surface [2], the use of this model seems to be reasonable, but that doesn’t tell if the links between this perturbation and protein dynamics reproduce what happens in experiments. Nonetheless, with it comes a practical matter: if a simple water model is perturbed far from the protein surface, it means that we need a big amount of water molecules to prevent the protein from "seeing itself" through the periodic boundary conditions, which in turn limits the length of the simulation.

Another practical problem is the storage space necessary to save velocities of a big system every 4 fs. Indeed, several dozens of To would be needed for the whole trajectory and we had to discard water molecules in the saved trajectory to meet our available storage.

This approach can also realistically be compared to the experiment in the low THz range and be used to identify frequency shifts associated with changes in the atoms dynamics when the molecule is driven far from equilibrium due, for instance, to energy absorption from the external medium.

We stress also that a limit of these simulations concerns the absolute amplitude of the calculated frequencies: molecular dynamics simulations are based on a classical model while an accurate estimation of these amplitudes would require a quantum treatment. Moreover, experiments of THz spectroscopy in a protein solution is also tricky and can lead to artefacts [4], [6], [22], [23], and especially with this protein which can be considered "sticky". As a consequence, the comparison between simulations and experiments is still problematic and additional work is needed to achieve reliable physical information.

However, we want to emphasize that the calculation of the autocorrelation of the atoms’ velocities can nonetheless provide useful insights into protein dynamics when measured.
and calculated frequency bands are shifted, and the protocol here presented would allow this possibility.

Moreover, the protein here studied provides a working example that may lead to more calculations of molecular “noise” and comparison with experiments.

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Energy and Area Aware Digital Fingerprint Generator Using Intrinsic Randomness

Sandeepkumar Pandey
Department of Electronics Engineering,
Shri Ramdeobaba College of Engineering and Management,
Nagpur, India, 440013
Email:pandeys@rknec.edu

Jawar Singh and Pramod K Tiwari
Department of Electrical Engineering,
Indian Institute of Technology Patna,
Bihar, India, 801106
Email:(jawar, pktwari)@iitp.ac.in

Fig. 1. Self-correcting overdrive voltage tracking along with amplifier stage.

I. INTRODUCTION

Aggressive scaling of CMOS technology, two major sources of process variations manifest themselves in the form of sub-wavelength lithographic variation and variations resulting from vacillations in quantity and locality of dopant atoms [2]–[4]. Random Dopant Fluctuation (RDF) and Line Edge Roughness (LER) influence the threshold voltage (\(V_T\)) of the MOSFET, which in turn becomes a random function. This random function could be exploited to generate a hardware circuit and whose electrical properties are unique and driven by physical traits of MOSFETs called PUF. These are emerging hardware security primitive circuits widely used for secret key generation, identification, and authentication of electronic devices. Different approaches and various topologies for PUF realization have been studied and proposed in the literature. Published PUF implementations could broadly be classified as: SRAM based PUFs, arbiter PUFs, RO (Ring Oscillator) PUFs, and optical PUFs. [5]–[8].

However, main challenges associated with PUF designs include: relatively small challenge space, high resource utilization, power consumption, and susceptibility to modeling attacks. Particularly, the PUF architectures proposed in [9]–[11] have high resource utilization overhead. This presents serious bottleneck in resource constrained designs. We propose a PUF architecture which extracts and amplifies the process variations outlined above to generate secret keys. The proposed PUF is evaluated on important performance matrices of standard PUF design and it is observed that our design has extremely low power and resource utilization.

II. SELF-CORRECTING OVERDRIVE VOLTAGE TRACKING CIRCUIT ALONG WITH AMPLIFIER STAGE

We propose a PUF architecture which extracts and amplifies the random fluctuations or intrinsic randomness that arises during manufacturing of integrated circuits to generate secret keys. A sub-threshold CMOS PUF cell is proposed as shown in Fig. 1, where, transistor (\(M_1\)) is used to extract process fluctuations and made to operate in sub-threshold region to have lesser energy dissipation. However, main challenges associated with PUF designs include: relatively small challenge space, high resource utilization, power consumption, and susceptibility to modeling attacks. Particularly, the PUF architectures proposed in [9]–[11] have high resource utilization overhead. This presents serious bottleneck in resource constrained designs. We propose a PUF architecture which extracts and amplifies the process variations outlined above to generate secret keys. The proposed PUF is evaluated on important performance matrices of standard PUF design and it is observed that our design has extremely low power and resource utilization.

If we ensure constant \(V_{DS}\) and \(I_{DS}\) through \(M_1\) then, the overdrive voltage would be constant. However, random fluctuation in \(V_T\) forces the \(V_{GS}\) for self-correction to keep \((V_{GS} - V_T)\) constant. In other words, random fluctuation in \(V_T\) of \(M_1\) is translated to change in \(V_{GS}\) and later

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**Abstract**—Manufacturing variability/fluctuations is a major concern in integrated circuits and this has been exploited to design a hardware security primitive or digital fingerprint generator, referred as physical unclonable function (PUF) for resource constrained low power applications. The proposed architecture employs single transistor as PUF cell, one operational amplifier (OpAmp), and little peripheral circuitry, thereby making it very light and suitable for resource constrained applications. Simulation results show that the energy consumption of proposed design is 30fJ/bit which is 6.33× lesser than earlier reported design, it has 0.0025\(\mu\text{m}^2\) area/bit and the values of 1s and 0s are equi-probable with probability of a ‘1’ being 50.2%. The mean measured by inter HD plot is 49.72 which is very close to an ideal value of 50%. This affirms that the proposed PUF can supply unique identifiers.

**Keywords**—Physical Unclonable Function (PUF), Security, Low energy consumption, Digital fingerprint.
amplified by the amplifier stage for secret key generation. In Fig. 1, a constant current $I_{DM1}$ through $M_1$ is set and the source follower ensures fixed $V_{DS}$ across $M_1$. If the $V_{T}$ of the $M_1$ changes then so does the source voltage thereby keeping the term $(V_{GS} - V_{T})$ as constant. Since threshold voltage fluctuation is random in nature, we can say that $V_{GS}$ variation is random too. For array of such devices whose $(V_{T})$ is assumed to vary randomly due to process fluctuations and same is extracted for generation of secret keys. The extracted randomness in the form of source voltage variation in correspondence with $(V_{T})$ variation is further amplified by the presented amplifier stage comprising of $M_3$ and $M_4$. The $I_{DS}$ in sub-threshold region changes exponentially with $V_{GS}$. Therefore, $V_{S}$ gets amplified at $V_{out}$. Node voltage $V_{S}$ corresponds to the sensed change in threshold voltage which is amplified by amplifier stage. The voltage variation at $V_S$ are such that it never goes beyond the threshold voltage of $M_3$. This means that $M_3$ always operates in sub-threshold region. The amplifier stage is designed to improve the voltage swing available at $V_S$. The gain of the amplifier has been obtained analytically using small signal equivalent model of amplifier in [18], as shown in Fig. 2. Due to small variation at node $V_S$, the gain is expressed as:

$$\frac{V_{out}}{V_S} = g_{mM3}(r_{oM3}/r_{oM4})$$

(1)

Because of sub-threshold operation, the current is expressed [19]:

$$I_{sub} = A \times e^{(V_{gs} - V_{th} - \gamma V_{ds} + \eta V_{ds})/mV_{T}} \times \left(1 - e^{-V_{gs}/V_{T}}\right)$$

(2)

where,

$$A = \mu_0 C_{ox} \frac{W}{L_{eff}} v_{T}^{3} e^{1.8}$$

(3)

$\mu_0$ is carrier mobility, $\eta$ is the drain induced barrier lowering (DIBL) coefficient, $\gamma$ is the body effect coefficient, $m$ is the sub-threshold swing coefficient of the transistor and $V_{T}$ is the thermal voltage. From (2) the small signal parameters i.e $g_{m}$, $r_{o}$ are calculated as

$$g_{mM3} = \frac{\partial I_{ds}}{\partial V_{gs}} = \frac{\partial I_{ds}}{\partial V_{S}} \frac{I_{ds}}{m_{nM3}v_{T}}$$

(4)

$$r_{oM3} = \frac{\partial I_{ds}}{\partial V_{ds}} = \frac{1}{m_{nM3}v_{T}}$$

(5)

$$r_{oM4} = \frac{\partial I_{ds}}{\partial V_{S}} = \frac{1}{m_{nM4}v_{T}}$$

(6)

by combining (1), (4), (5), (7), we get:

$$\frac{V_{out}}{V_S} = \frac{m_{nM4}}{m_{M3}r_{M3} + m_{M4}r_{M4}}$$

(7)

The length of the transistor $M_3$ and $M_4$ is used for gain enhancement of amplifier stage since it modifies the respective DIBL coefficients.

III. THE PUF ARCHITECTURE OF SECRET KEY GENERATION

The complete PUF architecture of secret key generation is shown in Fig. 3. This is based on the self-correcting excess voltage tracking circuit described in Fig. 1. Challenges are applied to row and column decoders based on which a particular device is selected. The applied row challenges select the gate of row transistors $R_i$ (i=1,2,3,..N), such that the source voltage $V_{SX}$ (x=1,2,3,..N) gets applied to the input of source follower circuit. The same gets applied to the amplifier stage for amplification. The output of source follower circuit is applied to the non-inverting input of the OpAmp. The other input (inverting) of the OpAmp is obtained from $V_{DS}$ (x=1, 2, 3, 4, 5...N). For example the applied challenge to the row decoder and column is such that it internally selects row-0 and column-4 so that $C_0=0$, $C_{40}=1$ and $R_0=1$. This selects the green colored transistor in dashed ellipse, as shown in Fig. 3. Fixed constant current $I_{DSM}$ is made to flow through the path OpAmp-$V_{DS1}$-ground. This is shown with red arrow in Fig. 3, and corresponding flow of current is as shown in Fig. 1 with same red color arrow. Source follower and OpAmp together ensure that the constant $V_{DS}$ is applied to the green transistor (or any other selected device). Also the current source $I_{DMN}$ ensures constant current is applied through the selected transistor. Hence, the random process variations in threshold voltage are available at node $V_{S1}$ in the Fig. 3 that is further amplified by amplifier stage and given to key storage register through buffer circuit. The switching threshold of the buffers is designed at half of the swing available at the output of the amplifier stage. Complete process for secret key generation is shown step-by-step in Fig. 4 with the help of a flow chart. Therefore, random but consistent outputs are generated out of the buffer circuit which is stored in the secret key storage register.

More popular PUF architectures such as delay-based PUFs are prone to modeling attacks, because their basic building topologies can be mathematically represented by model whose unknown delay coefficients can be estimated by machine learning techniques from the gathered challenge-response pairs. Our proposed PUF directly uses a random amount of $V_{T}$ fluctuation available in manufacturing process. This will make it more resilient to attacks. The single transistor selection for random $V_{T}$ extraction employs a two dimensional array structure addressed by row and column decoders. This increases the challenge response space. For proposed PUF with M challenge bits applied at the row decoder for row selection and N challenge bits applied at the column decoder for column selection, there could be $2^N \times 2^M$ challengeresponse pairs. This satisfies the criterion for strong PUF. The proposed architecture selects one
transistor at a time for secure bit generation. Thus, required variable key lengths could be generated based on platform requirements and computational constraints.

The threshold voltage ($V_T$) fluctuations or the mobility ($\mu$) fluctuations during fabrication are the two MOSFET parameters which could be effectively used to realise PUFs. Since these parameters show local fluctuations due to statistical fluctuations. Our PUF working in the sub-threshold region is characterized by the $V_T$ variations of the selected MOSFETs. The $V_T$ can be expressed as a function of temperature as [20]:

$$V_T(T) = V_T(T_0) \cdot (1 + TCV_T \cdot (T - T_0))$$

where $V_T$ is measured at the temperature $T_0$ and the temperature coefficient of $V_T$ is defined as:

$$TCV_T = \frac{1}{V_T} \cdot \frac{\partial V_T}{\partial T}$$

In our PUF design, temperature dependence can be compensated by varying the bias current with temperature. Thus, $V_{GS}$ changes corresponding to $V_T$ changes in selected transistors could be made to stay constant with varying temperature.

IV. SIMULATION RESULTS

All the simulation results are obtained from 50 nm CMOS technology models with $V_{DD}=1V$. The energy consumption $E_{bit}$ of the proposed PUF is evaluated as: $E_{bit} = I_{drawn} \times V_{DD} / f_{clk}$, where, $I_{drawn}$ is the current drawn from supply to produce the response bit, $V_{DD}$ is the supply voltage, and $f_{clk}$ is the applied clock frequency. The circuit draws 30nW for 1us for one bit generation thereby giving $E_{bit}$ of 30fJ/bit which is 6.33X lesser than [12]. A comparison of energy efficiency of the proposed PUF design with the earlier reported PUF designs is shown in Fig. 5. At the nominal operating condition, the proposed PUF achieves an $E_{bit}$ of 30 fJ/bit.

Uniqueness is another very important security metric for PUF. The quality of a given PUF instance to provide an unambiguously distinct behavior in comparison with different other PUFs with the identical topology implemented on different chips is measured by this quantity. Suppose the two PUFs (each with a challenge response pair (CRP) of 6 bits are to be realized on two distinct chips. When both the PUFs are subjected to same challenge say (010101) then the response obtained from each PUF is distinct. Assuming the Hamming distance between obtained response is 2, which implies that 25% of the response bits differ. Ideally Uniqueness should be close to 50%. Fig. 7 shows the inter Hamming distance (HD) of die for one thousand runs. The mean measured by inter HD plot, as shown in Fig. 7 with mean of 49.72 which is very close to an ideal value of 50%. This affirms that the proposed PUF can supply unique identifiers.
The proposed PUF design relies on the concept of the self-correcting overdrive voltage tracking circuit. The design is low power and the only power hungry component used is OpAmp. It could be switched off after particular transistor outputs are identified. Thus, significantly reducing the power consumption. The PUF cell used as source of randomness is minimum sized transistor making the area per bit matrix of proposed cell extremely small. The proposed PUF architecture has low hardware overhead in terms of input/output and processing circuitry. The proposed architecture employs single transistor circuitry thereby making it ultra-light weight which is well suited for resource constrained applications. The circuit draws 30nW for 1us for one bit generation thereby giving energy suited for resource constrained applications. The circuit draws 30nW for 1us for one bit generation thereby giving energy suited for resource constrained applications.

V. CONCLUSION

The proposed PUF design relies on concept of the self-correcting overdrive voltage tracking circuit. The design is low power and the only power hungry component used is OpAmp. It could be switched off after particular transistor outputs are identified. Thus, significantly reducing the power consumption. The PUF cell used as source of randomness is minimum sized transistor making the area per bit matrix of proposed cell extremely small. The proposed PUF architecture has low hardware overhead in terms of input/output and processing circuitry. The proposed architecture employs single transistor as PUF cell, only one OpAmp and very little processing circuitry thereby making it ultra-light weight which is well suited for resource constrained applications. The circuit draws 30nW for 1us for one bit generation thereby giving energy dissipation of 30fJ/bit.

REFERENCES

Effects of mechanical stress on electrical parameters and noise of supercapacitors

Arkadiusz Szewczyk, Łukasz Lentka, Janusz Smulko
Faculty of Electronics, Telecommunication and Informatics
Gdańsk University of Technology
Gdańsk, Poland
szewczyk@eti.pg.edu.pl

Abstract — Results of noise and electrical parameters measurements of prototype electrochemical double layer capacitors (EDLC) are reported at the presence of selected mechanical stress. This issue is of great importance due to future applications in wearable technology. The measurement results are compared, and we may conclude than flicker noise is more sensitive to any stress than other considered electrical parameters.

Keywords— EDLC, supercapacitor, noise, mechanical stress

I. INTRODUCTION

Electrochemical double layer capacitor (EDLC) is a device that is capable of storing a relatively high amount of electrical energy in comparison to its mass. Additionally, it may work effectively for a long time. The charge is stored in a double electrical layer, formed between the porous electrode and the electrolyte solution interface. The interface is very fragile to any changes in the EDLCs due to the low size of pores, even below 1 nm, where the ions preserve electrical charge.

The device may be applied as an energy supplier in wearable electronic systems of low power consumption. It means that it should be cheap and resistive to mechanical stress during exploitation. We present measurement results of selected electrical parameters of exemplary EDLCs when mechanically bent. These parameters were compared with low frequency noise measurements (in flicker noise and white noise regions).

II. SAMPLES

In our experimental studies, the prototype EDLC cells were used. The cell comprises of two electrodes with a porous carbon layer with ion permeable separator between them that is typical for such elements. The internal structure of the EDLC is shown in Fig. 1.

The electrolyte solution fulfills the space between electrodes. The structure is enclosed in a hermetically welded pouch cell. Electrodes are led out on both sides of the pouch to assure electrical contacts. The pouch cell is flat and flexible and may be attached to clothes. Its size is approximately about 8 cm x 9 cm. The applied electrolyte is organic and the nominal voltage is 2.7 V. The cell capacitance $C \approx 7 \text{ F}$, which is sufficient to supply energy for tiny electronic systems. As the single EDLC cell is relatively thin (approx. 0.5 mm), cells could be stacked to reach a higher value of capacitance.

Mechanical stress applied to the cell, especially bending, could cause shifting of inner layers of the EDLC structure (electrodes and separator) and possible changes in distance between them. The movements of inner layers shouldn’t cause a change in capacitance, as it results from electrode-electrolyte interface, unless the space between electrodes remain fulfilled by the electrolyte. The bending of the cell could result in cracks in the active carbon layer and possible electrical short-circuits between positive, and negative electrodes. Those failures should be visible in changes of capacitance and equivalent series resistance values, as well as in low-frequency noise level observed during element operation.

III. EXPERIMENT

The measurement set-up comprises of programmable potentiostat and galvanostat to assure the charging-discharging process of the tested EDLC, and noise measurement channel utilizing low-noise amplifier and 24-bit analog-to-digital converter [1, 2]. The block diagram of the measurement set-up is shown in Fig. 2.
The specimen was charged to different voltages from 0.5 V to 2.5 V and back. At each voltage, the specimen was kept for 2 hours (this stage is called floating). Next, the specimen was discharged through the loading resistance 100 Ω, and the noise was recorded. During noise recording, the sample was disconnected from the charging-discharging circuit to avoid interferences. After discharge, the specimen was five-time charged and discharged and capacitance C, and ESR were estimated from the current and voltage of the fifth discharge by the method described in [3]. The time waveform of voltage over EDLC contacts is shown in Fig. 3.

Recorded discharge signals with DC and noise component were subjected to a procedure of DC component removal [4], and the power spectral density of the noise component was estimated.

During the measurement, the tested EDLC was placed on a flat surface and pressed by 1 kg mass to stabilize its geometry. The same experiment was repeated by bending the samples using the selected rigid forms (Fig. 4), fabricated by 3D printing technology. Two forms were used: first with a radius equal 55.2 mm which corresponds to 20 mm of bending, and second with a radius equal 117.8 mm which corresponds to 10 mm of bending. Each form consists of two parts what allows to place EDLC pouch cell between them and press to force uniform bending. Contact electrodes protrude out of form to allow contact to measurement set-up. The form with EDLC inside was pressed by 1 kg mass to stabilize bending. Each sample was tested in two deformations. Specimen denoted JP5.4 was first bent by 20 mm, then by 10 mm, whereas the one denoted JP5.5 was first bent by 20 mm, then by 10 mm. Forms with specimens mounted inside are shown in Fig. 5.

We observe some changes of C and ESR values at different polarizing voltages. The changes are not significant, except the ESR values of specimen JP5.4, which increased much more than for specimen JP5.4.

For each floating voltage the power spectral density of the discharge voltage were estimated. Typical curve of power spectral density of discharge voltage fluctuations is shown in Fig. 8.

To compare the noise intensity at various experiment voltages the mean of power spectral density multiplied by frequency square, $f^2$, for 1/f region (0.02÷0.1 Hz) and mean value of power spectral density for white noise region (3÷10 Hz) were calculated. Results are shown in Fig. 9 and Fig. 10 for specimens JP5.4 and JP5.5 respectively.

The noise level increases slightly with voltage for 1/f region. Better reproducibility is observed for specimen JP5.5, however, the dispersion of mean values is within half of the decade.
The same procedure was repeated for bent specimens. As we mentioned before, each sample was tested while bent with both forms (10 mm and 20 mm). Results of capacitance and ESR estimation of specimens JP5.4 and JP5.5 are shown in Fig. 11 and Fig. 12 respectively.

![Fig. 9. Mean value of power spectral density of JP5.4: a – 1/f noise region, b – white noise region](image)

![Fig. 10. Mean value of power spectral density of JP5.5: a – 1/f noise region, b – white noise region](image)

We observe good reproducibility of C and ESR values for various voltage values. When the specimen is more bent, it shows lower capacitance. Resistance ESR seems not to depend on bending. The noise results are presented in Fig. 13 and Fig. 14 for the specimens JP5.4 and JP5.5 respectively.

![Fig. 11. Capacitance and ESR of JP5.4 when bent: a – capacitance, b – ESR](image)

![Fig. 12. Capacitance and ESR of JP5.4 when bent: a – capacitance, b – ESR](image)
Fig. 13. Mean value of power spectral density of bent specimen JP5.4:
 a – 1/f noise region, b – white noise region

Fig. 14. Mean value of power spectral density of bent specimen JP5.5:
 a – 1/f noise region, b – white noise region

Noise results of the bent specimens show better reproducibility than for flat sample measurements (Fig. 9, Fig. 10). We observe some increase of noise intensity at 1.5 V when the sample is bent for the first time (JP5.4: 2 cm, JP5.5: 1 cm). The change in the bending curve seems not to influence the noise of specimens.

V. CONCLUSIONS

We have observed some changes of C and ESR values at different polarizing voltages and bending. Noise intensity depended on polarizing voltage as well and changed when bent or when we repeated the experiment. The mechanical pressure and bend stabilize the noise and electrical parameters of EDLC.

We may conclude that noise seems to be more sensible than electrical parameters to any mechanical stress. Mean value of the product of power spectral density of voltage fluctuations and frequency square at low-frequency range is one order higher for bent specimen than for the flat one, while the capacitance dropped 5% and 9% only after the first bend.

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Noise in electrical double-layer capacitors (EDLCs)

Janusz Smulko
Faculty of Electronics, Telecommunication and Informatics
Gdańsk University of Technology
Gdańsk, Poland
http://orcid.org/0000-0003-1459-4199

Arkadiusz Szewczyk
Faculty of Electronics, Telecommunication and Informatics
Gdańsk University of Technology
Gdańsk, Poland
https://orcid.org/0000-0002-9407-4064

Łukasz Lentka
Faculty of Electronics, Telecommunication and Informatics
Gdańsk University of Technology
Gdańsk, Poland
https://orcid.org/0000-0001-7985-4356

Abstract—We present methods and problems of noise measurements in electrical double-layer capacitors (EDLC). Detailed noise equivalent electronic circuit is considered, and two possible ways of observations of random processes generated in the EDLCs structures are studied. We conclude that noise is a useful tool for characterization of the EDLC structures and their state-of-health, as in other materials and electronic devices. Eventual, practical applications of noise measurements are proposed to determine the state-of-health of the EDLCs.

Keywords—electrical double-layer capacitor, flicker noise, white noise, state-of-health, reliability

I. INTRODUCTION

The ability to collect electrical energy is crucial for developing electric vehicles and other ecological equipment, requiring reliable and distributed power sources. The electrical double-layer capacitors (EDLC) may be used for this aim when fast delivery of energy has to be secured. These capacitors are also known by different names such as supercapacitors or ultra-capacitors. EDLCs are used to power mobile devices or to preserve energy produced by wind power stations because they are able to store a large amount of energy than that of ordinary capacitors and to supply the energy in a faster way than that of batteries, as presented by Ragon plot comparing different technologies of electrical energy storage systems [1], [2]. Unfortunately, the EDLCs are operating at low voltages and are very sensitive to any overvoltage or overheating [3]. Therefore, we have to monitor the processes in their structures to study the applied technology for its further development or to avoid damages of the applied elements.

The EDLC comprise of two porous carbon electrodes and electrolyte with a separator between these electrodes, protecting them from contacting (Fig. 1a). A charge is stored in an electrical double layer formed by applying a relatively low voltage to the terminals connecting the carbon electrodes. Ions are drawn to the surface of the electrical double layer of the EDLC when charged (Fig. 1b) and moved away when discharged. The EDLCs have a large active surface, due to the pores in the carbon of a diameter even of a few nm only. Such structure is very fragile and may lose its ability to charge collecting. Overheating or increased voltage accelerate the chemical reaction and result in pores blocking and reducing active charging area [4].

Noise is a well-known tool applied to assess state-of-health or reliability of the elements or systems. Flicker noise is observed in semiconductor devices, sensors, and electrochemical systems for this aim [5], [6]. It may also be used for the same aim for batteries [7], capacitors [8] and EDLCs as well [9]. A thorough review of low-frequency noise sources existing in the electrode-electrolyte interface can be found elsewhere [10].

Noise measurements in EDLCs are not evident due to high capacitance and necessity of measurements in the low-frequency range, below 1 Hz. It means that the measurements require a long time of data acquisition, about a few hours, and stability to avoid any drifts induced by slow environmental changes like temperature. We consider these issues in the paper, and propose how to observe noise, and how to interpret the recorded data, and how to use it for a future practical application. Similar issues have been considered by others as well [11], [12].

II. THE ELECTRICAL MODEL OF THE EDLCs

The porous structure of the carbon electrodes collects the charge when the ions move into the pores driven by an electric field. The electrical double layer is formed on a porous surface at the interface between the electrolyte and the carbon electrode. Thus, a state-of-health (SOH) of the EDLCs depend on the processes taking place in this small area. The SOH may be monitored by impedance spectroscopy, changes in capacitance and resistance or other EDLCs parameters [13], [14]. We may suppose that noise generated there sheds light on the smoothness of charging and discharging mechanisms. The pores may clog due to, e.g., ions disintegration which results in capacitance reduction. These processes may occur locally due to voltage or temperature distribution within the EDLC structure. We may suppose that the pores before clogging influence smoothness of charge movements and should modify noise level. Similar conclusions were reported for batteries elsewhere [6], [15]. The main problem of confirming this assumption is if the noise is sufficiently intense and may be observed by using the measurement setup and how should be selected the conditions for the polarization of the investigated EDLCs during the measurements.
The equivalent circuit of the EDLC comprises of a ladder of resistances and capacitors, representing the processes of ions penetrating the pores. In general, the EDLCs may be modeled by a ladder circuit of at least two branches (Fig. 2) [6]. More detailed models of the EDLCs were proposed elsewhere [16], [17]. A thorough review of EDLCs modeling is presented in [18]. These models include new RC branches as presented in Fig. 2. We propose to focus on two branches only because it should be sufficient to model the dominant two physical phenomena. The first RC branch represents the process of charging and discharging which is relatively fast. The second RC branch represents the process of charge redistribution inside the EDLC structure which is much slower than charging and discharging process.

Capacitor C1 and resistor R_{\text{ESR}} represent the so-called Helmholtz capacitance with an equivalent series resistance (ESR) while capacitor C2 and resistor R_D represent processes induced by diffusion and present during charge redistribution [6]. Both capacitances, C1 and C2, have similar values. Additional resistor R_L represents a resistance responsible for a leakage current – the phenomenon of self-discharge. Resistance R_{\text{ESR}} is significantly lower than R_D, even about two orders of the magnitude. It means that C1 is charged to a specified voltage much faster than C2. The same remark is valid for discharging process. The difference in charging or discharging velocity means that we should be able to differentiate between noise sources related to both capacitances.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{equivalent_circuit.png}
\caption{Electrical equivalent circuit of the EDLC with voltage noise sources.}
\end{figure}

Noise generation in the EDLCs structures is related to ions migration in porous electrodes. Movements of ions within the pores of diameter even less than 1 nm take part in processes of electrical random signal generation. Electric field or diffusion phenomenon forces ions to penetrate the pores of the electrode material. Electric field may accelerate ions and energy dissipation is far from thermal equilibrium. The diffusion process is in thermal equilibrium, and the thermal noise sources may represent noise generated within the electrode-electrolyte interface. These two reasons (electric field or diffusion) should be responsible for the observed noise. Noise sources are exposed to the equivalent electrical circuit presented in Fig. 2. All voltage noise sources relate to the existing capacitances or resistors:

- C1, noise source e_{n1};
- C2, noise source e_{n2};
- R_D, thermal noise source 4kT R_D;
- R_{\text{ESR}}, thermal noise source 4kT R_{\text{ESR}};
- R_L, thermal noise source 4kT R_L.

All resistances are the sources of thermal noise, and at thermal equilibrium, their intensity is constant or changes due to alteration of these resistances (e.g., induced by EDLCs aging process). The noise sources e_{n1} and e_{n2} represent thermal noise of the capacitances C1, C2, and other additive noise, like shot noise (nonequilibrium fluctuations induced by charge transfer) or flicker noise (1/f-like noise) as well.

We may conclude that fluctuations are generated due to dissipative energy conductors and electrochemical interactions. When no current flows through the interface, the thermal equilibrium white noise dominates. Some low-frequency noise may be observed as well due to inherent noise of the measurement set-up and 1/f-noise of the RC elements of the electrical equivalent circuit of the EDLCs.

Local overheating or electrochemical reactions may block the pores and noise generated there should be very sensitive to this phenomena. The process of blocking pores modifies intensities of the noise sources e_{n1} and e_{n2}, which should be observed in the experiments. We may expect some changes in flicker noise as observed in various materials when changes in their structures induce a change of Hooge constant [19], [20]. This semi-experimental Hooge model of flicker noise generation by independent charges in the given bulk predicts such changes. We observed an increase of flicker noise intensity after aging in similar structures, the so-called smart windows, having nanoparticles electrode-electrolyte interface [21].

When we observe noise during the EDLCs discharging we may expect the shot noise component. Its power spectral density S(f) depends on discharging current I and the ratio of the resistances, representing the charge transfer resistance R_c in the electrode-electrolyte interface and the bulk solution resistance R_b [10]:

\begin{equation}
S(f) = 2q^2[(R_c/(R_c+R_b))^2]
\end{equation}

where q is the electron charge. The intensity of shot noise may change when the ratio of the distinguished resistances evaluates during aging of the EDLCs. The presented eq. (1) is valid when the response time of the circuit between the terminals is much faster than the charge transfer and the kinetics of mass relocation within the electrode-electrolyte interface. In the opposite case, the proposed model of shot noise does not include the coefficient of the resistance ratio R_c and R_b [10]. In the first case, we may suppose that aging (e.g., by pores blocking) increases the resistance R_c and therefore change the intensity of shot noise component.

III. MEASUREMENT SET-UP

The measurement set-up for low-frequency noise observations in EDLCs consists of current/voltage source with galvanostatic/potentiostatic ability to assure the EDLCs charging/discharging process. Noise is recorded by using low noise amplifier and analog-to-digital 24-bits converter at different polarizing conditions to expose selected noise sources [9].

Two measurement procedures are proposed to expose two different noise sources. The first one requires charging with a constant current to the specified voltage. Next, the specimen
is discharged by loading resistor, and the voltage across this resistor is recorded. The voltage noise component is identified by subtracting exponential-like discharging curve (trend), as presented elsewhere [22]. This component is related mainly to charges stored in the capacitor $C_1$ and only partially in $C_2$ because the time constant $C_1R_{\text{eq}}$ determines the discharging process. The second procedure requires again charging with a constant current to the given voltage. Next, the sample is short-circuited for a selected time after which the sample terminals are connected to the loading resistor and the voltage drop across this resistor is recorded due to charge migration from still partially charged $C_2$ to $C_1$. Noise component in the recorded time series is determined mainly by charge fluctuations in capacitance $C_2$.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

Two mentioned above measurement procedures were used to investigate the noise in the selected samples, produced by applying materials and methods similar to commercially available EDLCs. The samples had capacitances of about 10 F, and the specific method of their preparation was presented elsewhere [9].

The samples were aged by floating at 2.5 V. Noise was measured each time after a few hours of aging by floating. The noise was recorded during discharging the sample through the loading resistance $R = 100 \, \Omega$. Selected power spectral densities of voltage fluctuations across the loading resistance presents Fig. 3. We observed in the experiment:

- flicker noise at low-frequency range, usually below 1 Hz,
- the white noise of different intensities at frequencies higher than a few Hz.

Change of flicker noise intensity was somewhat limited, but other studies suggested that flicker noise may increase more during aging by floating then in the reported experiment. We suppose that this change of flicker noise level may be induced by pores blocking or local overheating and delamination between the metal collector and porous carbon electrode.

More intense noise change was observed in the white noise region. After aging, shot noise increased more than tenfold times. Rapid noise increase was observed after about ten hours of aging. More intense noise preserved in the next stages of aging with some variations. This effect may be explained by an increase of the resistance $R_2$ in an electrode-electrolyte interface and as a result the change of the fraction $R_2/(R_2 + R_1)$ in the eq. (1). An increase of noise intensity about ten times means that $R_2$ and $R_1$ should change respectively to result in a change of shot noise intensity. Although we don’t know the exact values of both resistances $R_2$ and $R_1$ we should expect that noise measurements allow shedding light on the relation between these resistances and further assess the state-of-health of the electrode-electrolyte interface, and the tested EDLC in general.

![Fig. 3. Power spectral density $S_u(f)$ of voltage fluctuations observed across the loading resistance $R = 100 \, \Omega$ versus frequency $f$ when the sample JP12.1 was discharged by a constant current $I = 650 \, \text{mA/g}$ to 0 V and next short-circuited for 2 h, followed by attaching the loading resistance $R$ to the terminals before noise measurements. Voltage fluctuations were recorded after 4 h (solid line) and after 24 h (dotted line) of aging by floating at 2.5 V.](image)

![Fig. 4. Power spectral density $S_u(f)$ of voltage fluctuations observed across the loading resistance $R = 100 \, \Omega$ versus frequency $f$ when the sample JP12.1 was discharged by a constant current $I = 650 \, \text{mA/g}$ to 0 V and next short-circuited for 2 h, followed by attaching the loading resistance $R$ to the terminals before noise measurements. Voltage fluctuations were recorded after 4 h (solid line) and after 24 h (dotted line) of aging by floating at 2.5 V.](image)

The second method of noise observation was applied when the sample was discharged – the charge collected in the capacitance $C_1$ was removed, but some charge was still preserved inside the capacitance $C_2$ (Fig. 2). After discharging and short-circuiting for 2 h, the loading resistor $R = 100 \, \Omega$ was attached to the terminals of the investigated EDLC. Then, the capacitance $C_2$ charged $C_1$ slowly and generated a voltage between the terminals of the EDLC. We observed that the voltage between the terminals slowly increased reaching the maximum value over a dozen mV after about 15 min. from the moment of attaching the loading resistor $R$. Then, we measured voltage fluctuations and determined their power spectral densities after the trend removal procedure. Power spectral densities $S_u(f)$ of voltage noise across the resistor $R$ are shown in Fig. 4. The intensity of $S_u(f)$ changed slightly after aging at the white noise region only. We may conclude that the aging process does not affect the capacitance $C_2$ and therefore we don’t observe changes in noise level.

We should underline that noise measurements of the EDLCs require a few hours of voltage time series recording in both applied methods. Moreover, noise intensity is relatively low and requires low-noise measurement set-up which may...
limit future applications of the proposed method to monitor a SOH of the EDLCs.

V. CONCLUSIONS

We propose an electrical noise circuit of the EDLC which explains the results of our experimental studies. Moreover, selected problems of noise measurements in the EDLCs are presented. Measurement results were considered to point out future practical applications for their SOH evaluation.

We conclude that noise may be used to assess the state of the electrode-electrolyte interface. Any changes within this area determine the intensity of shot noise, which was confirmed by our measurement results. Additionally, some changes in flicker noise may also be present but of lower intensity than for shot noise. We argue that noise measurements may be utilized in practice for the EDLCs assessment.

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REFERENCES

Ultra-low noise, single JFET voltage pre-amplifier for low frequency noise measurements

Graziella Scandurra
Dept. of Engineering
University of Messina
Messina, Italy
gscandurra@unime.it

Gino Giusi
Dept. of Engineering
University of Messina
Messina, Italy
ggiusi@unime.it

Carmine Ciofi
Dept. of Engineering
University of Messina
Messina, Italy
c ciofi@unime.it

Abstract—We propose an approach to the design of voltage amplifiers for low frequency noise measurements applications that is based on a single transistor (JFET) amplifier as the first stage. With respect to differential stages, a single transistor input stage has the advantage, among others, of allowing for a lower background noise. The problem of the dispersion of the gain, in the absence of a passive feedback loop, is addressed by introducing auxiliary operational amplifier based stages in parallel to the very low noise amplifier chain and using cross correlation among the outputs at a conveniently high frequency for obtaining the accurate estimate of the gain of the main amplifier in a short time and while measurement is in progress. With the approach we propose no adjustment of any kind is required on the circuit parameters of the main amplifier prior to any new measurement.

Keywords—Low noise voltage amplifier, cross correlation, noise measurements

I. INTRODUCTION

Performing meaningful noise measurements on electron devices and systems requires that the Background Noise (BN) of the measurement chain be much smaller than the noise to be detected [1]. In the case of voltage noise measurements, the BN of the system can be reduced to the Equivalent Input Voltage Noise (EIVN) of the voltage amplifier directly connected to the Device Under Test (DUT). A possible approach to the design of very low noise voltage amplifiers is to add a differential stage based on discrete, very low noise, devices in front of a solid state high gain stage. In this way, the entire system can be regarded as a very low noise operational amplifier whose gain can be set and stabilized by means of passive feedback [2-6]. This approach has the added advantage that, because of the input differential stage, coupling to the DUT down to DC is possible, which can be relevant in the field of Low Frequency Noise Measurements (LFNM) where the frequencies of interest may extend well below 1 Hz [1]. While the lowest level of EIVN are obtained by employing BJT based input stages [1,3], JFET input stages are preferred in all those cases in which noise to be measured on nodes where a significant DC component is also present. This is often the case in the field of noise measurements on electron devices, since measurements need to be performed on biased DUTs. Using BJTs, AC coupling down to the tens of mHz range would be made quite difficult because of the large bias currents and equivalent input noise current of these devices.

A feedback amplifier design approach allows to compensate for the spread in the discrete device characteristics so that a stable and known gain can be obtained regardless of the individual device being used. This simplification, however, comes at a price. Compared to a Common Source (CS) single transistor stage, the EIVN is at least doubled because both transistors in the input stage equally contribute to the BN [5]. Moreover, obtaining stability when designing with an operational amplifier based on a discrete component input stage is never easy and frequency compensation often results in reduced bandwidth and BN increase at higher frequencies [5, 6]. It is for these reasons that we devoted our attention to the possibility of designing a low noise voltage preamplifier based on a single transistor input stage that could be effectively employed in the field of LFNM.

II. PROPOSED APPROACH

The simplest possible implementation of a low noise voltage amplifier based on a single transistor stage for application in the field of LFNM is reported in Fig. 1. The active device is a low noise JFET that insures that we can obtain AC input coupling down to very low frequencies. In our prototype we employ a IF3601 large area JFET by InterFet, capable of providing transconductance gains \(g_m\) in the order of a few tens of mAV with a drain current in the order of a few mA. This device is characterized by an equivalent input noise that can be as low as 0.3 nV/√Hz at 100 Hz. As it is characteristic of JFET devices, DC and AC parameters are quite spread. The datasheet for IF3601 lists a pinch off voltage \(V_{GS_{p}}\) as follow: for a typical value for the saturation current at DC \(I_D\), the particular \(V_{GS_{p}}\) is certainly smaller than 2 V means that for \(V_{GS}\) equal to 6 V or more, the bias current \(I_D\) can be set with a reasonable error (typically less than 10%) by the ratio \(V_{GS}/R_S\). In other words, with the simple bias configuration in Fig. 1, the effect of parameter dispersion is modest, as far as the bias point is concerned. Note, however, that this does not insure in any way

![Fig. 1. Schematic of a low noise voltage amplifier using a Common Source (CS) JFET amplifier as a first gain stage.](image-url)
that we obtain a predictable and known gain for the amplifier, independent of the particular device being used. Dealing with the problem of a largely unpredictable overall gain without resorting to time consuming calibration procedures that require the modification of circuit parameters, is the main subject of the paper. However, if a circuit like the one reported in Fig. 1 is to be used for LFNM, we must insure a passband extending well below 1 Hz, a result that is not easily obtained. The small signal equivalent circuit for gain and noise calculation in the circuit in Fig. 1 is reported in Fig. 2. In the circuit in Fig. 2, $e_{in}$ is the source representing the thermal noise of the resistance $R_s$ (we assume that all resistors in the circuit are excess noise free resistors); the parameter $g_m$ is the transconductance of the JFET corresponding to the bias point in Fig. 1; $e_x$ is the noise source representing the noise generated by the JFET and $e_{o1o2}$ is the equivalent input noise voltage of the solid stage amplifier cascaded to the first stage (we assume, for the sake of simplicity, negligible equivalent input current source). For the time being, let us assume that all noise sources are turned off (replaced by short circuits) so that we can proceed with the evaluation of the transfer function. Applying standard circuit solving techniques we obtain:

$$A_v = -A_c \frac{st_A}{1 + st_s} + \frac{st_D}{1 + st_s} \frac{g_m(R_D)(R_B)}{1 + g_mR_S}$$

(2)

where

$$\tau_A = R_sC_s; \tau_D = (R_D + R_B)C_o; \tau_S = R_sC_s$$

$$\tau'_S = \frac{R_s}{1 + g_mR_S}; R'_S = \frac{R_s}{1 + g_mR_S}$$

(3)

From (2) it is apparent that, since $(\tau'_S < \tau_s)$, a constant frequency response is obtained for frequencies larger than the one corresponding to the pole with the largest magnitude. With resistances $R_1$ and $R_2$ in the order of a few MΩ and $C_s$ and $C_o$ in the order of a few tens of μF, the pole frequencies to $\tau_A$ and $\tau_D$ can be easily set to be below 10 MHz, but in the case of the pole frequency corresponding to $\tau'_S$ we have the problem that $R_1$ must be in the order of 1 kΩ to obtain bias currents in the order of a few mA according to (1). Moreover, since $g_m$ is in the order of a few tens of mA/V, $R'_S$ in (3) may be as low as a few tens of Ω. Therefore, the only way to obtain a value of $\tau'_S$ in the order of a few seconds or tens of seconds (to obtain a pole frequency well below 1 Hz) is to resort to a capacitor value in the order of 1 F or more for $C_s$. While up to a few years ago this would have been largely impractical, nowadays supercapacitors in very compact size are easily available with capacitances ranging from a few mF to a few F. Note that, because of the circuit configuration in Fig. 1, the DC voltage drop across the capacitor $C_s$ is $-V_{gs}$ that, in the case of the JFET we employ in our design, is typically below 1 V. Unlike other high specific capacitance devices (electrolytic capacitors) supercapacitor have been shown to be compatible with instrumentation intended for LFNM applications [7-9]. From (2) we can calculate that the passband gain $A_{PB}$ (assuming $R_p >> R_B$) is:

$$A_{PB} = -A_c g_m R_D$$

(4)

We can now evaluate the Power Spectral Density (PSD) of the equivalent input voltage noise of the amplifier using Fig. 2. We will limit our analysis to frequencies well into the passband (all capacitors replaced by short circuits) in the reasonable hypothesis of uncorrelated noise sources. As before, we will assume $R_p >> R_B$. We have:

$$S_{ENV} = S_{EN} + \frac{4kT R_D}{(g_m R_D)^2} + \frac{S_{ENAG}}{(g_m R_D)^2}$$

(5)

where $S_{EN}$ is the PSD of the overall equivalent input noise of the amplifier, $S_{ENAG}$ is the equivalent input noise of the solid state amplifier with gain $A_c$, $k$ is the Boltzmann constant and $T$ is the absolute temperature. With reference to (5) it is important to note that even assuming conservative values for $g_m$ and $R_D$ (say $g_m=10$ mA/V, $R_D=1$ kΩ), the noise coming from the resistance $R_D$ and the equivalent input noise of the second stage would be divided by a factor 100, so that neither of them, with proper design of the solid state amplifier, contribute significantly to the overall equivalent input noise. The $ENVM$ of the amplifier can be therefore reduced to the equivalent input noise of the JFET alone. While the extremely low noise represents an advantage with respect to feedback configurations [5], we are left with the problem of a gain that is directly proportional to $g_m$ that, as we have noted before, does change considerably from one device to another. Our approach to address this issue is illustrated in Fig. 3.

With reference to Fig. 3, the uppermost amplifier ($A_1$) in the figure represents a very low noise amplifier with an unknown gain $G_1$, while the other two ($A_2$ and $A_3$) are Operational Amplifier (OA) based amplifiers with stable and well known gains $G_2=\frac{G}{G_1}=G$. Let us assume that we can perform cross correlation among all the three output channels. For the sake of simplicity we will assume all gains to be real. Ideally, the equivalent input noise of the amplifiers are cancelled out in the cross correlation process so that:

$$S_{o13} = S_G G_1$$

(6)

$$S_{o23} = S_G G^2$$

where $S_{o13}$ is the cross spectrum between outputs $V_{o1}$ and $V_{o3}$ and $S_I$ is the PSD of the input $V_I$. Using (6) we have:

$$G_1 = \frac{S_{o13}}{S_{o23}}$$

(7)

that means that we can calibrate the unknown gain $G_1$ from the known gain $G$ and the measurement of the cross spectra between channels 1 and 2 and 2 and 3 independently of the shape of $S_I$. At a first sight, the following obvious question may arise: why bother with the design of a low noise amplifier?
if one can obtain the correct estimate of the input noise by cross correlation between channels 2 and 3 regardless of the level of equivalent input noise, as suggested in (6)? The fact is, as it is very well known, that reducing the uncorrelated noise contribution in cross correlation measurements requires averaging the results over several independent time records [10]. The level of uncorrelated noise decreases with the square root of the number of averages and therefore, the time required to uncover the correlated component (the one we want to measure) from the uncorrelated ones depends on the level of uncorrelated noise with respect to the correlated one. Suppose the uncorrelated noise is 10 dB above the noise to be measured; in order to reduce it to 10 dB below the noise to be measured the averaging of about 10^4 records is required. In FFT based spectrum analyzes, the duration of the record used for elaboration is the inverse of the resolution bandwidth Δf used for analysis. Especially in the case of the analysis of flicker spectra, the resolution bandwidth required to insure the absence of systematic errors must be significantly smaller than the minimum frequency of interest [11]. If the minimum frequency of interest is 100 mHz, as it can be the case in measurements on electron devices [1], the resolution bandawidth Δf must be in the order of 10 mHz and therefore the record length is in the order of 100 seconds (1/Δf). This means that performing 10^4 averages, as in the example above, would require more than ten days of uninterrupted measurement. If we go back to the approach we propose, assuming that the gains of the amplifiers in Fig. 3 be constant over a sufficient frequency interval, we can use a conveniently larger resolution bandwidth since we are only interested in calculating the gain G1 of the amplifier A1 according to (7). This allows to considerably reduce the time required to reach the correct estimate of the cross spectra: with Δf=100 Hz, for instance, 10^4 averages can be performed in less than two minutes. With these considerations in mind, our approach can be summarized as follows: we design a low noise amplifier (main amplifier) whose first stage is based on a single active device for obtaining the lowest possible BN at very low frequencies; we add two auxiliary OA based amplifiers to obtain the configuration in Fig. 3; we employ cross correlation with a large resolution bandwidth among the outputs in Fig. 3 in order to obtain an accurate estimate of the gain of the main amplifier; the estimate gain is used to calculate the PSD of the noise at the output of the main amplifier form the measurement of the PSD at the output V03 in Fig. 3.

III. SYSTEM PROTOTYPE AND EXPERIMENTAL RESULTS

The prototype of the system used for the demonstration of the approach we propose is reported in Fig. 4. With respect to the circuit in Fig. 1, the output stage is modified in order to allow to employ a transeristance amplifier based on the low noise OP27 as a gain stage. The configuration we employ is similar to the one in [12]. The overall passband gain of the amplifier is g_m R_s and can reach values in the order of 60 dB for g_m close to 20 mAV. C_L is chosen in such a way that the associated pole frequency remains below 10 mHz regardless of the actual value of g_m in the expected interval between 10 and 50 mAV. In this way the lower frequency corner is set by C_D together with R_0 (10 mHz). This means that we can expect an essentially flat response starting from 100-200 mHz, depending on the position of the pole that set by g_m. The amplifiers A2 and A3 are nominally identical voltage amplifier based on TLC070 operational amplifiers as in Fig. 5. They are characterized by extremely low input bias current and equivalent input current noise (so that they do not interfere appreciably with the main amplifier circuit), have a passband gain of 61 dB and a low and high frequency corner of 16 mHz and 100 kHz, respectively. The equivalent input noise at 1 kHz is in the order of 10 nV/√Hz, and therefore much larger than the E/VN of the low noise amplifier that is expected to be below 1 nV/√Hz. Noise measurements are performed using a 4 channel PCI-4462 DSA board. The software for data acquisition and spectra and cross spectra elaboration has been developed around the QLSA library [13]. QLSA, as discussed in [13], allows to perform spectral estimation using a quasi logarithmic frequency approach. In the simplest terms, QLSA behaves as a set of conventional spectral analyzers in parallel, each covering a different frequency range with proper resolution bandwidth. This allows to estimate spectra and cross spectra at higher frequencies with large resolution bandwidth, and spectra at lower frequencies with lower and lower resolution bandwidths. Using QLSA is particularly useful in this application since cross spectra calculated at higher frequencies (with larger resolution bandwidths) can be used to calibrate the amplifier gain, while at the same time measurement in the very low frequency range are being performed with the required much smaller resolution bandwidth. In principle, using QLSA, gain calibration and corresponding spectra correction can be performed on line, during a single measurement session. As a first test we used a 50 kΩ resistance as a DUT. The thermal noise of the DUT at room temperature is 29 nV/√Hz, well above the expected equivalent input noise of main amplifier down to the hundreds of mHz range. The result of the estimation of the PSD of the noise at the output V01 in Fig. 4 is reported in Fig. 6 (curve labeled S1) and demonstrates a flat response down to the hundreds of mHz range. The decrease in the spectrum at frequencies above 2 kHz is due to the effect of the equivalent capacitance at the gate of the JFET that, because of the large area, is in the order of 0.5 nF, as can be verified in the datasheet. The spectrum at the output of one of the auxiliary
amplifiers is also shown in Fig. 6 (curve S33) together with the cross spectra $S_{13}$ and $S_{23}$ required to calibrate the amplifiers, two more OA based preamplifiers characterized by stable and known gain.

The equivalent input noise as measured after gain calibration when a 50 kΩ resistor is used as a DUT is much smaller than the EIVN of the auxiliary amplifiers (inset in Fig. 7), corresponding to the expected value for the JFET alone. By employing supercapacitors for bias network decoupling, the passband can easily extend down to the hundreds of mHz range and below. No adjustment to the circuit parameters is required to calibrate the gain: the value of the gain is obtained during the actual measurement on the DUT by resorting to the properties of cross correlation applied to a proper configuration including, besides the main amplifier, two more OA based preamplifiers characterized by stable and known gain.

Fig. 6. Spectra and cross spectra as evaluated at the outputs of the circuit in Fig. 4 when a 50 kΩ resistor is used as a DUT.

Fig. 7. Equivalent input noise as measured after gain calibration when a 50 kΩ, 100 Ω and 0 Ω are at the input of the amplifier. The insets report the noise level (measured at the outputs) and used for gain calibration.

The equivalent input noise as measured after gain calibration when a 50 kΩ resistor is used as a DUT is much smaller than the EIVN of the auxiliary amplifiers (inset b in Fig. 7). As it is apparent from Fig. 7, the resulting equivalent input noise after gain calibration is larger than the one expected form a 100 Ω resistance (dashed line) and this is due to the contribution of the EIVN of the main amplifier that is no longer negligible. In order to estimate the EIVN of the main amplifier, we measured the PSD at the output $V_{th}$ and calculated the equivalent input PSD using the value of the gain measured in the case of $R_{DUT}=100$ Ω. As reported in Fig. 7, the EIVN of the amplifier that has been build is below 1 nV/√Hz above 2 Hz, and approaches 0.5 nV/√Hz for frequencies above 1 kHz.

IV. CONCLUSIONS

We proposed an approach for the realization of a very low noise voltage amplifier for applications in the field of LFNMe using a single JFET CS stage as the input stage. The approach we propose results in extremely simple hardware and in the minimum possible level of background noise for a given device, as it is reduced to the one introduced by the JFET alone. By employing supercapacitors for bias network decoupling, the passband can easily extend down to the hundreds of mHz range and below. No adjustment to the circuit parameters is required to calibrate the gain: the value of the gain is obtained during the actual measurement on the DUT by resorting to the properties of cross correlation applied to a proper configuration including, besides the main amplifier, two more OA based preamplifiers characterized by stable and known gain.

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Low frequency noise characterization and modeling of SiGe HBT featuring LASER annealing in a 55-nm CMOS node

J. El Beyrouthy, A. Vauthelin, B. Sagnes, F. Pascal, A. Hoffman, M. Valenza
IES, Univ Montpellier, CNRS
Montpellier, France
bruno.sagnes@ies.univ-montp2.fr

S. Haendler, A. Gauthier, P. Chevalier, D. Gloria
STMicroelectronics, 850 rue Jean Monnet
38926 Crolles Cedex, France

Abstract— This work presents Low Frequency Noise (LFNoise) characterization and modeling performed on DPSA-SEG SiGe HBT integrated in a 55-nm CMOS node. The aim of this study is to evaluate the advantage brought by the implementation of a Dynamic Surface Annealing (DSA) in addition to the well-known Spike Annealing process.

The HBTs are supplied by STMicroelectronics Crolles and present transit \( f_t \) and maximum oscillation \( f_{\text{MAX}} \) frequencies in the 320-370 GHz range.

Spectra can be affected by the presence of generation-recombination (GR) components. The 1/f noise amplitude is modeled following the SPICE compact model, and the 1/f parameters \( k_B \) and \( A_f \) are calculated. The extracted figure of merit \( k_B = k_j A_r \) has a very good value of \( 6.8 \times 10^{-10} \mu \text{m}^2 \) for transistors processed using the DSA technique.

Keywords— HBTs, BiCMOS technologies, Low Frequency Noise, 1/f Noise, Laser annealing

I. INTRODUCTION

Over the last ten years, there has been an increasing demand for RF, MMW, optoelectronics and for future TeraHz applications. In order to provide integrated circuits in such high frequency range, elementary bipolar transistors should reach very high frequency performances. If at the beginning III-V transistors seemed to be the most promising in terms of frequency performance, the Si/SiGe base-emitter Heterojunction Bipolar Transistors (HBT) appeared in the early 2000s as one of the best candidates. In addition to its excellent performances, its compatibility with conventional CMOS technologies also makes this technology essential.

The HBT devices studied in this work are developed by STMicroelectronics for RF, MMW and THz applications in the framework of the ongoing European TARANTO project [1]. They are based on the structure of reference used in the B55 technology [2] and showing 320 GHz \( f_t \) and 370 GHz \( f_{\text{MAX}} \) performances.

In order to achieve that objective, the HBTs process includes a Double-PolySilicon Fully-Self-Aligned architecture associated with a selective SiGe:C epitaxy for the base. In particular, to control the DC and HF transistor performances, it is essential to control the emitter-base junction doping level. For instance, to prevent the out-diffusion of Boron and thus to control the base doping profile, some carbon (< 1 %) is incorporated in the SiGe layer. Still focused on the optimization of the E-B junction, another important step process is the optimization of the thermal budget used to activate the dopants (note that effective activation of dopants is also important to lower sheet resistances). Thus, in order to improve the performance of the transistors while remaining compatible with CMOS technology, the thermal budget should be carefully adjusted.

This paper presents a comparison of low frequency noise (LFNoise) characteristics in Si/SiGe:C HBTs processed using two distinct dopants activation techniques. The first one used a classical thermal activation technique (spike annealing), referred to as B55THA, while the second one used a DSA (Dynamic Surface Annealing) process, referred to as B55DSA, in addition to a lower spike annealing temperature. This is done in order to limit the dopant diffusion while providing a higher activation level [3]. The main goal of this work is to evaluate the advantage brought by the implementation of a Dynamic Surface Annealing. DC characteristics will be compared but, due to its sensibility, a focus will be made on LFNoise (spectral analysis and 1/f noise modeling). If most of the LFNoise works on modern bipolar transistors lead to the conclusion that 1/f noise sources are located in the emitter-base region (mainly in its intrinsic part), investigations on different technological parameters are needed in order to go further [4] [5] [6]. In this frame, the comparison of HBTs developed with and without the DSA process could also be of interest.

II. EXPERIMENTAL SETUP AND DEVICES

The measurements are performed using a manual probe station. On-wafer contacts are performed using coplanar probes. I-V characteristics (Gummel-plots) are measured using a Keithley 4200 semiconductor characterization system. LFNoise measurements are performed in the 10 Hz-100 kHz frequency range. HBTs are biased in a common emitter configuration. 3 emitter areas are studied for the B55THA process (\( A_e = 1, 3 \) and \( 4.2 \mu \text{m}^2 \)), and 4 for the B55DSA one (\( A_e = 0.6, 1, 2.5, \) and \( 4.2 \mu \text{m}^2 \)). For each area, measurements are performed on several dies. The input base current noise spectral density, \( S_m \), is measured in high impedance configuration (Figure 1), with a spectrum analyzer (HP89410A) and a low noise transimpedance amplifier (EG & G 5182). The transistors are placed in a...
Faraday cage and biased by batteries to avoid external disturbances.

Fig.1: LFNoise experimental set-up used for the measurement of the input base current spectral density, $S_{Ib}$.

III. DC CHARACTERISTICS

Figure 2 and 3 show typical Gummel plots obtained on both B55THA and B55DSA processes. For each technology, 3 devices with an emitter area of 4.2 μm² are plotted. Same base and collector current components are observed. The die to die dispersion (measured over 10 dies) is ranging between 4 and 7%. The maximum current gain $h_{FE}$ is of 2300 and 2500 for B55THA and B55DSA transistors respectively.

IV. LOW FREQUENCY NOISE CHARACTERISTICS

We measured $S_{Ib}$ as a function of the base bias current and the emitter area $A_e$. Representative spectra obtained on both type of transistors are reported in Figures 4 and 5. For instance, concerning the spectra obtained on B55DSA (Figure 5), 11 HBTs with an emitter area of 1 μm² were tested. 5 exhibit spectra composed of a 1/f noise component followed by the shot noise $2qI_b$, the others are affected by the presence of generation-recombination (GR) components more or less pronounced. The 1/f noise amplitude dispersion is estimated to be less than one decade. More generally, almost 40% of the tested components are GR free whatever the type of transistors.

As reported in [11], we found that most of the GR components are related to RTS noise. To take into account the presence of GR components, a statistical approach (i.e. average spectrum done over a large number of measured spectrums) was proposed in Poly-Emitter BJTs [7] and in SiGe HBTs [8].

- Fig. 2: Gummel plot and current gain of 3 HBTs of the B55THA process, $A_e = 4.2 \mu m^2$.

- Fig. 3: Gummel plot and current gain of 3 HBTs of the B55DSA process, $A_e = 4.2 \mu m^2$.

- Fig. 4: Examples of spectra obtained on 6 HBTs of emitter area $A_e = 4.2 \mu m^2$ and for $I_b = 100 nA$ (B55THA).

- Fig. 5: Examples of Spectra obtained on 6 HBTs of emitter area $A_e = 1 \mu m^2$ and for $I_b = 100 nA$ (B55DSA).
Now, focusing on the 1/f noise component only, its amplitude is modeled using the usual 1/f SPICE model [9]:

$$S^{1/f}_{IB} = K_F \frac{A_F}{f}$$ \hspace{1cm} (1)

where $K_F$ is the parameter representing the 1/f noise amplitude and $A_F$ its evolution with the base bias current.

The evolution of $S_{IB}$ at 1 Hz versus base current $I_B$ determines the $A_F$ coefficient. Figure 6 gives a representative example of $S_{IB}$ at 1 Hz versus $I_B$ for B55DSA HBTs with 3 emitter geometries. Base bias currents are 50, 100 and 500 nA. As can be seen, a quadratic dependence is found leading to a value of $A_F$ close to 2. Same behavior was observed for the B55THA process. This classic result was reported in all the works related to 1/f noise in modern Si-BJTs [10] and SiGe HBTs [5].

![Fig. 6: B55DSA 1/f noise level ($S_{IB}$ at 1 Hz) versus the base current $I_B$.](image)

The 1/f noise level is then studied as a function of the emitter area. In Figure 7, where the dimensionless coefficient $K_F$ is plotted versus $A_e$ for several dies, the general trend for the two types of processes is that the 1/f noise level is found to be inversely proportional to the emitter area. The 1/f noise dispersion observed in both Figures 4 and 5, for emitter areas of 4.2 and 1 $\mu$m² as examples, confirms that the dispersion is always less than one decade. Nevertheless, the mean 1/f noise level is slightly lower in the case of the DSA technique. As $K_F$ is found to be inversely proportional to $A_e$, the product $K_F \times A_e = K_B$ (in $\mu$m²) can be used as a very convenient figure of merit in order to compare different technologies [9] [11] or processes/parameter steps [6]. The comparison between the two types of transistors is in favor of the DSA technique with a value of $K_B$ of 6.8 $10^{-10}$ $\mu$m² versus 1.3 $10^{-9}$ $\mu$m² for the THA one.

![Fig. 7: SPICE parameter $K_F$ versus emitter area $A_e$.](image)

**V. DISCUSSION/CONCLUSION**

The impact of the DSA process on high frequency performances results in the improvement in HBT transit frequencies of 5 – 7 % (due to a decrease of As-dopant diffusion from emitter) and in a better MOSFET compatibility [3].

From our DC characterizations, we observed an increase of the current gain $h_{FE}$ in HBTs processed with the DSA technique compared to the THA one. In both types of transistors, a small dispersion of the DC results is observed.

Concerning the focus of this work (i.e the comparison of the LFNoise of 2 types of transistors), the global behavior of LFNoise remains unchanged: GR components are always present with characteristics (cut-off frequencies and plateaus) that can vary very significantly and randomly. The 1/f noise level is reduced by a factor 2 for the B55DSA HBTs compared to the B55THA ones. Hence the reduction of the 1/f noise is noticeable. In both cases, the 1/f noise dispersion remains reasonable. The approximate $K_F$ values of 6.8 $10^{-10}$ $\mu$m² and 1.3 $10^{-9}$ $\mu$m² for DSA and THA respectively are very good even if they do not reach the very best ones of 4 $10^{-11}$ $\mu$m² [12] and 6.7 $10^{-11}$ $\mu$m² (our result) [13], both obtained from mature 130 nm BiCMOS technologies. Note that these best values are obtained from a single die, no dispersion was taking into account in ref. [12], while we published in [8] extended works (statistical study based on 61 tested devices) leading to a mean value of 6 $10^{-10}$ $\mu$m². In order to take into account the high frequency and the LFNoise performances, a better figure-of-merit was introduced: the ratio $f_c/f_T$ [14] [11]. $f_c$ is the 1/f noise corner frequency, measured from the intercept of the 1/f component and the white noise on the $S_{IB}$ spectrum. This parameter is the image of the 1/f noise amplitude. $f_T$ is the transit frequency. The use of this figure-of-merit is clearly in favor of the DSA technique as both $f_c$ and $f_T$ are improved.

Concerning the localization of the 1/f noise sources, the combined effect of the 1/f noise level ($S_{IB}$ at 1 Hz) quadratic dependence on the base current and the 1/Ae dependence of $K_F$ indicates that the associated noise sources are uniformly distributed over the entire emitter area. This conclusion, which is found in all the papers concerning 1/f noise in...
recent bipolar transistors, is based on the study of several physical models referring to an $I_B^2$ law and an $A_e^{-1}$ dependence [9][15]. Also, the study of the voltage coherence function (between input ($v_i$) and output ($v_o$) fluctuations), gives a direct argument concerning the localization of the $1/f$ noise sources at the emitter-base junction [12][16]. Even if almost all the published work indicates that the $1/f$ noise sources are homogeneously distributed at the emitter-base area, there is no evidence of their exact localization. This is due to the sophisticated architecture of the emitter-base region with, in addition to the complex internal junction, the possible influence of the oxide spacer, emitter perimeter, series resistance, poly-silicon/silicon interface...

Concerning the observed reduction of the $1/f$ noise in the DSA transistors, the question is: can it be a direct proof of the effect of this technique and thus demonstrate the better structural or electrical quality of the deposit layer (more effective activation of dopants)? Not sure because it could be an indirect effect associated to the better control of the dopants, for instance a better control of the emitter-base junction depth can play a role. Effectively we have clearly demonstrated in [17] that when the electrical E-B junction depth increases (with respect to poly/mono interface), the $1/f$ noise level decreases. This could be attributed to the reduction of the interaction of the minority carriers with interface states located at the poly/mono interface.

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Low-Frequency Noise in InGaZnO Thin-Film Transistors with Al₂O₃ Gate Dielectric

Horacio Londoño Ramírez
imec, Neuro-Electronics Research Flanders and KU Leuven
Leuven, Belgium
Horacio.Londo_Ramirez@nerf.be

Yu-Chieh Chien
imec and National Sun Yat-Sen University
Leuven, Belgium & Kaohsiung, Taiwan
ycchien@ieee.org

Luis Hoffman
imec and Neuro-Electronics Research Flanders
Leuven, Belgium
Luis.Hoffman@nerf.be

Manoj Nag
Large-Area Electronics imec
Leuven, Belgium
Manoj.Nag@imec.be

Sebastian Haesler
Dept. of Neurosciences
Neuro-Electronics Research Flanders and KU Leuven
Leuven, Belgium
Sebastian.Haesler@nerf.be

Soeren Steudel
Large-Area Electronics imec
Leuven, Belgium
Soeren.Steudel@imec.be

Jan Genoe
Large-Area Electronics, Dept. of Electrical Eng.
imec and KU Leuven
Leuven, Belgium
Jan.Genoe@imec.be

Abstract—We present a study of the low frequency noise (LFN) behavior of amorphous indium gallium zinc oxide (a-IGZO) thin film transistors (TFTs) fabricated with Al₂O₃ as gate dielectric. The LFN follows a 1/f-spectrum and can be described using the carrier number fluctuation theory with correlated mobility fluctuations, with a calculated trap density (Nᵥ) of $4.1 \times 10^{13}$ cm⁻²eV⁻¹ and a scattering coefficient ($\alpha_s$) of 1.5 $\times 10^3$ Vs/C. For low drain currents, LFN originates from fluctuations in the flat band voltage, whereas in lineal regime the noise is dominated by mobility fluctuations. The extracted Hooge’s parameter ($a_H$) is ~1.1 $\times 10^3$ for devices with Al₂O₃ as gate dielectric, the lowest ever reported for a-IGZO TFTs using a high-k dielectric without any interfacial dielectric.

Keywords—Thin-film transistor, a-IGZO, Low-frequency Noise, Al₂O₃, Hooge’s parameter

I. INTRODUCTION

Metal oxide semiconductor thin-film transistors (TFTs), in particular amorphous indium gallium zinc oxide (a-IGZO) TFTs, have attracted interest for display and flexible electronics applications due to their low deposition temperatures, large area uniformity and good electrical characteristics, such as high electron mobility (~10 cm²/Vs) and low subthreshold swing [1]. Low frequency noise (LFN) can be used as a characterization technique to study defects in amorphous semiconductor and the quality of the semiconductor and gate dielectric interface. Additionally, for analog circuit applications, noise is an important issue to be considered as it sets the lower limit of the signal amplitude in the system. The use of high-k dielectrics as the gate dielectric has the potential to improve device performance by reducing the operational voltages and power consumption, but usually comes with a degradation in noise performance [2]–[6].

As previously reported in [3] and [6], the use of Al₂O₃ as gate dielectric had a detrimental effect on the noise of the TFTs. It was concluded that the high LFN was due to an increase of the electron-phonon scattering originating from the remote phonon modes in the high-k dielectric. To overcome this problem and obtain devices with low noise while using high-k dielectrics, SiO₂ or SiNₓ have been employed as an interfacial dielectric in between the semiconductor and the high-k dielectric, which can improve the interface quality or suppress the remote phonon scattering from the high-k dielectric [3], [4], [7]. However, this comes with an increase in the number of fabrication steps.

In this work, we characterize the LFN of a-IGZO TFTs with Al₂O₃ as gate dielectric, without any additional interfacial dielectric layer. The origin of the LFN is investigated and the carrier number fluctuation theory with correlated mobility fluctuations ($\lambda$-N–$\Delta\mu$) is employed to describe the measurements.

II. EXPERIMENTAL

Top-gate Self-Aligned (SA) a-InGaZnO TFTs with Al₂O₃ gate dielectric were fabricated following a process flow similar to the one reported in [8]. A schematic cross-section of fabricated TFTs is presented in Fig. 1 (a). Briefly, fabrication started with an Al₂O₃ buffer layer deposited by atomic layer deposition (ALD) on a glass substrate. As the active layer, 12 nm of a-InGaZnO was deposited by DC sputtering and the device’s active area was defined by wet etching. 20 nm thick Al₂O₃ was deposited by low temperature ALD as gate dielectric, followed by the sputtering of Mo/TiN as gate metals. The gate metal and gate dielectric were patterned using dry and wet etching, respectively. 150 °C, a SiN interlayer was deposited by plasma enhanced chemical vapor deposition (PECVD). A metal stack of sputtered Ti/Al/Ti was patterned by liftoff and used as source/drain contacts. A final annealing at 250 °C for 1 hour under N₂ ambient was performed. Low-frequency noise measurements were carried out using a dedicated LFN setup (Keysight’s E4727A) at room temperature. The LFN was measured in saturation for low drain currents and linear regime of operation, with $V_{DS}$ set to 0.5 V and the overdrive voltage swept between 0 V and 4.5 V.

![Fig. 1 (a) Schematic cross-section of the fabricated top-gate self-aligned (SA) a-IGZO TFTs with Al₂O₃ gate dielectric, (b) Transfer characteristics, $I_{DS}$ – $V_{GS}$ curve, in linear regime ($V_{GS}$= 0.1 V) for W= 15 µm and $L$= 5 µm.](image-url)
III. RESULTS AND DISCUSSION

Fig. 1 (b) shows the transfer characteristics (\(I_{DS} \text{ vs. } V_{GS}\)) and transconductance (\(g_m\)) of a fabricated a-IGZO TFT with 15 \(\mu\)m and 5 \(\mu\)m channel width (W) and length (L) respectively, measured in linear regime of operation, \(V_{DS} = 0.5\) V. Normally, for IGZO TFTs, \(g_m\) increases steadily. In this case however, the transconductance reaches a maximum, indicating the presence of contact resistance.

The correct extraction of the threshold voltage (\(V_t\)) is a critical step to assess the origin of the noise but presents special difficulties for metal-oxide thin-film transistors since \(V_t\) is not clearly defined and a standardized methodology for extracting it does not exist. In the past, this ambiguity has led to opposite interpretations of the origin of the LFN when analyzing data of the same devices, as different models could be applied depending on how the threshold voltage was initially calculated [9], [10]. In this paper, \(V_t\) was calculated using the \(I_f/g_m\) method described in [11], as it takes into account the mobility power-law parameter present in IGZO TFTs and it is insensitive to the presence of contact resistance, present in our TFTs. \(V_t\) was then found to be 0.17 V.

The normalized drain current noise power spectral density (PSD) (\(S_{ID} / I_{DS}^2\)) of the A12O3 TFT, biased in the linear regime, is presented in Fig. 2 for different \(V_{DS} - V_t\). \(S_{ID} / I_{DS}^2\) decreases for increasing gate voltages and the LFN follows the expected \(1/f^n\) noise spectrum, with \(n = 0.8 - 1.1\) in the frequency range of 1 Hz to 500 Hz for all biases.

Two main theories have been used to explain the origin of the LFN in transistors: the carrier mobility fluctuations model (\(\Delta n\)), in which the noise arises from carrier-phonon scattering in the semiconductor, and the carrier number fluctuations model (\(\Delta N\)), in which the noise is caused by the flat band voltage fluctuations, usually associated with carrier trapping and detrapping in the gate dielectric close to the channel-interface [12].

For the carrier mobility fluctuation theory, the following empirical formula for the normalized drain current PSD is suggested

\[
S_{ID} / I_{DS}^2 = \frac{a_m}{f^{2n}}
\]

where \(f\) is the frequency, \(N\) is the total number of carriers involved in the conduction, and \(a_m\), known as the Hooge’s parameter, is a technology constant and it is used as a quality indicator of the device [13].

In the carrier number fluctuation theory, the normalized drain current PSD is defined as

\[
S_{ID} / I_{DS}^2 = \frac{g_m^2}{I_{DS}} S_{VFB}
\]

where \(g_m\) is the transconductance, and \(S_{VFB}\) is the flat band voltage spectral density associated with fluctuations of the interface charge [14] and is defined as

\[
S_{VFB} = \frac{q^2 kT}{C_g W L} N_t
\]

where \(q\) is the electron charge, \(C_g\) is the gate insulator capacitance per unit area, \(k\) is the Boltzmann’s constant, \(T\) is the temperature, and \(N_t\) is the density of traps near the channel/dielectric interface.

A way of assessing the origin of the low-frequency noise and distinguishing between the different theories, is by evaluating the dependence of \(S_{ID} / I_{DS}^2\) on \(I_{DS}\) and comparing it to \((g_m/I_{DS})^2\), as shown in Fig. 3 (a). In the carrier number fluctuations model, both quantities are proportional, with \(S_{VFB}\) as the proportionality constant, as stated in eq. (2). From Fig. 3 it is observed that the normalized current PSD is in good agreement with the \(\Delta n\) theory for low drain currents. Deviation between the PSD and \((g_m/I_{DS})^2\) starts occurring at a drain current of about 1 \(\mu\)A, which also corresponds to the bias condition for which the transistor transitions from saturation to linear regime \((V_t - V_o = V_d)\). From Fig. 3 (a), \(S_{VFB}\) can be calculated as \(S_{ID} / g_m^2\) for values in which both...
graphs overlap, \( S_{V_{th}} \) is then found to be \( 1.45 \times 10^{-11} \text{ V}^2/\text{Hz} \). Applying (3) allows extracting the trap density, 
\[ N_t = \frac{2 e V_x}{3 e_{eff} g_{m}^2 S_{V_{FB}}} \]
where \( e_{eff} \) is the effective mobility and its sign depends on the nature of the traps: negative for acceptor-like traps or positive for donor-like traps, and \( \alpha_n \) is the Coulomb scattering coefficient [13]. Fig. 3 (b) shows the fitting of (4) to the measurements previously presented, with \( S_{V_{th}} \) having the same value as before and \( \alpha_n \approx 1.5 \times 10^{11} \text{ V}\text{s}/\text{C}. \) By incorporating the correlated mobility fluctuations, it is possible to model the LFN of the IGZO TFTs both in saturation with low drain currents and in linear regime of operation.

Another way of assessing the dominant noise mechanism is by studying the dependence of \( S_{th} / I_{DS}^2 \) on the gate overdrive voltage on logarithmic scale, as shown by Hooge, a theory that proposes that the main noise contribution stems from carrier-phonon scattering in the bulk [13]. This dependence of \( S_{th} / I_{DS}^2 \) on gate bias suggests that the mobility fluctuations in the semiconductor is the dominant noise source of the LFN in our TFTs with Al_{2}O_{3}, when biased in the linear regime.

The Hooge’s parameter offers a convenient way of assessing the quality of the channel and channel/dielectric interface, as well as the noise performance of the fabricated TFTs, independently of the device’s geometrical factors. The lower its value, the higher the noise performance of the device. From (1), the Hooge’s parameter can be calculated as
\[ \alpha_H = \frac{I_{DS}^2}{S_{th}} f N, \]
where \( f \) is the frequency of the noise and \( N \) the total number of carriers in linear regime is given by
\[ N = \frac{L W}{q} \frac{C_{ox}}{C_{th}} \left( V_{ds} - V_{th} - \frac{V_{th}}{2} \right) \]
Substituting (5) in (6) one obtains
\[ \alpha_H = \frac{I_{DS}^2}{S_{th}} \frac{f C_{ox} W L}{q} \left( V_{ds} - V_{th} - \frac{V_{th}}{2} \right) \]

Figure 4 (a) shows the extracted Hooge’s parameter, which is approximately \( 1.1 \times 10^{-8} \), against \( V_{gs}-V_{th}/2 \) for the studied TFT with aluminum oxide. This value is comparable to the lowest reported in the literature for IGZO.
TFTs, as shown in Fig. 4 (b), and is 3 orders of magnitude lower than the ones previously reported for TFTs with Al$_2$O$_3$ gate dielectric [3]. One reason that could explain the discrepancy between our results and values previously reported, is found in [13], where it is suggested that the LFN of the IGZO TFTs with aluminum oxide gate dielectric reported in [3], follows the $\Delta n$ model instead of the $\Delta \mu$ model. Additionally, the value reported here is the lowest ever presented for an a-IGZO TFT, using only a high-k gate dielectric without the use of an interfacial layer. Moreover, it is comparable to the value reported in [16], where IZO TFTs with anodized Al$_2$O$_3$ as gate dielectric were characterized.

IV. CONCLUSION

In this work, we presented the characterization of the LFN of IGZO TFTs with low-temperature ALD Al$_2$O$_3$ as gate dielectric. The LFN follows the carrier number fluctuation theory with correlated mobility fluctuations, with a calculated trap density of $4.1 \times 10^{11}$ cm$^{-2}$ eV$^{-1}$ and a scattering coefficient ($a_n$) of $1.5 \times 10^{-4}$ Vs/C. For low drain currents, the LFN primarily originates from the fluctuations in the flat band voltage, while for operation in the linear regime the noise is dominated by the mobility fluctuations. The reported Hooge’s parameter is approximately $1.1 \times 10^{-3}$ and is the lowest ever reported for an a-IGZO TFT, using only a high-k gate dielectric without the use of an interfacial layer. This result demonstrates that Al$_2$O$_3$ as gate dielectric does not necessarily cause a detrimental effect in the LFN of a-IGZO TFTs and shows that it can be used as dielectric for low noise thin-film transistors, contrary to what was previously reported [3].

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Adhesive Low Frequency Noise (LFN) in Charge Trap Transistors (CTT) for Neuromorphic Analog Processing

Alexios Birbas
Dept. of Electrical and Computer Engineering,
University of Patras,
Patras, Greece 26500
birbas@ece.upatras.gr

Abstract—Neuromorphic devices performing synaptic processes in the analog domain are preferable from their digital hardware counterparts for the implementation of Artificial Neural Networks (ANN). The reason is their reduced area and power requirements. Charge Trap Transistors (CTT) have been recently proposed as analog neural network computing engines due to their CMOS compatibility as well. Gate controlled differential change of the threshold voltage related to charge storage into the high-k dielectric layer (traps) is the transduction mechanism for the non-volatile device operation; the same mechanism serves the learning process when it is employed as a synaptic element. The stochastic nature of the trapping (de-trapping) procedure produces low frequency noise of RTS type and modulates the channel current. This non-desirable but unavoidable counter-intuitive effect (detrimental to the device performance) could be adhesive if this internal noise is used in order to induce stochastic resonance (SR) into the synaptic operation of the CTT. Indeed since the synaptic element is a non-linear system, SR at an engineered range of noise intensities can improve system performance (making information at the input visible at the output). This expands the learning capability of CTT into multi-layer fully connected neural networks with non-linear activation functions.

Index Terms— RTS noise, High-k dielectrics, neuromorphic devices, Stochastic resonance, Charge-trap transistors, Artificial Neural Networks

I. INTRODUCTION

CTTs [1] have recently paved their way into non-volatile logic devices (memory) implementation for SoC applications. Their structure comprises an HFO based high-k metal gate (HKMG) and is fully compatible with state of the art CMOS fabrication (SOI, FinFET). This gate dielectric is known to exhibit oxygen vacancy related traps and defects which are strongly influenced by bias stress (self induced temperature) [2]. A SiO$_2$ interface layer is imposed between Si and the HFO$_2$ layer for blocking/tunneling purposes; it additionally contributes to the device good charge retention characteristics. LFN characteristics of high-k-dielectrics have been studied for device quality characterization and appear to be significantly increased [3]. In this work we adopt a model for the channel current LFN calculation of a CTT in the sub-threshold region based on the generalized flat band variation [4], where a charge pumped modulated density of states $N_i$ and a $C_{ox}$ stemming from the equivalent two layer dielectric thickness ($E_{ox}$) are incorporated (1).

$$S_{vbeff} = \frac{q^2 \lambda k T N_{t,occ,trapmodulated}}{FWLC_{ox}}$$

Apparently the LFN noise as it appears in the channel current is direct representation of the spectral density of the flat band fluctuation and strongly dependant on the voltage bias conditions ($V_{ds}$, $V_{g}$). It appears that the channel current noise exhibits a 1/f behavior as expected if one considers the aggregated effect of the RTS noise originating from all the oxide traps and mobility fluctuation effects. To our interest is the noise level at the sub-threshold region assuming a charge pumping (through gate pulsing) pre-circle. Then, the oxygen traps near the conductance band are occupied (this corresponds to a gate threshold voltage increase ($\Delta V_{th}$)) while a long discharging time as well as the SiO$_2$ interface layer isolates those traps from the Si/SiO$_2$ interface.

CTTs have been proposed as analog neuromorphic computing engines [1] (convolutional neural networks for deep learning). They handle matrix multiplication (for both feed forward and error back propagation computations) by multiplying the $i_{DS}$ subthreshold channel current with the effective $G_{subth}$ off channel conductance after been modulated by the gate bias charge pulsing (trapped charge). In this work, we argue that LFN could be adhesive when CTT is employed as a neural synaptic element (where the threshold variation, due to the trap occupation, is used for learning). In general, externally induced LFN (of any color) in a neuromorphic analog synaptic element can provide an optimal level of noise (stochastic resonance of the neuron) [5] which can be beneficial in realizing a desired synaptic input-output relationship. Stochastic resonance is feasible even when internally produced noise is employed a resonance initiator[6]. This has been observed in ion-channel systems which operate alike a HMKG transistor in the sub-threshold regime.

This work has been supported by a grant (10073 FPGAs4SGs) of Western Region of Greece to the University of Patras under the RIS-3 program.
II. ARTIFICIAL NEURAL NETWORKS

Neural networks realize non-linear functions to provide optimal input-output relations as shown in Fig.1. In particular cases neural nets comprise hidden units employing threshold elements which in turn can be affected by noise. These threshold elements allow the implementation of threshold activation functions and in generally can be implemented by digital hardware. Activation functions can be replaced during learning by sigmoids and used as threshold functions at neural net runtime. In general, smoothing out the activation functions is a necessity during the learning period for performing gradient descent algorithms. It has been investigated that Stochastic Resonance (SR) can be very beneficial when the input-output relation is nonlinear in the sense that adding noise helps smoothing out the activation function. Of special interest in this work is the examination of threshold elements which’s inputs are modified by random fluctuations (noise) during ANN training and run time. The noise is not just added to the input-output pairs which train the network but instead are used to compute a distribution from which new noisy data are generated. In the implementation of a large NN on silicon, binary outputs with steep threshold functions are convenient in terms of the ease of construction.

The NNs in general face difficulties to convey with software implementations (given that the computational power of a human brain ranges at $10^{11}$ MFLOPs for a limited power budget of 10 W) so hardware implementations are heavily pushed alternatively. However, a new class of neuromorphic circuits have been lately evolved which can closer emulate the neurons operability and exploit their analog nature. The ANN general structure comprises a synapse (network wiring, synaptic weights and learning) and the neuron (neuron state-summation of weighted inputs and the activation function which is highly non-linear).

III. CTT

Charge trap transistors (CTTs) have been recently proposed as a compact continuously tunable non-volatile synapse devices (CMOS-compatible analog neural network computing engine) [7].

It is feasible today to fabricate CTTs in 14nm FinFET technology without extra process complexity and masks as shown in Fig.2. The enhanced and stabilized charge-trapping behavior of CTTs, allows their use into analog non-volatile memory (trap charge dissipates very slowly) and make them candidates for analog computing (neuromorphic) neural networks (learning). Device key characteristic is an interfacial layer $\text{SiO}_2$ followed by an $\text{HfSiO}_x$ layer as the high-k gate dielectric, common in modern CMOS technologies. The device gate threshold voltage $V_{th}$ is modulated by the trapped charge (long trapping and de-trapping procedure) into the gate dielectric depending on a pulse train applied on the gate. Such a device exhibits very low energy consumption per synaptic operation.

As shown in Fig.3, the CTT matrix implementing a synapse connects the output $Y_j(k + 1)(j = 1 \ldots N)$ with the input $Y_i(k)(i = 1 \ldots M)$ through a weighted MxN matrix with synaptic weight coefficients $W_{ij}$:

$$Y_j(k + 1) = \alpha \sum_{i=1}^{M} Y_i(k)W_{ij}$$

The CTTs are biased in the sub-threshold regime ($V_{GS}-V_{DS} < V_{th}$). The fast-reading and the slow nature of CTT discharging allows programming by storing the synaptic weights in the threshold voltages (through positive/trapping and negative/de-trapping voltage pulse trains.) A resistor connected to the sources of all the transistors in a row allows the collection of all the drain to source currents thus implementing the summation producing the $Y_j(k + 1)$ output. If the data
are available at the particular nodes at the same time the calculation is performed in one circle given that $V_{th,ij}$ has been pre-trained with the appropriate pulse train. For a given node, the inputs are multiplied by the weights in a node and summed together. This node value is referred to as the summed activation of the node. The summed activation is then transformed via an activation function ($\alpha$) and defines the specific output or activation of the node. The simplest activation function is referred to as the linear activation, where no transform is applied at all. A network comprised of only linear activation functions is very easy to train, but cannot learn complex mapping functions. Linear activation functions are still used in the output layer for networks that predict a quantity (e.g. regression problems). Nonlinear activation functions are preferred as they allow the nodes to learn more complex structures in the data. Ikemoto et al. [5] have recently propose non linear activation functions (threshold elements) that are modified by noise. The exploitation of the noise in this work is fundamentally different than the addition of noise to the input - output pairs for training purposes. This is a demonstration of noise assisted switching, wherein both the noise and the non-linear characteristics are exploited in order to enhance the flow of information to the output. RTN noise induced by single oxide trap is observed at the drain current of FETs with high-k dielectrics [8]. Normally the single-trap induced RTN is a typical Poisson process and the stochastic nature of RTN noise an indication that SR is achieved for a certain input noise as shown in Fig.4. The stochastic resonance is attained by adding common noise into the FET operating at a specific frequency band. Detecting signals buried in noise indicating stochastic resonance at different voltages and sub-threshold slopes (The Fig. has been reproduced from K. Nishigussi and A. Fujiwara ref. [9].

IV. STOCHASTIC RESONANCE

Stochastic resonance (SR) is a mathematical concept that in general refers to the increased sensor sensitivity of a system due to the external addition of a finite level of noise [4][5][10][11]. Generally a system under SR exhibits a maximum SNR upon an optimal noise level applied. A system with an internal noise level can exhibit SR upon inducing an external noise source into the system; this external noise source eventually induces SR. The system that exhibits SR will attain a maximum when an optimal level of noise is applied for a specific frequency band. Detecting signals buried in noise in FETs with strong current nonlinearity which overcomes the thermal limitation and dynamic bistability is achieved by adding common noise into the FET operating at the subthreshold region under a pulsed voltage [9]. The FET is driven by an external pulse train and a noise source. However, it has been shown that if the physical mechanisms producing internal noise are well defined and understood it would be possible to engineer the system so that the internal noise alone is able to attain SR. The effect has been observed in nanoscale biosensors (ion channel switch biosensors) which uses ions to transduce events of molecular recognition into detectable changes in impedance [6]. To improve the sensitivity of these sensors, one aims to lower the sensors limit of detection. This is accomplished by enhancing the SNR. It is worth mentioning that the internal noise in these systems arises from the probabilistic gating of the ion channels themselves.

In modeling these stochastic fluctuations as a noisy current component in the Hodgkin-Huxley equations, it has been shown that if the noise increases with noise, attains a maximum, and then asymptotically approaches zero, thus showing the signature of SR [6]. It is evident that one can achieve SR in such devices if the gated transductive device is properly engineered. In this work we employ CTT as a neuromorphic device for implementing ANNs. CTTs are CMOS compatible nanowire transistors driven by gate voltage train pulses aiming at trapping and detrapping of transistors into the dielectric. This gated structure exhibits SR [9] under the supply of external noise applied on the gate of the transistor along with a voltage train pulse with a specific offset. A correlation coefficient $C$ is defined as a metric to evaluate the information transmitted from input to output under stochastic resonance. By defining the $C$ coefficient it has been shown that this coefficient increases and then decreases as a function of noise an indication that SR is achieved for a certain input noise as shown in Fig.4. The stochastic resonance is attained to the bistable energy system diagram providing hysteresis characteristics and represents fluctuations of the barrier height.

![Fig. 4. The correlation coefficient C as a function of the increasing induced noise indicating stochastic resonance at different voltages and sub-threshold slopes](image)

V. DISCUSSIONS AND CONCLUSIONS

As was shown in section III, the inference and training operations rely on the matrix multiplication for either feed forward or error back propagation. Indeed the specific CTT operating at the sub-threshold regime executes the multiplication:

$$V_{oj} = R_{source}K_nW \sum_{i}^{M} V_{ds,ij}(V_{gs} - V_{th,ij})$$  \hspace{1cm} (3)

$V_{ds,ij}(V_{gs},V_{th,ij})$ can be relaxed to a direct product when an input related offset is removed at the digital domain post processing [7]. $R_{source}$ is the source series resistance connected to the CTTs sources of the entire row. $V_{oj}$ then corresponds to the $V_o/k$ neuronal cell and it is basically calculated as the $R_{source}G_{subth}$ product where $G_{subth}$ is the off subthreshold conductance. $V_{th,ij}$ is the threshold voltage which incorporates the threshold variation $\Delta V_{th}$ associated with the trapping density change during
initialization (trapping), programming (trapping) and erase (de-trapping) phases of the neural net training. $V_{th}$ depends both on the self heating (applied $V_{dd}$) but mostly on the $V_{gs}$ pulse train (voltage level and pulse frequency). It also depends on the trap location (1-distance from the channel). Moreover, it is well understood that LFN in nanoscale transistors, in the linear regime of operation, is associated with the stochastic oxide trap fluctuating occupancies. This holds true even for planar high-k metal gate MOSFETs with ultra thin HfO$_2$ gate dielectrics (i.e CTTs). This low frequency noise is of the RTS type (RTN) and a single-trap induced RTN comprises both a capture time constant $\tau_c$ and an emission time constant $\tau_e$ representing the average waiting time (to capture or emit) a carrier at the trap level from (to) the channel. RTN induced by $N$ traps give a colored noise spectral density for the channel current and the noise dependence on $V_{gs}$ is determined by:

$$l = \frac{E_{oxide}}{q} \frac{\partial \ln(\frac{\epsilon}{\epsilon_0})}{\partial V_{gs}}$$

(4)

$$E_{oxide} = E_{si} + \frac{\epsilon_{si}}{\epsilon_{hf}} E_{hf}$$

(5)

$E_{si}$ and $E_{hf}$ are the SiO$_2$ and HfO$_2$ thicknesses respectively. Equations (1), (4) and (5) indicate the close relation of RTN with the gate voltage pulse train (through the modulation of the effective trap density) and with the dielectric characteristics. Similarly, $V_{th}$ associated with the threshold voltage variation (during gate charging and erasing periods neural net learning), depends on the same parameters. The RTN noise peak current density in such devices can be as high as 6-7 % of the channel current and can even exhibit anomalous RTN behavior where RTN data exhibit two zones with identical amplitudes but reversal time constants (spectral density) for the source current and the noise [8]. Recalling back from section IV the discussion regarding stochastic resonance it is evident that in a CTT, the RTN is the internal dominant noise mechanism and under certain device conditions could induce stochastic resonance in the CTT. As was explained, both phenomena depend on the same parameters which mean that it is possible to engineer the CTT so that the input signal of the non-linear system (small to affect the output) becomes observable by adding appropriately the internal non-zero level noise to the system. Engineering parameters such as $E_{oxide}$, capturing constants, gate pulse characteristics (amplitude and frequency) and gate offset level can eventually create a CTT operating in a learning process under stochastic resonance conditions. The quantitative analysis needed is currently under research. In [11] it has been shown that regardless the fact that the external noise source, usually employed in order to induce stochastic resonance in a neural network, is white noise, the employment of a colored noise source (pink -1/f noise) can amplify the output signal by orders of magnitude. The synaptic CTT can then used for performing the synaptic calculations and the same time SR can provide the threshold activation functioning required by the hidden layers of the NN. This could also be achieved by using a separate CTT as a pass-transistor driven by the same gate voltage pulse train substituting the source resistance of the synaptic CTT. In conclusion: this work provides evidence that a Charge Trap Transistor is a suitable neuromorphic device for the expedition of synaptic calculations in the analog domain. The internally produced RTS noise associated with trapping and de-trapping of charges into the SiO$_2$/HfO$_2$ dielectric system is closely associated with the learning procedure since both are related to charge trapping in the high-k dielectric traps. The internally produced noise can eventually induce SR into the CTT system operating as a synapse calculation engine of the neural net. This SR, in turn, improves (decreases) the information loss of the non-linear system (CTT) by making it closer to a linear system.

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Low-frequency noise considerations for sensors based on manganites

Guillet Bruno
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
bruno.guillet@unicaen.fr

Flament Stéphane
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
stephane.flament@ensicaen.fr

Liu Shuang
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
shaung.liu@unicaen.fr

Roussseau Olivier
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
olivier.rousseau@unicaen.fr

Enger Luiz Guilherme
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
luiz-guilherme.enger@ensicaen.fr

Lebargy Sylvain
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
sylvain.lebargy@ensicaen.fr

Nascimento Vanuza Marques
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
marques-donascimento@unicaen.fr

Wu Sheng
Normandie Univ, UNICAEN, ENSICAEN, CNRS, GREYC
Caen, France
sheng.wu@unicaen.fr

Abstract—Low frequency noise considerations for sensors based on \( \text{La}_{0.33}\text{Sr}_{0.67}\text{MnO}_3 \) (LSMO) thin films are discussed in this paper. Thanks to special attention on the film quality, on the electrical readout electronics and on the patterned geometries, epitaxially grown LSMO thin films can show a very low level of low-frequency noise and can thus be used to fabricate high signal-to-noise ratio sensors such as uncooled bolometers and uncooled low-field magnetoresistances.

Keywords—low frequency noise, LSMO, bolometers, magnetoresistances

I. INTRODUCTION

Among manganites, the ferromagnetic composition \( \text{La}_{0.33}\text{Sr}_{0.67}\text{MnO}_3 \) (LSMO) with a Curie temperature of 360 K is particularly promising for sensors since the metal-to-insulator transition accompanying the ferromagnetic-to-paramagnetic transition occurs above room temperature, thus allowing room temperature applications [1].

The objective of the present paper is to show that before using LSMO thin films for sensors, we had to optimize the deposition conditions, the patterned geometries and the electronic readout in order to obtain very low electrical noise, as summarized in section II. In addition, we show the effect of film thickness on the structural and electrical properties. Section III and section IV briefly present two examples of high signal-to-noise ratio sensors based on LSMO thin films: uncooled bolometers on buffered silicon substrates [2,3], and uncooled low-field magnetic sensors on SrTiO\(_3\) (STO) substrates [4].

II. LSMO MATERIAL PROPERTIES AND CONTACT DESIGN

LSMO films of various thicknesses in the 5-150 nm range were epitaxially grown either on STO (100) substrates by Pulsed Laser Deposition (PLD) or on STO buffered Si (001) substrates by reactive Molecular Beam Epitaxy (MBE) [5].

Fig. 1. 0-20 out-of-plane X-ray Diffraction (XRD) and resistivity versus temperature measurements, respectively, for (a) and (c) LSMO / STO (001), (b) and (d) LSMO/STO/Si (001).

The lattice mismatch between epitaxial films and substrates can significantly influence the electrical and magnetic properties (DC properties, noise level, etc.). The structural and electrical properties were systematically checked using 0-20 out-of-plane X-ray Diffraction (XRD) and electrical resistivity versus temperature measurements, respectively, as shown in Fig. 1. From the out-of-plane lattice parameter of LSMO, named \( c_{\text{LSMO}} \), we could measure the LSMO out-of-plane lattice deformation \( c_{[0 0 1]} \) defined as

\[
e_{[0 0 1]} = \frac{c_{\text{LSMO}} - c_{\text{LSMOpc}}}{c_{\text{LSMOpc}}} \quad (1)
\]
where \( c_{\text{LSMO}} \) is the pseudo-cubic lattice parameter of bulk LSMO (0.3873 nm). \( \varepsilon_{0 0 1} \) is negative for the whole film thickness range, as expected in case of tensile strain, when grown on STO material, of lattice parameter equal to 0.3905 nm.

In addition to dedicated low noise electronics, a specific attention has been paid on the electrical contacts [2], as well as on the contact geometry in order to minimize parasitic effects, due to non-homogeneity in the current density distribution. Samples were patterned with different designs. More details will be given in a coming paper. The mask designs are made to avoid any deviation of the current lines. When the geometry is not well chosen, up to 3 orders of magnitude difference in \( \alpha_H/n \) values have been measured on the same sample.

Finally, we showed that a clear correlation exists between the 1/f noise level, the temperature of the metal-to-insulator transition and the thickness of the LSMO thin film as previously published in [9]. In case of well matched crystalline substrate (for example STO), higher noise was measured in films thinner than 10 nm, where higher electrical resistivity and higher lattice deformation were also measured. In the 20 – 80 nm thickness range, LSMO films are very probably relaxed and therefore show low noise values. At about 80 nm, which can be close to the critical thickness value for epitaxial films on STO, one can observe an increase of the noise levels, which can be related to the presence of inhomogeneous strain as stated above, or even cracks for these higher thickness values. On STO/Si substrate, the 1/f noise level can be larger by up to one order of magnitude but stayed at very low values. They do not show a clear dependence versus thickness as it was seen on STO. LSMO films on STO/Si were used for bolometers, in order to allow the use of standard micromachining techniques of silicon. LSMO films on STO were used for anisotropic magnetoresistances. In that case film thickness was kept below 25 nm so as to get a uniaxial magnetic anisotropy [10].

Figure 3 shows the dependence of the normalized Hooge parameters with the electrical resistivity of LSMO thin films of various thicknesses deposited on STO substrates and on buffered Si substrates. The normalized Hooge parameter could be lower than \( 10^{-18} \) m³ in a large number of cases.
III. PERFORMANCE OF LSMO DEVICES

A. LSMO uncooled bolometers

A bolometer measures the power of incident electromagnetic radiation via the heating of a material with a temperature-dependent sensor. The classical principle is based on the variation of an electrical resistance due to absorbed electromagnetic radiation. The bolometers are usually used in infrared radiation measurements since optical responsivity depends on the absorbed wavelength only through absorption. The performance of a bolometer is mainly determined by Temperature Coefficient of Resistance (TCR) and low thermal conduction.

LSMO is a promising candidate for fabricate uncooled bolometers thanks to the large TCR (around 2-3 % K\(^{-1}\)) close to room temperature and a low-noise level compared with other resistive materials such as semiconductors (a-Si, a-Si:H, a-Ge, poly SiGe) and other oxide materials (semiconducting YBCO, VO\(_x\), etc.). An example of the electrical resistivity and the derivative of R versus T noted dR/dT is shown in figure 4.

![Fig. 4](image)

Fig. 4. Electrical resistivity (left axis) and derivative of the electrical resistance (right axis) versus temperature of a suspended 150 μm long and 4 μm wide bridges patterned in a 75 nm thick LSMO thin films.

Non-suspended and free-standing LSMO bolometers have been investigated (Fig. 5) [2,3,11]. The substrate below the active area can be removed by using standard micromachining techniques, which reduce thermal conductance and therefore enables to fabricate sensitive and fast sensors, while keeping very low intrinsic electrical noise. The thermal conductance can be reduced by 3 orders of magnitude (down to \(10^{-7} \text{ W} \cdot \text{K}^{-1}\)) thus increasing the sensitivity by a factor of 1000.

![Fig. 5](image)

Fig. 5. Examples of non-suspended meander of width 18 μm and free-standing LSMO bolometers of width 4 μm and length 50 μm.

Thanks to the very low 1/f noise, and with appropriate geometries, ultra low Noise Equivalent Power (NEP) values around pW Hz\(^{-1/2}\) could be obtained at 300 K. An example of an ultra low NEP value is given in Fig. 6 for a phonon noise limited bolometer.

![Fig. 6](image)

Fig. 6. NEP of a 75 nm thick 2 μm wide suspended bolometer with different length.

B. LSMO uncooled low-field magnetoresistances

One major problem with low-field magnetic sensors is their excess noise at low frequency. Therefore, thanks to their low 1/f noise, LSMO may find important applications in magnetoresistive sensors. Moreover, LSMO exhibits a Curie temperature of about 350 K, which allows room temperature applications. An example of measured differential voltage of a Wheatstone bridge patterned in LSMO and relative magnetic field sensitivity as a function of the magnetic field \(μ_0H\) are shown in Fig. 7. The observed magnetoresistance is due to anisotropic effect in a ferromagnetic film showing uniaxial anisotropy.

![Fig. 7](image)

Fig. 7. (left axis) Example of differential voltage of a Wheatstone bridge (current perpendicular to applied magnetic field, bias current = 200 μA, \(R=8100 Ω\), \(T = 310 \text{ K}\) showing an uniaxial anisotropy, and (right axis) relative magnetic field sensitivity as a function of the magnetic field \(μ_0H\).

Some growth parameters such as the nature of the substrate, and the direction of the patterned stripes with respect to the easy magnetization axis, were modified in order to optimize the magnetic detectivity of the realized devices. Making use of uniaxial anisotropy in thin LSMO films on STO (001) and the Anisotropic MagnetoResistive effect (AMR), we could measure sensitivity of about 400%/T\(^{-1}\).

Promising results were obtained on Wheatstone bridge geometries patterned in LSMO thin films. Magnetic noise
was finally measured to be in the order of 200 pT·Hz$^{-1/2}$ at 310 K in a frequency range corresponding to neuron activities (Fig. 8).

Fig. 8. Example of detectivity versus frequency at 0.2 mT and at different bias voltage (current perpendicular to applied magnetic field, R=8100 Ω, T = 310 K).

IV. CONCLUSIONS

We have shown that LSMO thin films, associated with home-made readout and specific mask design, could be useful to fabricate state of the art uncooled bolometers and magnetoresistances.

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Abstract—Results of temperature and electric field dependencies of low-frequency noise power spectral density are presented.

Keywords—low-frequency noise, semiconductor structure, Schottky contact, deep level

I. INTRODUCTION

Investigation of deep levels (DLs) influence on the properties of semiconductor structures is still of interest. Low frequency noise (LFN) spectroscopy is a method that allows investigating semiconductor barrier structures and defining DL parameters.

II. SAMPLES AND INVESTIGATION METHODS

N-type silicon-based test structures in the form of Schottky diodes were manufactured according to the standard industrial technology. In the course of the study, a set of five samples was considered. All samples were performed as a single chip. Al Schottky contacts were created in the form of a square with the side of 2 mm.

At the present work an automated measurement complex was used to study the electrophysical parameters of materials and barrier structures including the Agilent E4980A RLC meter, low-noise preamplifier and voltage offset circuit with galvanic batteries (Fig. 1). As a measuring cell the Janis CCS-400/204N helium closed type cryostat working in the vacuum in the temperature range of 7-500 K was used [1, 2].

The experimental values of the parameters were recorded using the data acquisition board NI PCIe-6236 and processed by a special program executed in the LabVIEW environment.

In order to investigate the parameters of deep levels in semiconductor barrier structures current-voltage (I-V) and capacitance-voltage (C-V) characteristics, power spectral density (PSD) of LFN including at different temperatures, noise-voltage characteristics (N-V) were obtained.

The ionization energy of deep energy levels was determined by two independent methods, by plotting Arrhenius plots and by the temperature and electric field dependence of the noise PSD according to the activation-drift model (ADM) and the Pool-Frenkel effect.

EXPERIMENTAL RESULTS

I-V and C-V dependencies of the samples had the form typical for the metal – semiconductor type Schottky barrier structures (Fig. 2-3).

![Fig. 2. I-V characteristics of the samples at reverse bias](image)

Presentation of C-V characteristics presented in the Mott-Schottky coordinates allowed calculating the shallow donor concentration in the semiconductor according to the expression [3]:

$$
N = \frac{2}{qe_0\varepsilon^2} \int C^2 dU,
$$

where \( q \) is the elementary charge, \( e_0 \) is the electrical constant, \( \varepsilon \) is the dielectric permittivity of semiconductor, \( S \) is the area of the Schottky contact, \( C \) is the capacitance of the depleted layer, \( U \) is the reverse bias voltage.
Therefore the potential barrier height at the metal-semiconductor interface was 0.63 eV. The concentration of free electrons in silicon was $6 \cdot 10^{15} \text{ cm}^{-3}$.

Each type of DL has its relaxation time that depends on the temperature. In the theory of LFN spectroscopy the correspondence of frequency $\omega$ to relaxation time $\tau$ of a particular process is expressed by changing the slope of the spectrum curve and is manifested in the form of its "bend" which is expressed by the equality $\omega \tau = 1$.

According to Boltzmann law the relaxation time is

$$
\tau = \tau_0 \exp \left( \frac{\Delta W_t}{k_B T} \right), \quad (2)
$$

where $\Delta W_t$ is DL ionization energy, $T$ is the temperature, $k_B$ is the Boltzmann constant. Physical meaning of $\tau_0$ differs depending on applying model [4]. In works [5, 6] parameter $\tau_0$ is the characteristic time that is determined by the period of the natural oscillations of atoms in the crystal lattice of semiconductor and $\tau_0$ has the order $10^{-12}$ s.

In practice the ionization energy of deep trapping levels $\Delta W_t$ is determined using Arrhenius plots of experimental dependencies. DL ionization energy $\Delta W_t$ is found from the slope of the Arrhenius line which is represented as

$$
\ln \tau = \ln \tau_0 + \frac{\Delta W_t}{k_B T}. \quad (3)
$$

The values of the DL ionization energy in the studied barrier structures are determined from the experimental low-frequency noise spectra obtained at a bias voltage of -7.9 V and temperature values from 90 to 290 K (Fig. 4). Based on experimental dependencies Arrhenius plots were drawn in semi-logarithmic coordinates and the slope angle proportional to $\Delta W_t$ was found.

Spectrum analysis showed the presence of inflection points at a number of frequencies in the range of Hz units at three temperature regions 100-140 K, 160-190 K and 270-290 K. This indicates the presence of three DLs. By splitting the noise spectra into two sections and approximating those with straight lines the values of the bend frequency were calculated and the Arrhenius plots for the low frequency region were drawn (Fig. 5). The values of the DL ionization energy for the sample obtained in this way are presented in the Table 1.

### TABLE 1

<table>
<thead>
<tr>
<th>Deep level</th>
<th>DL1</th>
<th>DL2</th>
<th>DL3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta W_t$ (meV)</td>
<td>101</td>
<td>107</td>
<td>279</td>
</tr>
<tr>
<td>$\tau_0$ (s)</td>
<td>$1.86 \cdot 10^{-6}$</td>
<td>$2.63 \cdot 10^{-5}$</td>
<td>$1.76 \cdot 10^{-7}$</td>
</tr>
</tbody>
</table>

The method of determining of the ionization energy of DLs developed in this work is based on the measuring of the experimental PSD noise spectrum at the constant temperature and determining the bend frequency of the LFN PSD curve.

According to ADM theory at a sufficiently high reverse bias all activated from DL charge carriers are transferred through the space charge region (SCR) of the barrier structure during the Maxwell relaxation time $\tau_M$ determined by the resistivity of the base [7].

According to the Poole-Frenkel model developed by the authors of this work the ionization energy of deep levels can be calculated according to the expression

$$
\Delta W_t = k_BT \ln \left( \frac{1}{4\pi \hbar c \cdot \tau_m} \right) + \frac{qN}{\pi \varepsilon_0} \sqrt{\frac{qN}{2\varepsilon_0} (U_{bd} - U)} \quad (4)
$$

![Fig. 3. C-V characteristics of the samples measured at 1 MHz](image)

![Fig. 4. PSD dependencies of the sample #1 at different temperatures in the range 90 – 290 K](image)

![Fig. 5. Arrhenius plots of the sample #1 in the low frequency range](image)
where $f_b$ is the frequency of the PSD curve bend, $U_k$ is the contact potential difference.

The bend frequency of the PSD curve at room temperature was about 350 Hz (Fig. 6) for five samples. The $U_k = 0.4$ V estimated from the C-V measurements was used to find $\Delta W_t = 0.54$ eV.

Investigation of LF noise PSD showed the presence of an additional inflection point in the frequency range from tenths to units Hz. It was found that the samples #2 and #3 had no bend in the low-frequency region, so that the values of LF noise PSD are almost the same. At the same time their I-V and C-V characteristics coincide.

For the samples #4 and 5 high-frequency regions of PSD respectively coincide, however low-frequency regions of PSD and I-V characteristic are noticeably different. The sample #1 has different from other samples of I-V and C-V characteristics, but its high-frequency region of the PSD dependencies coincides with that of the #2 and #3 samples which indicates similar properties of the semiconductor characterized by the presence of deep level defects.

For the sample #1 measurements of LF-noise PSD were carried out at different reverse bias voltages in the range of 1.4-20.2 V (Fig. 7). Several frequency values were selected from the obtained data array and N-V characteristics were measured (Fig. 8).

All characteristics had the region of initial increase which passed into a site of saturation after some value of tension. The functional dependence that best describes the N-V characteristics had the form

$$PSD = B \exp(U'),$$

where $B$ and $\gamma$ are characteristic coefficients.

It was found that the coefficient $\gamma$ was about 0.5 at all frequencies which corresponds to the Pool-Frenkel model [3].

One of the advantages of the functional dependence is that it satisfactorily describes the N-V characteristics when considered both on a linear and logarithmic scale.

For the sample #1 the dependence of the value of the deep level energy decrease on the electric field in the sample was obtained (Fig 9). The average value of the electric field in the SCR was determined from the expression [3]:

$$E(U) = 0.5 \sqrt{\frac{2qN(U_k - U - k_BT/q)}{e\varepsilon_0}}.$$  \hspace{1cm} (6)

The dependence shown in Fig. 9 indicates a proportional increase in the value of the decrease in the DL's ionization energy with an increase in the applied electric field.

The correlation between the value of the bend frequency and the applied voltage at the high-frequency section of the low-frequency noise spectrum was established while no such correlation was observed in the low-frequency region (Fig. 10).
The results shown in Fig. 9 and 10 confirm the model developed by the authors according to which the Pool-Frenkel effect proposed to describe the phenomenon of decrease in the activation energy of shallow energy levels at low temperatures. It can be applied to the description of relaxation processes involving DLs in the semiconductor barrier structure at elevated temperatures. In accordance with the experimental results LFN magnitude may differ by 2-3 orders for semiconductor structures manufactured in a single technological cycle and located in close proximity to each other. Still I-V and C-V characteristics of such structures may be compatible. The experiments proves a higher sensitivity, information content and prospects of LFN-analysis method application to predict potentially unreliable structures and threshold parameters of semiconductor devices in comparison with I-V and C-V methods.

**Fig. 9.** Dependence of the DL’s ionization energy reduction on the electric field in the SCR

**Fig. 10.** Dependence of the bend frequency of the PSD dependence on the applied voltage at the high-frequency section of the LF-noise spectrum

The developed physical model of low-frequency noise generation taking into account the influence of DLs is confirmed by experimental results. The Pool-Frenkel model that states potential barrier lowering in the vicinity of a defect that forms a deep energy level in the band gap of a semiconductor under the influence of a strong electric field was proved by experimental data. This model developed for the case of small energy levels at low temperatures [3] can be applied to the case of deep energy levels at relatively high temperatures.

Investigation and modeling of low-frequency noise PSD on voltage dependence is important in both theoretical and practical way. Firstly, this dependence reveals the electric field strength role in the formation of barrier structures current noise component.

The value of the electric field lead to span saturation of charge carriers activated from DLs in a specific structure is still unknown. According to [8] the average electric field strength in the barrier structures SCR must be at least 10⁴ V/cm to obtain reliable data on DL characteristics. At the same time average field strength should not exceed the value of 10⁶ V/cm when breakdown may occur. τₑ on SCR electric field strength dependence is still not clear. The solution of this issue is the goal for further research.

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Low-Frequency Noise in Low-Dimensional van der Waals Materials

Alexander A. Balandin
Nano-Device Laboratory
Department of Electrical and Computer Engineering
University of California – Riverside
Riverside, California 92521 USA
E-mail: balandin@ece.ucr.edu

Sergey Rumyantsev
Center for Terahertz Research and Applications
Institute of High Pressure Physics
Polish Academy of Sciences
Warsaw, 01-142 Poland
E-mail: roumis4@gmail.com

Abstract – The emergence of graphene and two-dimensional van der Walls materials renewed interest to investigation of the low-frequency noise in the low-dimensional systems. The layered van der Waals materials offers unique opportunities for studying the low-frequency noise phenomena owing to their properties, controlled by the film thickness, and their widely tunable charge carrier concentration. In this review, we describe unusual low-frequency noise phenomena in quasi-2D and quasi-1D van der Waals materials. We also demonstrate that the low-frequency noise spectroscopy is a powerful tool for investigation of the electron transport and charge-density-wave phase transitions in this class of materials.

Keywords – low-frequency noise, charge density waves, van der Waals materials, 2D materials, 1D materials

I. INTRODUCTION

The emergence of graphene and two-dimensional (2D) van der Walls materials renewed interest to investigation of the low-frequency noise in the low-dimensional systems [1-3]. This type of materials offers unique opportunities for studying the low-frequency noise phenomena owing to their properties, controlled by the film thickness, and their widely tunable charge carrier concentration. Practical applications depend on the ability to understand and reduce the noise in this new type of materials. From the other side, these materials offer new opportunities for addressing the fundamental problems of noise and fluctuations.

II. NOISE IN GRAPHENE

The first material of this class – graphene – revealed a number of interesting properties in the context of 1/f noise owing to its 2D nature, unusual linear energy dispersion for electrons and holes, zero-energy bandgap, specific scattering mechanisms and metallic-type conductance [1-15]. From one side, graphene is an ultimate surface where conduction electrons are exposed to the traps, e.g. charged impurities in a substrate or on its top surface, which can result in strong carrier-number fluctuations. From the other side, graphene can be considered a zero-bandgap metal, where mobility fluctuations resulting from the charged scattering centers in the substrate or surface can also make a strong contribution to 1/f noise. The ability to change the thickness of few-layer graphene conductors by one atomic layer at a time opened up opportunities for examining surface and volume contributions to 1/f noise directly [14-15]. Independent studies have shown that noise in graphene reveals an unusual gate-bias dependence [5-13]. In the vicinity of the Dirac point, the noise amplitude follows a V-shape dependence, retaining its minimum at the Dirac point where the resistance is at its maximum [10]. The unusual gate dependence of the noise amplitude in graphene supported the conclusion that 1/f noise in graphene devices does not follow the conventional McWhorter model used for complementary metal-oxide-semiconductor (CMOS) devices and other metal-oxide-semiconductor field-effect transistors (MOSFETs) (see Figure 1).

Fig. 1 Low-frequency noise in graphene. (a) Typical 1/f noise spectra in a graphene device. The inset shows a scanning electron microscopy image of a representative graphene device. (b) Unusual V-shape bias dependence of 1/f noise in graphene. The figures are reproduced from Ref. [1] with permission from the Nature Publishing Group.
III. NOISE IN 2D VAN DER WAALS MATERIALS

Recently, the exotic phenomena, such as charge density waves (CDW) in 2D van der Waals materials, attracted interest in the context of the low-frequency noise research [16-18]. The CDW phase is a macroscopic quantum state consisting of a periodic modulation of the electronic charge density accompanied by a periodic distortion of the atomic lattice [19]. One of the most interesting 2D CDW materials is 1T-TaS$_2$ [17, 20-23]. As the temperature increases above 180 K, the commensurate CDW phase in this material breaks up into a nearly-commensurate CDW phase that consists of ordered commensurate CDW regions separated by domain walls. This transition is revealed as an abrupt change in the resistance with a large hysteresis window in the resistance profile at 200 K. As the temperature increases to 350 K, the nearly commensurate phase melts into an incommensurate phase, in which the CDW wave vector is no longer commensurate with the lattice. This transition is accompanied by a smaller hysteresis window in the resistivity. Only at high temperatures of 500 K – 600 K the incommensurate CDW phase melts into the normal metallic phase of 1T-TaS$_2$.

We investigated the low-frequency noise in 1T-TaS$_2$ thin films as they were driven from the nearly commensurate (NC) to incommensurate (IC) CDW phases by voltage and temperature stimuli [17]. This study revealed that noise in 1T-TaS$_2$ has several pronounced maxima at the bias voltages, which correspond to the onset of CDW sliding and to the phase transitions (see Figure 2). The noise spectral density was more sensitive to the phase transitions and changes in the electron transport than the current-voltage (I-V) characteristics. We observed the unusual Lorentzian features and exceptionally strong noise dependence on the electric bias and temperature, leading to the conclusion that electronic noise in 2D CDW systems has a unique physical origin different from known fundamental noise types [17]. It was established that the low-frequency noise spectroscopy can serve as useful tool for understanding electronic transport phenomena in 2D CDW materials characterized by coexistence of different phases and strong pinning. The technique has been also used for the vertical 1T-TaS$_2$ CDW devices. It was found that the low-frequency noise spectral density revealed strong peaks, below the temperature of the commensurate CDW to nearly-commensurate CDW transition, possibly indicating the presence of the strongly debated hidden phase states [18].

The low-frequency electronic noise spectroscopy has been instrumental in monitoring the electric-field induced transition from the incommensurate CDW phase to the normal metal phase. The noise data were measured at the bias of $V_{bias}$=13 mV and frequency $f$=10 Hz. The decrease in resistance at $T_C$=160 K corresponds to the well-known commensurate to nearly-commensurate CDW transition. A diagram, depicting the reconstruction of Ta atoms into hexagonal clusters, is shown to illustrate the phase transition. A distinctive noise peak is observed at the same temperature $T_C$. Below the commensurate CDW – nearly-commensurate CDW phase transition temperature, one can see another step in the resistance with the corresponding peak in the noise spectral density. The figure is adapted from Ref. [18].

![Fig. 2](image-url) Low frequency noise in 2D charge-density-wave materials. (a) Current-voltage characteristics of thin-film 1T-TaS$_2$ device at room temperature. The inset shows the device schematic. The abrupt CDW phase transitions are seen at voltages $V_H$ (up scan) and $V_L$ (down scan). The red circles indicated the biasing points for the low-frequency noise measurements. (b) The normalized noise spectral density, $S_n/\langle I^2\rangle$ at 10 Hz as a function of the bias voltage, $V_n$. Two pronounced local maxima correspond to the de-pinning of the charge density wave and the phase transition between two CDW phases. The figures are adapted from Ref. [17].

We investigated the low-frequency noise in 1T-TaS$_2$ thin films as they were driven from the nearly commensurate (NC) to incommensurate (IC) CDW phases by voltage and temperature stimuli [17].
changes, which allow to accurately determine the transition (see Figure 4).

IV. NOISE IN 1D VAN DER WAALS MATERIALS
The concept of van der Waals materials has been extended to one-dimensional (1D) systems. In contrast to the layered quasi-2D crystals, quasi-1D materials, such as MX3 (where M = transition metals; X = Se and Te) consist of the atomic threads, which are weakly bound in bundles by van der Waals forces. As a consequence, the exfoliation of the MX3 crystals results not in 2D layers but rather in quasi-1D nanowires [24-28]. It has been shown that some of quasi-1D van der Waals materials reveal exceptionally high electrical current densities [24, 27]. These materials are very interesting from the low-frequency noise prospective as well (see Figure 5).

Specifically, we found that quasi-1D TaSe3 and ZrTe3 van der Waals nanowires, which possess exceptionally high current densities, have rather low levels of the low-frequency noise as compared to graphene [25, 28]. In ZrT3 nanowires, the noise reveals the 1/f behavior near room temperature but becomes dominated by the Lorentzian bulges at low temperature. Unexpectedly, the corner frequency of the Lorentzian peaks shows a strong sensitivity to the applied source–drain bias. The dependence on electric field can be explained by the Frenkel–Poole effect only in the scenario where the bias voltage drop happens predominantly on the defects, which block the quasi-1D conduction channels [28]. In TaSe3, 1/f noise becomes the 1/f2 type as temperature increases to ~400 K, suggesting the onset of electromigration. Using Dutta–Horn model, we determined that the noise activation energy for quasi-1D TaSe3 nanowires is ~1.0 eV, comparable to that for Cu and Al interconnects. Our results suggest that quasi-1D van der Waals metallic nanowires have potential for applications in the ultimately downscaled local interconnects [25]. While this review emphasized the use of low-frequency noise as a
spectroscopy tool for understanding the electron transport phenomena, the obtained results are also important for practical applications of 2D and 1D van der Waals materials in sensors or information processing [29-35].

V. CONCLUSIONS

We reviewed our recent results pertinent to the unusual noise phenomena in quasi-2D and quasi-1D van der Waals materials. We also discussed the prospects of the low-frequency noise spectroscopy which is a powerful tool for investigation of the electron transport and charge-density-wave phase transitions in this class of materials.

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Thermodynamics limits in Oscillators and Phase Locked Loops

A. Khakpour and A. Liscidini, Senior Member, IEEE

Abstract—In this paper the impact of thermal noise in time domain signal processing is reviewed with an emphasis on oscillators and phase locked loops. It will be shown that both oscillators and phase-locked loops display an upper bound in their efficiency that is fundamentally thermodynamic limited. Such upper bound will be used to derive two comparative figure-of-merits (FoMs) that resemble the ones already present in literature. Finally, an analysis of the state of the art of the designs already present in literature confirms the proposed analysis.

Index Terms—figure of merit (FOM), phase-locked loop (PLL), phase noise, signal to noise ratio (SNR), voltage-controlled oscillator (VCO), jitter, absolute jitter, time-domain

I. INTRODUCTION

The most common time-domain circuits in electronics are oscillators and phase locked loop (PLL) where the information is contained in a time difference between two events rather than the voltage drop between two terminals. Recently it has been shown that both time and voltage domain circuits are ultimately thermodynamically limited in a similar way, which leads to a straightforward relationship between power dissipation and signal to noise ratio (SNR) achievable [1]. In this paper, after an overview of the thermodynamic limits for time-domain circuits, it will be demonstrated that such fundamental limit is also behind the most common benchmarking figure-of-merits used for oscillators and PLLs. The presented derivation will allow to identify also thermodynamic upper bounds for such FOMs and intrinsic limitations when used in the attempt to provide a fair comparison between different designs.

The paper is structured as follows: in Section II an overview of the thermodynamic limit in time-domain signal processing is presented. In section III, thermodynamic limits of Oscillators and PLL are discussed. A relationship between the oscillator and PLL FOMs is derived in Session IV which is accompanied with a discussion about the restrictions in the use of such FOMs. Paper ends by introducing a new FoM to benchmark the architecture efficiency of the PLLs regardless of the oscillator and reference frequency adopted.

II. THERMODYNAMIC LIMIT IN TIME-DOMAIN SIGNAL PROCESSING

In voltage domain, the maximum and the minimum signals that can be processed are limited by the voltage supply and the thermal noise respectively. These boundaries define the signal-to-noise ratio (SNR) of the circuit and consequently its performance [1]. In time-domain, the jitter noise sets a lower bound in the time difference detectable, while the upper bound is set by the maximum delay generated by the circuit.

In the case of an inverter, the relationship between power (P) and the SNR achievable when used as an integrator has been evaluated by Enz et.al. in [2] as follows:

\[ \text{Power} = 8kTf_{BW}SNR = 4kTf_{BW}SNR_{\alpha_T} \]  

(1)

where \( k \) is the Boltzmann constant and \( T \) is the temperature of the circuit \( f_{BW} \) is the bandwidth of the circuit and \( \alpha_T \) a coefficient set to 2 and \( f \) is the bandwidth of the system. When the inverter is used as a delay stage, the jitter noise produced by the inverter and the maximum delay achievable can be used to achieve an expression of power versus SNR.

The expression of the jitter added by a CMOS inverter as a function of the generated delay was evaluated by Abidi in [6] and is given by

\[ \sigma^2 = \frac{8y_k T t_d^2}{C V_{DD}^2 (V_{DD} - V_I)} + \frac{4kT t_d^2}{CV_{DD}^2} \]  

(2)

where \( \sigma \) is the jitter, \( t_d \) the delay of the inverter, \( y_k \) the thermal noise coefficient for the transistor, \( V_I \) the transistor voltage threshold. Equation (2) can be used for both transitions (i.e. form \( V_{DD} \) to 0 and vice versa) [6]. Starting from (2) it is possible to define the SNR of the inverter working as a delay stage as

\[ \text{SNR} = \frac{t_d^2}{\sigma^2} = \frac{1}{\frac{8y_k T}{C V_{DD}^2 (V_{DD} - V_I)} + \frac{4kT}{CV_{DD}^2}} \]  

(3)

Notice that, the SNR of the inverter in time domain is independent of the delay generated, being a function of the capacitive load \( C \), the voltage supply and the transistor threshold. For an easier comparison with (1), (3) can be rewritten as

\[ \text{SNR} = \frac{C}{4kT \alpha_T} V_{DD}^2 \]  

(4)

with \( \alpha_T = \frac{2y_k C}{V_{DD}^2} + 1 \)  

(5)

Equation (4) looks very like (1) except for the factor \( \alpha_T \). In fact, while in voltage domain \( \alpha_T = 2 \), in time domain \( \alpha_T \) is technology dependent because it is a function of the ratio between the voltage threshold and the voltage supply. Since the power consumption of the stage in both cases is the same, the following relationship between power and SNR in time domain results:
Power = 4kTf_{BW}SNR\alpha_{T} \tag{6}

III. VCO AND PLL THERMODYNAMIC FoMs

From (6) it is possible to define an upper limit for SNR_{f_{BW}}/P given by 4kT\alpha_{T}, which limits efficiency in time-domain signal processing (without resonators). The relationship between the SNR achievable by a circuit for a given power dissipation P and bandwidth BW can be expressed by

\begin{equation}
\frac{P}{p} = 4kT\alpha \tag{7}
\end{equation}

Since in voltage domain the relationship between SNR, \( f_{BW} \) and P is constant and thermodynamically limited to 8kT, (7) has been used as basis in the definition of several FoMs which in the most trivial expression is given by

\begin{equation}
FOM = \frac{SNR_{f_{BW}}}{p} \tag{8}
\end{equation}

Since in time domain circuits SNR can be defined as

\begin{equation}
SNR_{p} = \frac{\Delta t}{\sigma_{p}} \tag{9}
\end{equation}

where \( \sigma_{p} \) is the jitter accumulated over the time \( \Delta t \) [1], by choosing \( f_{BW} = 1/\Delta t \), (8) can be rewritten as

\begin{equation}
FOM = \frac{\Delta t}{p,\sigma_{p}^{2}} \tag{10}
\end{equation}

and can be used to benchmark oscillators and PLLs from the point of view of the thermodynamic limit previously highlighted.

A. Oscillator FoM

In literature, the most common FoM used to characterize an electrical oscillator is given by

\begin{equation}
FOM_{osc} = \frac{f_{o}^{2}}{M_{f}\ell(\Delta \ell)} \frac{1}{p} \tag{11}
\end{equation}

Where \( f_{o} \) is the oscillation frequency, \( \Delta \ell \) is the frequency offset from \( f_{o} \), \( \ell(\Delta \ell) \) is the phase noise spectral density evaluated at an offset \( \Delta \ell \) and P is the power dissipated (usually normalized to 1mW). Although historically (11) has been derived by a simple rearrangement of the empirical Leeson’s model of the phase noise in an oscillator [4], it will be shown that (11) can be derived by starting from (10).

Since in a free running VCO, the jitter square accumulated in N clock cycles is ideally N times the jitter square accumulated in one cycle\(^1\), by setting \( \Delta t = N\tau_{o} \) gives \( \sigma_{p}^{2} = N\sigma_{p}^{2} \), where \( \tau_{o} = 1/f_{o} \) is the oscillation period and \( \sigma_{p} \) is the period jitter. Substituting \( \Delta t \) with \( N\tau_{o} \), and \( \sigma_{p}^{2} \) with \( N\sigma_{p}^{2} \) in (10) leads to the following

\begin{equation}
FOM_{osc} = \frac{N\tau_{o}}{N\sigma_{p}^{2}} \frac{1}{f_{o}\sigma_{p}^{2}} \tag{12}
\end{equation}

Based on [6], the period jitter, \( \sigma_{p} \), can be expressed in term of phase noise, \( \ell(\Delta \ell) \), as follows

\begin{equation}
\sigma_{p}^{2} = \ell(\Delta \ell) \cdot \frac{\Delta f^{2}}{f_{o}^{3}} \tag{13}
\end{equation}

By substituting (13) in (12) the traditional FOM_{osc} for the oscillator expressed by (11) is obtained.

Since the proposed derivation started from (8), an upper bound for its value is expected, as found for filters, amplifier and analog-to-digital converters. However, it should be noted that the limits imposed by (8) is intended for circuits where no resonances are present. In presence of a resonance such limits can be overcome as demonstrated in the analysis proposed in [7].

B. PLL FoM

In the case of PLLs, a very straightforward FoM, which relates the total integrated jitter accumulated, \( \sigma_{ip} \), and the overall power consumption, P, was derived by Gao et al. [4]. Such FoM is expressed as

\begin{equation}
FOM_{PLL_{dB}} = -10\log \left( \left( \frac{\Delta t}{\sigma_{p}} \right)^{2} \cdot \frac{1}{p} \right) \tag{14}
\end{equation}

where P is the PLL power normalize to 1mW. The above FOM_{PLL} was derived empirically through a detailed analysis on the relationship between phase noise and power consumption for the different building blocks constituting a generic PLL [4]. However, in this paper we will demonstrate that also such FoM is also a rearrangement of (10). Therefore, it is more general than what its original derivation could suggest.

As mentioned, for a free running oscillator, the jitter square accumulated in N clock periods is N times the period jitter square. Hence, the oscillators’ FoM derived by using (10) became independent of integration-time of the jitter (i.e. \( \Delta \tau \)). However, this is not the case for PLLs because the oscillator is locked, since its phase noise is actively suppressed within the PLL bandwidth. Because of this, unlike a standalone VCO, PLL’s integrated jitter does not diverge to infinity if assumed locked to an ideal jitter-free reference. This means that ideally the SNR expressed by (9) improves as \( \Delta \tau \) increases. Hence, for a fair comparison between different designs, a reference integration time \( \Delta \tau \) should be defined in such a way that allows for the phase noise suppression within the bandwidth of PLL to be considered. For example, setting \( \Delta t = 1s \) and substituting it into (10) leads to the following expression for the PLL FoM

\begin{equation}
FOM_{PLL} = \frac{\Delta t}{p,\sigma_{ip}^{2}} = \frac{1s}{\Delta \tau^{2}} \tag{15}
\end{equation}

where \( \sigma_{ip}^{2} \) is the jitter integrated from \( f = 1Hz \) to \( f = \infty \) which for a PLL converges to the jitter integrated over the whole frequency range. It should be noted that this expression is very similar to the one introduced by Gao in a logarithmic form (i.e. (14)). Equation (15) differs from (14) in two aspects: first, it is inversely proportional to jitter and power so that a larger number corresponds to a lower power consumption, and jitter. Second, in the proposed derivation the unit of the FoM is \( \text{J}^{2} \text{Hz}^{-1} \) (or dB\(^{2}\)) in a logarithmic form as for (11) and (8) (when power is not to normalized to 1mW).

\(^{1}\) This is true if only thermal noise is considered.
IV. FoM PLL THERMODYNAMIC LIMITS

In the previous section, it was shown that both \( FOM_{PLL} \) and \( FOM_{VCO} \) are transpositions of (10). Since any PLL is built around a voltage-controlled-oscillator (VCO), it is also useful to see how these two FOMs relate to each other.

By reusing (10), the FoM of the VCO closed into a PLL (\( FOM_{VCO} \)) can be expressed as function of the bandwidth of the PLL (\( f_{BWPLL} \)) and the jitter accumulated by the VCO outside the bandwidth of the PLL (\( \sigma_{vco}^2 \)) which corresponds to an integration time of 1/\( f_{BWPLL} \):

\[
FOM_{VCO} = \frac{\Delta t}{P_{VCO}} = \frac{1}{\sigma_{vco}^2 P_{VCO} f_{BWPLL}} \tag{16}
\]

where \( P_{VCO} \) is the power consumption of the VCO. By combining (16) with (15), \( FOM_{PLL} \) can be rearranged as function of \( FOM_{VCO} \) as follows

\[
FOM_{PLL} = \frac{\sigma^2_{vco} P_{PLL} f_{BWPLL}}{f_{VCO} P_{VCO}} \cdot FOM_{VCO} \tag{17}
\]

where \( P_{PLL} \) is the total power consumption of the PLL. This equation is very intriguing since it allows to identify an upper limit that only depends on the type of the oscillator used and the reference frequency adopted (\( f_{REF} \)). In fact, since \( \sigma^2_{vco} \) cannot exceed \( P_{VCO} < P_{PLL} \), and \( f_{BWPLL} \) cannot exceed the reference frequency, the upper limit on the FOM is given by

\[
FOM_{PLL,MAX} = FOM_{VCO} \cdot \frac{f_{REF}}{f_{VCO}} \tag{18}
\]

The presence of an upper limit that depends on both \( f_{REF} \) and \( FOM_{VCO} \) suggests that the FoM expressed by Gao in (14) and (15) cannot be used as a metric to compare different PLL architectures since merely starting from a better VCO and a higher reference frequency could lead to a higher \( FOM_{PLL} \) even with a less efficient architecture.

The fact that better oscillator and higher \( f_{REF} \) help to obtain a better FoM is confirmed by the plot in Fig. 1, where several PLL reported in literature are compared by using (15). As shown in Fig. 1, not only all the design reported in literature are below the upper bound given by (18), but also better FOMs are obtained for higher values of \( FOM_{VCO} f_{REF} \).

V. A FoMs for PLL ARCHITECTURES COMPARISON

The analysis proposed in the previous section has shown that the \( FOM_{PLL} \) expressed by (15) has some biases when used to compare PLL architectures. By assuming a flat in-band noise profile, Gao at. demonstrated that to minimize the overall jitter for a given power, the PLL bandwidth must be set to an optimal value (\( f_{BWPLL,OPT} \)) so that jitter, and power consumption of the VCO are half of the total [4]. By using equations (4) and (19) reported in [4], it is also possible to extrapolate \( f_{BWPLL,OPT} \) for any sub-optimal design to be the following:

\[
f_{BWPLL,OPT} = f_{BWPLL} \sqrt{\frac{P_{VCO}}{P_{LOOP}}} \tag{19}
\]

Where \( P_{LOOP} \) is the power consumed by all the components in PLL loop excluding the power consumed in the VCO. When \( \sigma^2_{vco} \) and \( P_{VCO} \) are both half of the total (17) can be rewritten as

\[
FOM_{PLL,OPT} = \frac{1}{2} f_{BWPLL,OPT} \cdot f_{REF} \cdot FOM_{VCO} \tag{20}
\]

The biasing of the classical \( FOM_{PLL} \) from the power distribution between the VCO and the rest of the PLL (\( P_{LOOP} \)) is confirmed by the plot in Fig. 2, where the \( FOM_{PLL} \) state of the art have been plotted as function of the ratio between \( P_{VCO} \) / \( P_{LOOP} \) (after a normalization by \( FOM_{VCO} \) and \( f_{REF} \), to eliminate the biasing produced by the use of different VCO and different reference frequency). As predicted by Gao et al. in [4], the plot confirms that the designs with better FoM are the ones that tend to equalize the two powers.

A. General FoMPLL for architecture comparison

By using the relationship expressed by (19), equation (20) can be rewritten as
TABLE I

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<td>Fout(GHz)</td>
<td>4</td>
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<td>3.83</td>
<td>4.42</td>
<td>11.8</td>
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<td>11.72</td>
<td>2</td>
<td>2.3</td>
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<td>0.7</td>
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<td>0.21</td>
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<td>0.05</td>
<td>0.20</td>
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<td>6</td>
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<td>182.7</td>
<td>183</td>
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<td>-19.6</td>
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<td>-20.3</td>
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1 Power normalized to 1mW (classical FOM VCO used in literature)

\[ FOM_{PLL_{opt}} = \frac{1}{4} \sqrt{\frac{P_{VCO}}{P_{loop}}} \cdot \frac{f_{BW_{PLL}}}{1Hz} \cdot FOM_{VCO} \]  
(21)

This expression represent maximum FOM<sub>PLL</sub> (eq. (15)) that a design would reach if the power were equally distributed to minimize the overall jitter. In this way it is possible to compare the different design regardless of the constraints that lead to an unbalanced power distribution between loop and VCO. However, as shown previously, for a fair comparison among the different PLL architectures, (21) needs to be normalized by FOM<sub>VCO</sub> and \( f_{REF} \). This yields the following FOM:

\[ FOM_A = 10 \log \left( \frac{1}{4} \sqrt{\frac{P_{VCO}}{P_{loop}}} \cdot \frac{f_{BW_{PLL}}}{1Hz} \right) \]  
(22)

FOM<sub>A</sub> (expressed in logarithmic form) can be used to compare architectures independently of the PLL optimization realized by the designer and without being biased by reference frequency and the VCO performance.

The closer FOM<sub>A</sub> is to zero, the closer the PLL is to its optimal point. Additionally, this FOM assumes the basic phase noise profile shown in [4] which is only valid for single PLLs and not the cascaded ones.

In table one, some of the most performant PLLs present in literature are reported and compared by using the traditional FOM<sub>PLL</sub> expressed by (15) and the new FOM<sub>A</sub> introduced in this paper. Among all the designs the three best values for each FOM are highlighted.

VI. CONCLUSIONS

The traditional FOM for both oscillators and PLL has been analyzed and derived in a more general form by demonstrating the presence thermodynamic upper bonds. Furthermore, a new FOM has been introduced for the comparison of PLL’s architecture, by eliminating the several biases affecting the traditional FOM, such as performance of the VCO, reference frequency and power distribution.

REFERENCES

Statistics of low-frequency noise in MOSFETs

NXP Semiconductors, High Tech Campus 46, 5656 AE Eindhoven, The Netherlands;
†Universidade Federal de Pelotas (UFPel), Pelotas-RS, Brazil; ‡Universidade Federal de Santa Maria (UFSM), Santa Maria-RS, Brazil; *Universidade Federal do Rio Grande do Sul (UFRGS), Porto Alegre-RS, Brazil.

Introduction. The study of MOSFET low-frequency noise variability is important for two reasons. First, due to technology scaling, device dimensions are getting ever smaller, and the low-frequency noise variability may span over many decades (see Ref. [1] and also Fig. 1(a)). Obviously, this huge variability has to be taken into account in noise-sensitive applications. A second reason to study low-frequency noise variability, is that it gives a lot of information about the underlying mechanism [2].

Results. Fig. 1(a) shows an example of the huge low-frequency noise variability encountered in today’s CMOS technologies. The average of such a collection of spectra is remarkably close to $1/f$ [3,4]. However, all the individual spectra are very different, and have “bumps” at different frequencies. To quantify the difference in shape between the various spectra, we study the correlation between noise spectral density $S_{D}$ at two frequencies $f_1$ and $f_2$. In Fig. 1(b)), where the correlation coefficient w.r.t. to the center frequency of $f_1 = 1$ kHz is shown. If we assume that the low-frequency noise is a sum of Lorentzians, the correlation coefficient $c$ is predicted to be [2]

$$c = 2 \cdot f_1 \cdot f_2 \cdot \frac{\ln(f_1/f_2)}{f_1^2 - f_2^2},$$

which is in almost exact agreement with the measured data; see Fig. 1(b). It is important to notice that the shape of $c$ only depends on the spectra of the fundamental noise sources and their superposition; there is no fitting parameter whatsoever in Eq. (1). If, for instance, the observed low-frequency noise would be the sum of microscopic $1/f$ noise sources with a perfect slope of $-1$, $c$ would be equal to 1 for all frequencies. Therefore, this correlation plot forms a signature of the frequency dependence of the fundamental noise sources underlying the low-frequency noise. Thus, the analysis in Fig. 1(b) provides strong evidence for low-frequency noise theories based on the summation of individual RTS signals [5,6]. Interestingly, exactly the same result is found for p-channel MOSFETs [2], the low-frequency noise of which is sometimes claimed to be of different nature [7]. Also, the same characteristic bell-shaped $c = f$ curve is found for long-channel devices, for which the individual noise spectra look much more $1/f$-like [2]. In conclusion, our correlation analysis strongly supports the picture of MOSFET low-frequency noise as a superposition of individual Lorentzians. This is true for NMOS as well as PMOS, and for both short- and long-channel devices.

Figure 1: (a) Low-frequency noise spectral density $S_{D}$ as a function of frequency, measured on a population of 320 small-area ($W = 0.3 \, \mu m, L = 40 \, nm$) n-channel MOSFETs in a 40-nm CMOS technology. (b) Measured (blue markers) and predicted (red line, Eq. (1)) correlation coefficient between $S_{D}$ at a certain frequency $f$ and $S_{D}$ at $f = 1$ kHz. For a few measurement points, error bars (black crosses) are added to indicate measurement uncertainty.

Random Telegraph Signal Fluctuations of Dark Count Rate in CMOS SPAD Structures

F. Di Capua, M. Campajola, D. Fiore, C. Nappi, E. Sarnelli

Abstract — In this paper we report the observation of a peculiar behaviour of Single Photon Avalanche Diode pixels operating in a proton radiation environment. The irradiated SPAD cells, in dark environment, start to switch between two or more levels of the Dark Count Rate pedestals. The investigation of main characteristics of such discrete switching makes it attributable to a Random Telegraph Signal phenomenon. We studied the Dark Count Rate discrete fluctuations as a function of the temperature. The measurements of the time constant of the phenomenon and of corresponding activation energy are reported.

I. INTRODUCTION

Single-Photon Avalanche Photodiodes (SPADs) are solid-state sensors with the capability to detect individual photons as well their arrival time with excellent resolution [1]. The implementation of SPADs in CMOS technology [2] has brought this kind of device to a growing interest thanks to the integrated readout and on-chip data processing [3]-[4]. In several fields where single-photon sensitivity and good timing resolution are required, pixel sensors based on SPADs are employed, like vision camera and lidar implementation for space applications [5], in microscopy and biomedical [6] and quantum cryptography [7].

In space and high-energy physics research applications, SPAD devices are often required to operate in a radiation environment. Radiation-induced defects in silicon structure, to which new energy levels in the bandgap are associated, can seriously worsen SPAD performances, since they cause the generation of carriers in depletion regions through both thermal and tunnelling processes [8, 9, 10] by increasing the Dark Count Rate (DCR). Other than the increase of DCR, the presence of the defects degrades the performance of SPAD devices inducing dark counts discrete fluctuations, known as Random Telegraph Signal (RTS) [11,12]. In this paper we report on the RTS phenomena of two different SPAD structures fabricated in a 150 nm CMOS process after the irradiation with protons. The test chip used contains two layouts: one structure based on P+/Nwell junction enclosed in a low-doped region in order to create a guard-ring to avoid premature edge breakdown; a second structure is constituted by Pwell/Niso junction. More junction details are described in [13-15].

II. THE PROTON IRRADIATION

The devices have been irradiated with protons by using a 14 MV Tandem accelerator and a Superconducting Cyclotron (SC) able to accelerate protons up to 62 MeV we irradiated the devices. Both irradiations have been performed at Laboratori Nazionali del Sud (LNS) - Istituto Nazionale di Fisica Nucleare (INFN) in Catania (Italy). The proton fluence has been measured in line by integrating an ionization chamber current during the whole irradiation time. The ionization chamber has been previously calibrated with a Faraday cup before to irradiate the device. A collimator at the end of beam-line defines the beam shape. The irradiation setup is shown in Fig. 1.

Fig. 1: Test beam setup for SPAD irradiation at LNS Cyclotron;

The dosimetry has been additionally verified with a Gafchromic film EBT3 type [16] positioned at the same position of device under test.. Three samples have been irradiated to similar Displacement Damage Dose (DDD) but with three different proton energies. One sample has been irradiated to a lower DDD of about one third. Direct ionization of protons is responsible for a certain accumulated Total Ionizing Dose (TID) level reported in Tab. 1.
Previous studies on the same device [17] shown a quite good tolerance to TID up to 1 Mrad, therefore the accumulated TID level during this study can be considered negligible.

### III. RTS OBSERVATION

The measurement setup is made by a motherboard providing the power supply to the read-out circuit on chip and the SPAD bias voltage. The output signal is sent to an oscilloscope and to a digital counter. The architecture of the device allows selecting the output of a single SPAD pixel.

The DCR output of each pixel has been acquired for a time of 600 s limiting our research for RTS and time constants at this time scale. The devices have been analysed before the irradiation and two months following the irradiation test to search for the presence of DCR switching pixels. The measurements of DCR show an increase up to two orders of magnitude, as shown in Figure 2 for a SPAD pixel.

![Fig. 2 Dark count rate as a function of applied bias voltage, before and after exposure to displacement damage dose of 304 TeV/g.](image)

After the irradiation a large fraction of pixels in the devices shows RTS behaviour. In Figure 3 the DCR for an irradiated SPAD pixel is plotted as a function of the observation time showing a switching behaviour between two DCR levels. In many other cases the fluctuation of DCR has been found on more than two levels (Fig. 4).

Sometime many levels could be present making difficult to distinguish them. A time lag plot is a scatter plot of data sampled at the $i$th time interval, $t_i$, versus data sampled at $i+1$ time interval, $t_{i+1}$. In such kind of plot the RTS levels appear as clusters along the diagonal (Fig. 5).

![Fig. 3 Two-level DCR fluctuations](image)

![Fig. 4: A three-level RTS in one SPAD pixel](image)

![Fig. 5: Time lag plot relative to RTS pixel shown in Fig. 4](image)

![Fig 6: A four-level RTS in one SPAD pixel: each of two main levels contain two sub-levels](image)

Fig 6 shows a SPAD pixel with a switching behaviour between two main levels which seems to be formed by two sub-levels. In the corresponding time lag plot (Fig. 7) the four populations are displayed.
In Table 2 the RTS occurrence probability for the irradiated sample n.1 has been reported distinguishing between two, three and multi-level RTS. The analysis proves that two-levels RTS are less probable and the number of RTS pixels with more than two levels is higher, as observed also in [18]. A higher RTS occurrence probability has been observed in P+/Nwell with respect to Pwell/Niso layout. On a sample of irradiated SPADs with a DDD of 376 TeV/g we observed a fraction of pixel with a RTS behaviour of 85±3% and 58±3% for PN and PWNISO, respectively.

A similar result has been obtained on other two irradiated samples exposed to a DDD of 300 TeV/g but with different proton energy. For a sample exposed to a DDD of 115 TeV/g we observed RTS occurrence fractions 55±5% and 39±3% for PN and PWNISO, respectively (Table 3).

The results indicate that RTS occurrence depends on the accumulated DDD rather than some peculiar energy-dependent proton interaction cross-section.

### IV. RTS Characterization

In the following we focused on two-level RTS by investigating in more detail some RTS characteristics. The distribution of times in which the system is in the top (or in bottom) DCR level, follow an exponential distribution (Fig. 8) as foreseen in Poisson distribution of random switching events [19]. The time constants of RTS two-levels have been found to be dependent on the temperature (Fig. 9), namely the switching probability is higher by increasing the temperature. In order to investigate it, continuous DCR measurements on the time scale from few hours up to several days have been performed by varying the temperature from 5°C to 40°C.

![Fig. 8: Time distribution for high and low levels in a SPAD with a two-level RTS fluctuations](image1)

![Fig. 9: RTS two-level switching as function of the temperature for a given SPAD pixel](image2)

Time constants ($\tau_{up}$ and $\tau_{down}$) are seen to vary with temperature according to the law

$$\frac{1}{\tau} = C \exp(-E_{act}/KT)$$

An example of the trend of the measured RTS time constant as a function of the temperature, Fig 12 and Fig 13 show the Arrhenius plot for the high and low DCR levels, respectively. In each graph is reported the measurement of a single two-level RTS found in P+/Nwell and Pwell/Niso layouts. The values obtained for the time constant activation energy $E_{act}$ are in the range 0.8-1.0 eV and are in agreement with the calculated kinetics of the reorientation of the vacancy relative to phosphorous atom in a P-V center defect [20]. A phosphorus-vacancy center has been already indicated as a possibility to explain RTS behaviour in [21] and [22]. In a phosphorus-doped device, the vacancies created by proton-induced silicon displacement, migrating in the lattice give rise to a P-V center defects (Fig 11).
The P-V defect can reorient its axis when the vacancy moves in one of four Si atoms close to P atom. This new position corresponds to a new energy level, which is at the origin of the RTS behaviour.

V. CONCLUSIONS

This work reports the RTS observations on two different SPAD layouts, implemented in 150 nm CMOS technology. The devices were irradiated with proton beams of different energies at LNS INFN in Catania. The RTS measurements reported in this work could support the hypothesis that attributes the RTS behaviour to the reorientation of the phosphorus-vacancy (P-V) center. The identification of defects responsible of RTS and the understanding of its evolution in different conditions of temperature and bias voltage could be very useful to limit such effect on the devices operating in a radiation environment. Further investigations are certainly necessary to enlarge the knowledge on RTS and to recognize each defects or cluster of defects involved in this mechanism.

REFERENCES


A kTC Noise Analysis in a Passive Switched-Capacitor CMS Circuit for CIS

Raffaele Capoccia, Assim Boukhayma, and Christian Enz
École Polytechnique Fédérale de Lausanne (EPFL), CH-1015, Switzerland
Email: raffaele.capoccia@epfl.ch

Abstract—This work presents a noise analysis of a passive switched-capacitor correlated multiple sampling (CMS) circuit based on a kTC noise charge method, developed for periodic switched capacitor (SC) circuits. Two case studies are presented, before a generalized formula for the total output noise voltage is given. Calculated values from the derived formula are compared with noise simulations performed with Spectre® circuit simulator.

Index Terms—Noise analysis, Switched Capacitor circuits, Correlated multiple sampling, CMOS image sensors

I. INTRODUCTION

The CMS is a technique widely used in low noise CMOS image sensors (CISs) and it is used to reduce the thermal, the flicker and the RTS noise [1]. This operation embeds an average of the input signal samples into the correlated double sampling (CDS). The averaging operation reduces the contribution of the thermal noise originated from previous stages in the readout chain [2]. In addition to this effect, the CMS introduces a zero in the transfer function, which decreases the contribution of the flicker noise generated by the in-pixel amplifier [3]. The latter is the dominant noise source in modern ultra low noise CISs for low light applications [2]. Different implementations of the CMS are reported in literature and both analog and digital techniques are exploited. A typical analog implementation of the CMS takes advantage of an integrator element to cumulate the sum of \( M \) consecutive samples of the input signal [1]. However, this solution may suffer from a dynamic range (DR) reduction due to this accumulation process. The digital implementations of the CMS [4, 5] require multiple analog-to-digital conversions, performed by a fast and power-hungry analog-to-digital converter (ADC). A SC CMS is based instead on the charge-domain, as shown in [6]. This circuit uses a minimum number of switches and capacitors to average the reset and signal samples, without the need of an additional circuitry and without an impact on the DR.

SC circuits are used for a variety of different applications, among which are signal processing and filter design, where they replace components like large resistors and inductors. The various nonidealities of these components are avoided, like the mismatch, the area and the non-linearity within a range of frequencies. The SC circuits can be implemented either with an active or with a passive implementation. Active SC circuits include an operational amplifier, which can introduce issues with stability and sensitivity, while passive SC circuits require only capacitors, switches and digital control circuits. The main limitations of the passive solution are the analog switches nonidealities, the charge injection, the mismatch and the \( kT/C \) noise. The latter is the charge noise variance on a capacitor due to thermal noise, where \( k \) is the Boltzmann constant, \( T \) the temperature expressed in Kelvin and \( C \) the value of the capacitor. The theoretical estimation of this SC noise can be arduous, since these circuits are a linear time-variant (LTV) system and the noise transfer functions for each noise source have to be recalculated during each phase [7]. This results in a difficult derivation for many SC circuits. The aim of this work is to provide a complete noise derivation of the total output noise of a SC CMS circuit. This derivation makes use of a charge-based method to define all the involved noise terms. The derivation is first proposed for a CDS circuit and then for a CMS of order four. The obtained analytical formulas and the conclusions drawn by this analysis are verified with noise simulations performed with Spectre® circuit simulator.

This paper is organized as follows: in Section II, the operation principle of the SC CMS circuit is briefly described. In Section III, the noise analysis is presented with a detailed step-by-step derivation of the total output noise. The enlargement of the proposed analysis for a CMS of generic order \( M \) is shown in the same Section. In Section IV, the noise simulation results are shown. Finally, the conclusions of this work are drawn in Section V.

II. PASSIVE SWITCHED-CAPACITOR CMS CIRCUIT

The schematic of the SC CMS and the timing diagram of the control signals (from \( S_1 \) to \( S_{10} \)) applied to the switches are shown in Figs. 1a and 1b. In Fig. 1a, the CMS schematic is divided into two red blocks, which are named the averager and the subtractor. The averager is where the input voltage level, \( V_{in} \), is sampled and the averages are progressively built. The subtractor is the part where the difference between the two average is implemented. The implementation of the subtractor requires only two source follower (SF) buffers, four switches and a bootstrap capacitor, \( C_B \). The averager takes advantage of the charge sharing principle between equal capacitors (from \( C_1 \) to \( C_5 \)). If the initial voltages of two equal capacitors are respectively \( V_1 \) and \( V_2 \), when connected together the final voltage across them will be equal to \( (V_1 + V_2)/2 \). This is due to the charge sharing between the two capacitors. If \( M \) consecutive samples with a sampling period, \( T_{CMS} \), are stored on \( M \) different capacitors, connecting them all together at the instant \( M \cdot T_{CMS} \), results in averaging the \( M \) samples. If the
initially, the switches opened a sample, and the exact same operations are allowed to store the average of eight samples on two equal capacitors are needed for averaging \(2^n\) samples. In a low noise CIS, the two averages correspond respectively to the reset and the signal levels, after the transfer of the photogenerated charge from the pinned photodiode (PPD) to the sense node (SN) [2]. In the schematic of Fig. 1a, five equal capacitors are used and the two final averages are stored on capacitors \(C_4\) and \(C_5\), while the timing diagram in Fig. 1b assumes \(M = 8\). Initially, the switches \(S_1, S_2\) and \(S_3\) are closed; when \(S_3\) is opened a sample, \(V_1\) is stored in \(C_4\). Then, \(S_1\) is also opened after a time \(T_{CMS}\) and the next sample, \(V_2\), is stored in \(C_1\). \(S_3\) is pulsed while \(S_1\) is opened and \(S_2\) is closed, and the average voltage is stored in \(C_3\). The exact same operations are iterated between capacitors \(C_1\) and \(C_2\), and then the average between fours samples is stored on \(C_4\) by closing \(S_3\) and \(S_5\). Other two iterations on \(C_3\) and on \(C_2\) allow to store the average of eight samples on \(C_4\). The latency introduced by the CMS is minimized if the sampling and the settling time between two consecutive samples are optimized. In a CIS, the additional area of this circuit is not significant, since the control signals for the switches are common to all the implemented readout columns and the logic circuit can be unique. To evaluate the impact of the CMS on the temporal readout noise, the squared absolute value of the CMS transfer function, \(|H_{CMS}(f)|^2\), has to be evaluated. The latter is plotted for different values of \(M\) in Fig. 2. The area delimited by \(|H_{CMS}(f)|^2\) reduces with the increased value of \(M\), hence the result of the integration with the constant power spectral density for the thermal noise is inversely proportional to \(M\). At low frequencies, the CMS introduces a zero in the transfer function. However, the maximum of \(|H_{CMS}(f)|^2\) moves to lower frequencies by increasing \(M\). Hence, the \(1/f\) noise is also expected to be reduced by increasing \(M\) and a plateau for the residual flicker noise can be predicted [2].

III. NOISE ANALYSIS IN SC CMS CIRCUIT

To evaluate analytically the total \(kT/C\) output noise of the proposed SC CMS, the noise analysis follows the following steps: first, the noise calculation method based on the charge-domain is explained; then, this method is applied on the basic example of the CDS circuit and to the SC CMS averager of order four; the noise analysis is then extended to a generic order \(M\) of the SC CMS. Finally, to derive the formula of the total output noise variance, the contribution of the subtractor is also added.

A. Noise Calculation Method

The charge-domain noise calculation method was first introduced in [8], where it was applied to a passive SC low pass filter and to a \(N\)-path filter. In periodic SC networks, each capacitor can be either connected to a voltage source or to another capacitor, through a switch. When connected to a voltage source, the switch resistance, \(R_{on}\), is responsible for the thermal noise injected in the capacitor, from which it is originated the well-known noise voltage variance, \(kT/C\). When connected to another capacitor, a phenomena of charge sharing between the two components takes place. The thermal noise is an uncorrelated type of noise, hence each generated noise term is independent from the others. Each noise contribution can be considered separately, from the generation during the voltage sampling to the propagation, where all operations are ideal since no extra noise is added. In this method, all the different phases in a SC circuit are first determined. Then, all the noise terms are distinguished and propagated in time through all the phases. Finally, all the contributions are summed to obtain the final output noise variance.

![Graph showing the term \(|H_{CMS}(f)|^2\) as a function of the normalized frequency \(f/T_{CMS}\) for four different values of the CMS order.](image)

\(|H_{CMS}(f)|^2\) as a function of the normalized frequency \(f/T_{CMS}\) for four different values of the CMS order.
B. Correlated Double Sampling

The schematic of the SC CDS is shown in Fig. 3. In a CDS operation, the difference of two input voltage samples is implemented [9]. In this circuit, it is possible to define three phases: φ1, where the first sample is stored on C1, φ2, to store the second sample on C2, and φ3, to implement the difference between the two samples. The noise charge variance on C1 at the end of φ1, $Q_1^2$, is equal to $kT/C_1$. When expressed in terms of voltage noise variance, $V_{\phi1}^2$, is equal to $kT/C$. This noise value will not be influenced by φ3, while in φ3 the charge sharing between C1 and C2 will lead to

$$V_{\phi3}^2 = \frac{kT}{C_1 + C_2}. \quad (1)$$

Similarly, the voltage noise variance due to the noise contribution generated during φ2 and shared during φ3, $V_{\phi2}^2$, can be written by inverting $C_1$ with $C_2$ in $V_{\phi1}^2$. Finally, the last noise contribution is generated during φ3 and can be expressed as

$$V_{\phi3}^2 = \frac{kT}{C_1 + C_2}. \quad (2)$$

All the computed terms can be summed to give the total output noise voltage variance, $V_{n,\text{out}}^2$, given by

$$V_{n,\text{out}}^2 = \frac{kT C_1}{(C_1 + C_2)^2} + \frac{kT C_2}{(C_1 + C_2)^2} + \frac{kT}{C_1 + C_2}. \quad (3)$$

If we consider the typical case where $C_1$ and $C_2$ are both equal to $C$, the total voltage variance results into $kTC$.

C. Correlated Multiple Sampling

The schematic of the SC averager circuit for a CMS order equal to four implemented with a minimum number of capacitors is shown in Fig. 4a. The minimum number of phases for sampling and averaging is equal to six (φ1 to φ6) and they are described in Fig. 4b. The noise generated in φ1 is equal to $kT/C_3$ and the charge sharing between $C_1$ and $C_3$ will determine a voltage variance equal to $kTC_3/(C_1 + C_3)^2$. By multiplying the previous expression with $C_3^2$, it is possible to define a charge noise variance. This noise term will then face a second operation of averaging, which will lead to the following expression

$$Q_{\phi1}^2 = \frac{kT C_3^2}{(C_1 + C_3)^2} C_3^2. \quad (4)$$

Both of these variances are equal to $kTC/8$, in the case of equal capacitors. The last noise contribution to be taken into account is the one generated by the average implemented during φ6, which results in

$$Q_{\phi6}^2 = \frac{kT C_3^2}{C_1 + C_3}. \quad (5)$$

The latter is equal to $kTC/2$ when $C_1 = C_3$ is assumed. All the noise terms can be summed to obtain the total output charge noise variance, which is equal to $kTC$. The noise analysis for the SC averager can now be generalized for an order $M$ of the CMS. As shown in Fig. 5, every averager in a CMS of order $M$ can be divide into the average of two averagers of order $M/2$. Based on the previous examples, if this process is iterated, each averager can be considered to generate a noise voltage variance equal to $kTC$. Under these assumptions, the total noise output voltage, $V_{n,\text{avg}}^2$, is given by

$$V_{n,\text{avg}}^2 = \frac{1}{4} \left( \frac{kT}{C} + \frac{kT}{C} \right) + \frac{kT}{2C} = \frac{kT}{C}. \quad (6)$$

If we consider all capacitors to be equal to $C$, $Q_{\phi0}^2$ is equal to $kTC/16$. The same noise contributions can be obtained for the other three samples, generated during φ2, φ3 and φ4. In φ3 and φ2, two different averages are generated, which are averaged a second time during φ6. The two additional noise contributions, $Q_{\phi3}^2$ and $Q_{\phi6}^2$, can be written as

$$Q_{\phi3}^2 = \frac{kT C_3^2}{C_1 + C_2}, \quad Q_{\phi6}^2 = \frac{kT C_3^2}{C_1 + C_3}. \quad (7)$$

Fig. 5. Generic CMS order $M$ splitted into two blocks of order $M/2$ and a simple averager.
The first noise term in (7) is originated by the noise sharing, while the second is the contribution of the average block. $V_{n,\text{out}}^2$ is equal to $kT/C$ for each order $M$ of the passive SC CMS circuit. The output noise contribution of the averager is independent from the number of input voltage samples that are processed. To give a formula for the total output voltage noise of the SC CMS circuit, the impact of the subtractor and the noise generated by the subtractor itself have to be added. The operation of subtraction doubles the input noise variance, while each voltage buffer will contribute with a noise variance equal to $\gamma kT/C_0$, where $\gamma$ is the noise excess factor [10]. The latter assumes a low output impedance equal to $1/G_{\text{in}}$ for the voltage buffers. Finally, the contribution from the switching operation is equal to $kT/C_0$, and the final output voltage variance results into

$$V_{n,\text{out}}^2 = \frac{2kT}{C} + \frac{(2\gamma + 1)kT}{C_0}. \tag{8}$$

IV. SIMULATIONS RESULTS

The proposed formula for the output voltage noise of the SC CMS is verified with periodic noise (PNOISE) and transient noise simulations in the Spectre® circuit simulator. These simulations are performed with a time between the samples, $T_{\text{CMS}}$, equal to $1 \mu s$. Figs. 6a and 6b show the root-mean-square of the CMS output voltage, $V_{n,\text{out}}$, in Fig. 6a, $V_{n,\text{out}}$ is computed for four different values of the averager capacitor, $C$, from $200\, \text{fF}$ to $1\, \text{pF}$. The noise contributions from the averager and the subtractor are shown separately and in dotted lines. The average noise reduction with an increase in $C$ is predicted by the $kT/C$ term in (8). The noise of the subtractor is instead not influenced by the value of $C$. In Fig. 6b, the output voltage noise is evaluated for four different CMS orders. The simulation results confirm that the noise contribution of the SC CMS is independent of $M$, hence it is not influenced by the needed number of samples. The PNOISE and transient noise simulations results show a good matching and validate the presented noise analysis results.

V. CONCLUSION

A method for $kT/C$ noise calculation based on the charge-domain noise is used to derive an analytical expression for the output noise of a passive SC CMS circuit. This method is applied to the CDS and the CMS with $M = 4$, before enlarging to the CMS of a generic order $M$. The derived formulas are validated by the good matching between the calculated values and the Spectre® PNOISE and transient noise simulation results. The analytical and simulation results show the noise contribution of this SC CMS circuit is not influenced by the CMS order.

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Nanoscopy of charge kinetics via terahertz fluctuation

Qianchun Weng1,2, Zhenghua An3, Wei Lu2, and Susumu Komiyama4,5

1Surface and Interface Science Laboratory, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
2National Laboratory for Infrared Physics, Shanghai Institute of Technical Physics, The Chinese Academy of Sciences, Shanghai 200083, China
3Department of Physics, Fudan University, Shanghai 200433, China
4Department of Basic Science, The University of Tokyo, Komaba 3-8-1, Meguro-ku, Tokyo 153-8902, Japan
5Terahertz Technology Research Center, NICT, Nukui-Kitamachi 4-2-1, Koganei, Tokyo 184-8795, Japan

Abstract—Hot electron distribution is visualized with a novel microscope, called scanning noise microscope (SNoiM), in which local current fluctuation or current noise in conductors is mapped on nanoscales. The detected noise frequency (~ 20 THz) is high enough to image ultrafast electron dynamics in nanoscale devices. The basic concept of SNoiM will be described along with several experimental results.

I. INTRODUCTION

Most material generates fluctuating electromagnetic evanescent field on its surface all the time due to current fluctuation (or the noise), thermally agitated (Nyquist noise) in equilibrium or non-thermally excited (excess or shot noise) in non-equilibrium conditions [2]. In either case, the local current fluctuation carries nanoscopic information of what is taking place at the given point of the material [3]. Reported here is an ultra-sensitive scattering-type scanning near-field optical microscope (s-SNOM) [4], called scanning noise microscope (SNoiM) [5,6], in which current fluctuation (or noise) is locally detected and mapped by picking up the fluctuating electromagnetic (EM) evanescent field that non-equilibrium electrons generate on the conductor surface (Fig. 1).

II. RESULTS

Successful applications of SNoiM include, so far, the imaging of thermal current fluctuation in metals [5,6], the mapping of concentrated Joule-heating in bended narrow metal wires [7], and the visualization of non-local energy dissipation of hot electrons in GaAs/AlGaAs quasi-two dimensional electron devices (Fig. 2) [3]. In the presentation, the uniqueness of SNoiM will be demonstrated by highlighting the hot electron systems and comparing the experimental results with those obtained with other techniques of scanning probe microscope such as active near-field optical microscopes (SNOMs) [3], scanning thermal microscope, and pump-probe microscopes.

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Low Frequency Noise Spectroscopy of GaN Bow-Tie THz Detectors

Sandra Pralgauskaitė
Institute of Applied Electrodynamics and Telecommunications, Vilnius University, Vilnius, Lithuania
e-mail: sandra.pralgauskaite@ff.vu.lt

Jonas Matukas
Institute of Applied Electrodynamics and Telecommunications, Vilnius University, Vilnius, Lithuania
e-mail: jonas.matukas@ff.vu.lt

Evaldas Kazukauskas
Institute of Applied Electrodynamics and Telecommunications, Vilnius University, Vilnius, Lithuania
e-mail: evaldas.kazukauskas@ff.vu.lt

Irmantas Kašalynas
Terahertz Photonics Laboratory, Center for Physical Sciences and Technology (FTMC), Vilnius, Lithuania
e-mail: irmantas.k@ktl.mii.lt

Vytanuiąs Janonis
Terahertz Photonics Laboratory, Center for Physical Sciences and Technology (FTMC), Vilnius, Lithuania
e-mail: vytautas.janonis@ftmc.lt

Jonas Matukas
Institute of Applied Electrodynamics and Telecommunications, Vilnius University, Vilnius, Lithuania
e-mail: jonas.matukas@ff.vu.lt

Evaldas Kazukauskas
Institute of Applied Electrodynamics and Telecommunications, Vilnius University, Vilnius, Lithuania
e-mail: evaldas.kazukauskas@ff.vu.lt

Paweł Prystawko
Institute of High Pressure Physics, UNIPRESS, Warsaw, Poland
e-mail: pprysta@unipress.waw.pl

Abstract—The low frequency (10 Hz - 20 kHz) noise characteristics of GaN-based bow-tie (BT) diodes designed for room temperature terahertz (THz) detection have been studied in the temperature range from 77 K to 320 K. Noise spectroscopy revealed the influence of the defects as charge carrier capture centers to the THz detector operation. The low frequency noise characteristics of the BT diodes are comprised from 1/f and Lorentzian type spectra. The observed fluctuations are caused by charge carrier capture and release processes in the centers with (0.19-0.55) eV activation energy. The diodes with different apex width demonstrated an increase of 1/f type fluctuations with decrease of the apex size.

Keywords—detector; GaN; noise; THz; trap

I. INTRODUCTION

Nowadays terahertz (THz) imaging and spectroscopy showed great prospects in a broad variety of applications and stimulated the development of THz technology [1]. Antenna coupled field effect transistors (TeraFETs), microbolometers, Schottky diodes have been proposed for efficient THz sensing at room temperature [2-4]. The main features for THz detectors employed in the THz imaging and spectroscopy systems are high sensitivity, low noise and short response time. Bow-tie (BT) diodes have been suggested as compact, room temperature THz detectors [5-7]. The BT diode is semiconductor layer processed in a BT shape with metalized one of its two leaves. Device operation is based on the THz wave rectification due to the non-uniform carrier heating in a trapeze-shaped semiconductor layer [5, 6]. The BT diodes have benefit of simplicity of fabrication process comparing to other THz detector concepts. Also they demonstrate very high resistance to an electrical and mechanical stress.

GaN, AlGaN and other nitride based materials are attractive wide bandgap semiconductors possessing high critical breakdown field, good thermal conductivity, large electron velocity, etc. However, lattice constant mismatch is often obtained in multilayered nitride based heterostructures. And this leads to a higher density of defects [8]. Defects formed charge carrier trapping centers cause random carrier capture and release processes determining fluctuation of free charge carrier number in the structure. Such fluctuation is a limiting factor for the detector performance [9, 10]. It is observed that long characteristic times are characteristic for the defect’s centers in the nitride based structures and carriers can be trapped in deep-level defects for a very long time [8, 9]. Therefore, instability in time of GaN-based diode operation characteristics and hysteresis behavior have been observed [8]. Also, defects play a key role on the detector’s reliability and rapids its failure [10-12].

In this paper the low frequency noise was measured in order to understand physical mechanisms of the charge carrier trapping and detrapping processes and influence of those processes to the THz detector performance. Low frequency noise spectroscopy is well known method for investigation of charge carrier transport and their trapping in defects’ formed centers [13-16]. Noise intensity determines the noise level of biased THz detector and its sensitivity [13, 17]. Furthermore, low frequency noise characteristic analysis reveals device quality and enables prediction of its reliability [14, 18, 19]. Thus, obtained results enable improvement of the operation characteristics of GaN-based BT detectors.

Here we present a comprehensive analysis of low frequency (10 Hz – 20 kHz) noise characteristics of GaN-based BT diodes designed for room temperature THz detection. The noise was measured to figure out an origin of the charge carriers’ trapping processes, their influence to the performance and quality of the THz detector.

II. DEVICES AND MEASUREMENT DETAILS

The BT diodes were fabricated of Al0.25Ga0.75N/AlN/GaN/SiC HEMT structures possessing the 2DEG density of 9 × 1013 cm–2 and the electron mobility of 1.7 × 104 cm²/Vs and 15 × 104 cm²/Vs at room and liquid nitrogen temperatures, respectively [20]. Ohmic contacts including one of the BT leafs metallization were fabricated of Ti/AI/Ni/Au compound annealed in nitrogen environment at 830 °C. The BT diodes with different apex width: 2 μm, 5 μm, and 7 μm, are studied in this paper.

The low frequency voltage fluctuation characteristics were measured in frequency range from 10 Hz to 20 kHz. The constant current operation was guaranteed by choosing appropriate load resistance. Noise measurements were performed in a specially shielded room (Faraday cage) to avoid interferences from electrical and communication
networks. The voltage noise spectral density was evaluated by comparing to the thermal noise of the standard resistor.

Current-voltage (resistance) characteristics were measured using semiconductor parameter analyzer Keysight Technologies B1500.

The investigation was carried out at room temperature and in the temperature range from 78 K to 320 K.

III. RESULTS AND DISCUSSION

Current-voltage characteristics of BT diodes are nonlinear and asymmetrical (Fig. 1), what is explained by a non-uniform carriers’ diffusion caused by the non-uniform electric field distribution in the trapeze-shaped semiconductor: electric field is concentrated in the vicinity of the apex [5, 6]. Also the resistance size was found to be dependent on the apex width. We noticed that investigated BT diodes demonstrated hysteresis in the resistance, which depends on the measurement cycle and the direction of voltage sweep. Also, an absolute value of the resistance varied with time. Such hysteresis and characteristics’ instabiliy with long (in order of few seconds) characteristic time show that used heterostructures contain deep defects acting as charge carrier trapping centers. Detailed experimental resistance instabilities were described in [8].

Measured voltage noise spectral density at room temperature is shown in Fig. 2. The noise was found directly proportional to voltage square at negative bias up to -2 V and at positive bias up to 0.2 V. Proportionality of the noise spectral density to voltage square indicates that observed voltage noise is determined by the resistance fluctuations. Dependence of the noise spectral density on the bias voltage polarity is caused by the same phenomena as for current-voltage characteristic. The lowest noise spectral density was observed at zero bias demonstrating optimum operation regime for BT diodes as THz detectors [5]. Noise intensity correlates with resistance of the diode: devices with larger resistance have higher noise level. Samples of lower resistance and with lower noise level over all investigated bias range demonstrate larger 1/f noise drop approaching to zero bias. As the resistance of the detectors with 2 μm apex width was greater comparing to the resistance of 5 μm or 7 μm apex diodes, the detectors with narrower apex also have demonstrated worse noise characteristics.

Noise spectra of the investigated BT diodes comprise from the thermal noise, 1/f fluctuations, and Lorentzian type components (Fig. 3). Fluctuations with the 1/f type spectrum usually are caused by the superposition of the charge carrier trapping and detrapping processes in defects [21]. The Lorentzian type “bumps” in the noise spectra were observed at characteristic temperature and/or bias current (Figs. 3-5). Lorentzian type “bumps” indicate presence of charge carrier trapping centers and appear, when the Fermi energy level coincides with the energy of the center: at particular operation conditions distinct charge carrier trapping centers are active.

The investigated BT diodes with larger resistance (no matters with the same or different apex width) also demonstrated larger voltage noise spectral density. Larger noise spectral density mainly is caused not by the higher intensity of the thermal noise, but by increase of the 1/f type fluctuations. For example, compare diodes with 7 μm and

![Fig. 1. Typical current-voltage characteristics of GaN-based BT diodes with 2 μm and 7 μm apex width at room temperature (inset shows electrical connection of the BT diode).](image1)

![Fig. 2. Dependencies of voltage noise spectral density on voltage at room temperature of GaN-based BT diodes with different apex width: 2 μm, 5 μm, and 7 μm (curves at “+” are at the positive bias and curves at “-” are at the negative bias).](image2)

![Fig. 3. Voltage noise spectra at room temperature of GaN-based BT diode with 2 μm apex at positive bias (solid line corresponds to the thermal noise at zero bias).](image3)
2 μm apexes: resistance increase from 1.1 kΩ to 1.8 kΩ causes thermal noise increase from $1.7 \times 10^{-12}$ $\text{V}^2\text{s}$ to $2.9 \times 10^{-11}$ $\text{V}^2\text{s}$ (1.7 times) while measured 1/f noise increased from $4.1 \times 10^{-13}$ $\text{V}^2\text{s}$ to $1.3 \times 10^{-12}$ $\text{V}^2\text{s}$ (3.2 times) (at positive 1 V bias, noise spectral density at 1 kHz). So, larger bias causes more intensive 1/f type fluctuations and just minor change of the thermal noise.

Temperature characteristics of the investigated BT diodes are presented in Fig. 4. Noise spectral density rises with temperature increase at temperatures below 250 K, where resistance of the diode almost does not change. At temperature above 250 K, the noise spectral density decreases when the resistance of the device starts to increase due to the decrease of carrier mobility.

As it was mentioned, at particular temperature the Lorentzian type fluctuations were observed. The regions, where the Lorentzian type fluctuations occurred, are marked by the lined areas (see Fig. 4). An example of voltage noise spectra representing activity of one trapping center is shown in Fig. 5. The Lorentzian type spectra were more evident at the positive bias than at the negative one. The observed spectra can be modeled by one or a few Lorentzians (each of them represents charge carrier trapping and detrapping process in a single center with characteristic time and activation energy), 1/f noise component and thermal noise base. The characteristic time of the observed trapping centers is found in the range (0.05-1) ms (see Fig. 6). Activation energies were calculated for those centers, which voltage fluctuation spectra can be approximated by a single Lorentzian type spectrum. The obtained activation energies are of 0.19 eV, 0.45 eV, and 0.55 eV (Fig. 6). The correlation between BT diode apex width and the trapping center parameters (characteristic time, activation energy) was not identified. And the influence of the defects, that also cause appearance of Lorentzian-type “bumps” in the noise spectra, is more adverse for the BT diodes with narrower apex.

### IV. CONCLUSIONS

Low frequency noise characteristics of GaN-based bow-tie diodes with different apex width have been investigated in temperature range of 78-320 K. The noise spectral density dependence on bias voltage was found asymmetrical and it was similar to the asymmetrical current-voltage characteristic of the device caused by the hot carriers’ diffusion under non-uniform internal electric field formed in the apex area of the trapezoid semiconductor.

Resistance instability in time and Lorentzian type noise spectra of the GaN-based BT diodes are explained by charge carrier trapping and detrapping processes in deep centers formed by defects. Low frequency noise spectroscopy has shown that centers have the characteristic time and the activation energy of (0.05-1) ms and (0.19-0.55) eV, respectively.

Correlation between the apex width of the investigated BT diodes and the observed trapping center parameters was not found. But influence of the defect centers was more evident in noise characteristics of the diodes with narrow apex.
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1/f Noise Model of 980 nm InGaAs/GaAs Laser Diodes based on Parasitic Parameters under Low Injection Current

1st Xiaojuan Chen  
School of Electronic and Information Engineering,  
Changchun University of Science and Technology-No.7089, Weixing Road,  
Changchun, China  
*Corresponding author: cxj_neiep@126.com

2nd Chang Qu  
School of Electronic and Information Engineering,  
Changchun University of Science and Technology-No.7089, Weixing Road,  
Changchun, China  
2018200079@mails. cust.edu.cn

Abstract—It is found that low frequency noise is always as a fast, easy-to-use, accurate and non-destructive tool to characterize the performance and the reliability of materials and electrical devices. In this paper, the noise equivalent circuit of laser diode with parasitic parameters under low injection current is developed. Based on theory of the carrier number fluctuation, 1/f noise model in 980 nm InGaAs/GaAs laser diode under low injection current is developed. This model suggests the low frequency noise of the device is caused by the fluctuation of surface nonradiative recombination current, which depends on the surface oxide traps and lattice dislocation. The low injection current were performed from one-thirtieth up to very close to the threshold current. The model can explain well the experimental results.

Index Terms—1/f noise model, laser diodes, low injection current, nonradiative recombination current, parasitic parameters

I. INTRODUCTION

980 nm InGaAs/GaAs laser diodes (LD) have been widely utilized in a plurality of fields, such as optical fiber communication, laser processing, assembly industrial and many other applications due to their high output power, stable optical spectrum, high photoelectric transformation efficiency, low electrical consumption, high performance and potential low-cost [1], in which area, high quality and functional reliability are required. As optical devices become more widely used and the performance requirements gradually improved, their reliability issues are of concern. Low frequency noise is always a fast, easy-to-use, accurate and non-destructive tool to characterize the performance and the materials and electrical devices. Low frequency 1/f noise, also as known as flicker noise, depends on the defects and impurity concentrations in the microscopic structure of the device to reflect the inherent quality and reliability of the device [2]. So, research on low frequency noise characteristics of laser diodes is of great significance.

The 1/f noise characteristics in laser diodes has been investigated for many years by many researchers and different 1/f noise models of semiconductor laser diodes have been developed and tried to explain the physical mechanism and the origins of 1/f noise [3]. In the below-threshold region the characteristics of the active region dominate the electrical behavior of the LD. However, comparison of the I-V characteristics of the LDs, the low frequency 1/f noise characteristics under low injection current contain more microscopic information that can reflects the internal defects of the device. Meanwhile, processes, observed at the low injection current (below laser diode threshold), could be used as precursors of device quality and degradation mechanism which affects LD reliability [4].

In the present paper, a 1/f noise model in 980 nm InGaAs/GaAs laser diodes under low injection current is presented, which takes into account the effects of parasitic parameters and nonradiative recombination. The noise equivalent circuit model of laser diodes with parasitic parameters which includes the parameters of package part, laser chip part and the active region of LD is developed. This noise equivalent circuit model is derived from the semiconductor physics and the electrical characteristics of LD. According to the noise theory (mainly the mobility fluctuation model and the carrier number fluctuation model), the recombination 1/f noise model based on parasitic parameters in the package surface and active region of LD under low injection current will also be described.

II. DEVICE DETAILS AND EXPERIMENTAL ANALYSIS

A. The device details

The device used is InGaAs/GaAs MQW high-power laser diodes lasing at 980 nm. The device structure was grown by means of metalorganic chemical vapor deposition (MOCVD) on a GaAs substrate and consisted of the active layer using an InGaAs double quantum well structure with two InGaAs well layers about 7-9 nm, and 100 nm composition AlGaAs barrier layer between the wells about 100 nm. In order to improve the anti-COD of the device, a 2.1 μm ultra-large cavity waveguide layer is adopted, in which the waveguide layer and cladding...
layer are Al\textsubscript{0.1}Ga\textsubscript{0.9}As and Al\textsubscript{0.3}Ga\textsubscript{0.7}As, respectively. The waveguide layer is asymmetric, that is the thickness of the layer is 300nm larger than the thickness of the upper waveguide layer to reduce the loss of the waveguide layer and suppress the higher-order mode. After the epitaxial growth, wide strip ridge device is fabricated by first etching a ridged plate with 300 nm height, sputtering 200 nm of SiO\textsubscript{2} and forming a 95\textmu m p-type electron hole by photoetching, using Ti/Au as p-type electrode and AuGeNi/Au as n-type electrode. The cavity length of the chip cleavage is 400\textmu m, and the stripe width is 95\textmu m. The threshold current and the slope efficiency were about 700 mA and 1.12 W/A at 25 °C.

B. Noise analysis of laser diode under low injection current

In order to investigate the noise source of the laser diode and its \(1/f\) noise source under low injection current, a noise equivalent circuit of 980 nm InGaAs/GaAs laser diodes with parasitic parameters under low injection current according to the low frequency noise analysis methods of Van der Ziel, Harder et al. is established. This circuit model can provide useful information for establishment of the laser diode \(1/f\) noise model under low injection current (below threshold). The noise equivalent circuit model is shown in Fig.1. This model is a two-port electrical network and consists of two parts i.e. LD parasitic parameter part and active region parameter part, of which LD parasitic parameter part includes LD chip parasitic parameters and package parasitic parameters. The meaning of the parameters and formula derivation in the noise equivalent circuit will be described in detail in the subsequent paper [5].

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{Fig.1}
\caption{The noise equivalent circuit model with parasitic parameters under low injection current}
\end{figure}

III. \(1/f\) NOISE MODEL OF 980 NM INGAAS/GAAS LASER DIODES UNDER LOW INJECTION CURRENT

Semiconductor laser diode is a type of laser with semiconductor material as optical gain medium, which is excited by p-n junction injection current to generate stimulated emission in a chip and establish optical oscillation, thus produce laser. In the present paper, the laser diode is restricted to below-threshold (under low injection current) operation and stimulated emission is therefore neglected.

In general, the surface of laser diode has a certain modulation effect on its performance, the \(1/f\) noise of laser diode depends on the surface composition of its operation current. In general, the current through the laser diodes generally has radiative recombination current \(I_r\) and the surface recombination current \(I_{nr}\), which is generated by the non-radiative recombination. Current-voltage characteristics in the low bias range are governed by the surface non-radiative recombination current if p-n junction reaches the surface [6]. Therefore, under low injection current, the low frequency noise of laser diode mainly comes from \(1/f\) noise caused by the fluctuation of surface non-radiative recombination current, which has a similar mechanism and a good corresponding relationship with the surface non-radiative recombination current.

When the LDs is under forward bias, the difference between the energy quasi-Fermi level of electrons \(E_{F_n}\) and the quasi-Fermi level of holes \(E_{F_p}\) is related to [7]

\[ E_{F_n} - E_{F_p} = qV_j \]

where \(q\) is the electron charge, and \(V_j\) is diode junction voltage.

The carrier concentrations in the space charge region are given by

\[ n(x) = n_i \exp \left(\frac{E_{F_n} - E_{F_{ix}}}{k_0 T}\right) \]  

\[ p(x) = n_i \exp \left(\frac{E_{F_p} - E_{F_{ix}}}{k_0 T}\right) \]

where \(E_{F_{ix}}\) is the intrinsic Fermi level in the space region. \(n_i\) is the intrinsic carrier concentration, \(k_0\) is the Boltzmann constant. \(T\) is the absolute temperature.

At the edge of the space charge region, the non-equilibrium minority concentrations of the p side \(p_n'\) and the n side \(n_n'\) are respectively

\[ n_p = n_{p0} \exp \left(\frac{qV_j}{k_0 T}\right) = n_{p0} \exp \left(\frac{qV_j - qV_D}{k_0 T}\right) \]

\[ n_n = n_{n0} \exp \left(\frac{qV_j}{k_0 T}\right) = p_{p0} \exp \left(\frac{qV_j - qV_D}{k_0 T}\right) \]

where \(n_{p0}, p_{p0}\) are the equilibrium carrier concentrations of p region, and \(n_{n0}, p_{n0}\) are the equilibrium carrier concentrations of n region, respectively. \(V_D\) is barrier potential of p-n junction.

The carrier concentrations in p region and n region are given by the following equations:

\[ P_p = N_A = n_i \exp \left(\frac{E_{F_p}}{V_T}\right) \]

\[ n_n = N_D = n_i \exp \left(\frac{E_{F_n}}{V_T}\right) \]

From eqn. 1, the diode junction voltage can be written as

\[ V_j = \frac{1}{q}(E_{F_n} - E_{F_p}) = \frac{k_0 T}{q} \ln \left(\frac{n_{p0}}{n_{n0}}\right) = \frac{k_0 T}{q} \ln \left(\frac{N_D N_A}{n_i^2}\right) \]

According to the equation above, the quasi-Fermi potential for electrons \(V_{F_n} = \frac{k_0 T}{q} \ln \left(\frac{N_A}{n_i}\right)\) and the quasi-Fermi potential for holes is \(V_{F_p} = \frac{k_0 T}{q} \ln \left(\frac{n_i}{N_D}\right)\). \(N_D N_A\) are doping concentrations.
From the reference, we can define the intrinsic Fermi level for the intrinsic $E_{i0}$ as zero. From the energy band diagram of p-n junction under forward bias (shown in Fig. 2), the intrinsic Fermi level can be described approximately [7]:

$$E_{ix} = E_{i0} - E_{iW} \cdot \frac{x}{W} = q \left( V_{j} - V_{Fn} - V_{D} \right) \frac{x}{W}$$

(9)

where the origin of the $x$ axis is set at the p side of the surface region. $W$ is the total width of the space charge region, and the value range is [0, $W$].

Under low and forward injection current, a part of the injected electron and hole is confined as a minority carrier at its quantized ground state within the surface space charge region. Each minority carrier diffuses along the surface in the cladding layer until the recombination takes place, where recombination rate is determined by the surface recombination velocity across section, etc [8]. This same analysis can also be applied to the interface of the investigated laser diodes, so the recombination current per unit length under low injection current at the interface of space charge region can be expressed by the following equation:

$$I_r = qA \int_0^W U dx = qA \int_0^W s \cdot n_s p_s - n_i^2 \frac{x}{n_s + p_s + 2n_i} dx$$

(10)

where $U$ is the recombination rate of carriers through the recombination center, and $A$ is the cross section. $s = v_{th} \gamma N_{it}$ is the surface recombination velocity, $v_{th}$ is the electron thermal velocity, $\gamma$ is the trap cross section and $N_{it}$ is interface trap density, $n_s$ is the surface electron concentration, and $p_s$ is the surface hole concentration.

The diode current arises from nonradiative recombination along the stripe edges and at the heterointerface under low bias [9]. It is assumed that the nonradiative recombination rate is proportional to the electron density $n$, and is characterized by a nonradiative carrier lifetime $\tau_{nr}$. The nonradiative recombination rate $r_{nr}$ is given by:

$$r_{nr} = \frac{n}{\tau_{nr}}$$

(11)

The laser diode non-radiative recombination current $I_{nr}$ is obtained by multiplying $r_{nr}$ by the recombination current under low injection current $I_r$:

$$I_{nr} = r_{nr} qA \int_0^W s \cdot \frac{n_s p_s - n_i^2}{n_s + p_s + 2n_i} dx$$

(12)

The carrier concentrations at the surface of the space charge region can be written by

$$n_s = n(x) \exp \left( \frac{V_s}{k_0T} \right) = n_i \exp \left[ \frac{q(V_j + V_{Fn} - V_D)}{k_0T} \right]$$

(13)

$$p_s = n(x) \exp \left( - \frac{V_s}{k_0T} \right) = n_i \exp \left[ \frac{q(V_j + V_{FP} - V_{Ex})}{k_0T} \right]$$

(14)

where $V_s$ is the surface potential of diode.

According to eqn. 10, the surface recombination rate can be expressed by

$$U(x) = s \frac{n_s p_s - n_i^2}{n_s + p_s + 2n_i}$$

(15)

Substituting eqn. 13 and eqn.14 into eqn.15 is written by

$$U(x) \approx \frac{1}{2} n_i \exp \left( \frac{qV_j}{k_0T} \right) \cos \left( \frac{q}{k_0T} \left( V_j + V_{FP} + \frac{V_j}{2} - q \frac{V_j + V_{Ex}}{W} \right) \right)$$

(16)

Substituting eqn. 16 into eqn. 12, the diode nonradiative recombination current at the surface of the space charge region can be expressed by

$$I_{nr} = \frac{1}{2} r_{nr} qA v_{th} \gamma s N_{it} n_i \exp \left( \frac{qV_j}{k_0T} \left( \frac{k_0T}{q} \left( \frac{W}{V_j + V_{FP}} \right)^{-\gamma} \right) \right)$$

(17)

where $\gamma = \arctan \left( \exp \left( \frac{q}{k_0T} \left( V_j + V_{FP} + \frac{V_j}{2} - q \frac{V_j + V_{Ex}}{W} \right) \right) \right)$

For the laser diodes operating under low injection current, the low frequency noise of the device is caused by the fluctuation of surface nonradiative recombination current, which depends on the surface oxide traps density and lattice dislocation. At the same time, the surface defects, impurities, dislocations and other factors of LDs cause the surface non-radiative recombination current. It can be seen that 1/f noise has a similar mechanism to the surface recombination current. Therefore, we relate the 1/f noise of laser diode under low injection current to the fluctuation of the surface nonradiative recombination current. According to eqn.17, the fluctuation of the surface nonradiative recombination current is given by

$$\frac{\delta I_{nr}}{I_{nr}} = \frac{q}{k_0T} \frac{\delta N_{it}}{N_{it}}$$

(18)

According to eqn.18, the 1/f noise spectral density of the surface nonradiative recombination current can be written by

$$S_{I_{nr}}(f) = \frac{P_{nr}^2}{k_0T} \frac{1}{A N_{it}} \left( \frac{q}{k_0T} \right)^2 S_{N_{it}}$$

(19)
Where $S_{N_{not}}$ is the oxide-trapped charge spectral density, which can be given by

$$S_{N_{not}} = \frac{q^2 k_0 T N_t \lambda}{f}$$  \hspace{1cm} (20)

where $N_t$ is the number of oxide-trapped charge per unit per unit energy ($/eV/cm^3$), $\lambda$ is the attenuation tunneling distance.

The $1/f$ noise spectral density of the surface nonradiative recombination current can be expressed by

$$S_{I_{nr}}(f) = \frac{I^2_{nr}}{k_0 A T^2 N_{not}^2} q^4 N_t \lambda$$  \hspace{1cm} (21)

IV. RESULTS AND DISCUSSION

Fig. 3 show the surface nonradiative recombination current power spectral density $S_{I_{nr}}(f)$ of a typical LD sample at five different low injection currents, which is 20.50mA, 255mA, 385mA and 530mA, respectively. It shows that the $1/f$ noise PSD of the sample increases with the injection current. In Fig. 4, shows the magnitude ($B$) of $1/f$ noise PSD versus the injection current below threshold.

Equation (22) shows that the oxide-trapped charges lead to a decrease in the surface nonradiative recombination current noise.

The surface recombination velocity increases with the induced interface states, which can bring about an increase in surface nonradiative recombination current. $1/f$ noise of InGaAs/GaAs laser diode is due to an increase in surface recombination velocity and it is moderated by oxide charges. The cavity quality of the laser diode can be evaluated by measuring the low frequency noise of the device under low injection current.

V. CONCLUSIONS

In this paper, it is thought that the fluctuation of the surface nonradiative recombination current of 980 nm InGaAs/GaAs laser diode under low injection current causes the $1/f$ noise. A noise equivalent circuit model with parasitic parameters under low injection current is developed which identifies the parameters and describes the source and composition of noise. Based on the theory of carrier number fluctuation and the model of surface nonradiative recombination current, $1/f$ noise model is developed. The model can explain well the experimental result.

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